Annals of Pure and Applied Mathematics
Vol. 9, No. 1, 2015, 23-33
ISSN: 2279-087X (P), 2279-0888(online)
Published on 1 January 2015
www.researchmathsci.org

Total Domination in Fuzzy Graphs Using Strong Arcs

***O.T.Manjusha*[1] and *M.S.Sunitha*[2]**

[1]Department of Mathematics, Kerala Govt. Polytechnic College, Westhill, Calicut Kozhikode-673005, India. E-mail: manjushaot@gmail.com
[2]Department of Mathematics, National Institute of Technology Calicut, Kozhikode - 673 601, India. E-mail: sunitha@nitc.ac.in

Received 5 November 2014; accepted 17 November 2014

Abstract. In this paper we introduce the concept of strong total domination in fuzzy graphs. We determine the strong total domination number γ_{st} for several classes of fuzzy graphs. A lower bound and an upper bound for the strong total domination number in terms of strong domination number is obtained. Strong total domination in fuzzy trees is studied. A necessary and sufficient condition for the set of fuzzy cut nodes to be a strong total dominating set is discussed. Also it is established that in a non trivial fuzzy tree each node of a strong total dominating set is incident on a fuzzy bridge .

***Keywords*:** fuzzy graph, strong arcs, strong total domination,fuzzy trees

AMS Mathematics Subject Classification (2010): 05C69

1. Introduction

Fuzzy graphs were introduced by Rosenfeld [14]. Rosenfeld has described the fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness and established some of their properties [14]. Bhutani and Rosenfeld have introduced the concept of strong arcs [7]. Different parameters like sum distance in fuzzy graphs and chromatic number of fuzzy graphs were discussed in [5,13]. The work on fuzzy graphs was also done by Akram, Samanta, Nayeem, Pramanik, Rashmanlou and Pal [1, 2, 3, 4, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27]. It was during 1850s, a study of dominating sets in graphs started purely as a problem in the game of chess. Chess enthusiasts in Europe considered the problem of determining the minimum number of queens that can be placed on a chess board so that all the squares are either attacked by a queen or occupied by a queen. The concept of domination in graphs was introduced by Ore and Berge in 1962 ,and further studied by Cockayne and Hedetniemi [10]. Somasundaram and Somasundaram introduced total domination in fuzzy graphs using effective edges [29]. It is further studied by Depnath[18]. In this paper, we define the total domination in fuzzy graphs using strong arcs.

This paper is organized as follows. Section 2 contains preliminaries and in section 3, the strong total domination of a fuzzy graph is defined in a classic way(Definitions 3.2,3.4). It is shown that the strong total domination number of complete fuzzy graph and complete bipartite fuzzy graph is two times the minimum arc weight of the fuzzy graph (Propositions 3.7,3.8). A necessary and sufficient condition for the strong total domination number of a fuzzy graph G to be the order of G (Theorem 3.10) is established. An upper bound for the sum of the strong total domination number of a fuzzy graph and its complement both having no isolated nodes is obtained (Theorem 3.18). A lower bound and an upper bound for the strong total domination number of fuzzy graphs in terms of strong domination number are given (Theorem 3.19). Finally, in section 4 strong total domination in fuzzy trees is studied. A necessary and sufficient condition for the set of fuzzy cut nodes to be a strong total dominating set (Theorem 4.6) is established. It is shown that in a non trivial fuzzy tree each node of a strong total dominating set is incident on a fuzzy bridge (Theorem 4.7).

2. Preliminaries

It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'Fuzzy Graph Model'. We summarize briefly some basic definitions in fuzzy graphs which are presented in [6,7,11,14,15,16,28,29,30].

A fuzzy graph is denoted by $G:(V,\sigma,\mu)$ where V is a vertex set, σ is a fuzzy subset of V and μ is a fuzzy relation on σ. i.e., $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$ for all $x,y \in V$. We call σ, the fuzzy node set of G and μ, the fuzzy arc set of G, respectively. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e.,$\mu(x,x) = \sigma(x)$, for all x) and symmetric (i.e., $\mu(x,y) = \mu(y,x)$, for all (x,y)). In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph by $G^*:(\sigma^*,\mu^*)$ where $\sigma^* = \{u \in V: \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in V \times V: \mu(u,v) > 0\}$. Throughout we assume that $\sigma^* = V$. The fuzzy graph $H:(\tau,\nu)$ is said to be a partial fuzzy subgraph of $G:(\sigma,\mu)$ if $\nu \subseteq \mu$ and $\tau \subseteq \sigma$. In particular we call $H:(\tau,\nu)$, a fuzzy subgraph of $G:(\sigma,\mu)$ if $\tau(u) = \sigma(u)$ for all $u \in \tau^*$ and $\nu(u,v) = \mu(u,v)$ for all $(u,v) \in \nu^*$. A fuzzy graph $G:(V,\sigma,\mu)$ is called trivial if $|\sigma^*| = 1$. A node u is said to be isolated if $\mu(u,v) = 0$ for all $v \neq u$.

A path P of length n is a sequence of distinct nodes $u_0, u_1, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1,2,\ldots,n$ and the degree of membership of a weakest arc is defined as its strength. If $u_0 = u_n$ and $n \geq 3$ then P is called a cycle and P is called a fuzzy cycle, if it contains more than one weakest arc. The strength of a cycle is the strength of the weakest arc in it. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x,y)$.

A fuzzy graph $G:(V,\sigma,\mu)$ is connected if for every x,y in $\sigma^*, CONN_G(x,y) > 0$. An arc of a fuzzy graph is called strong if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted . Depending on $CONN_G(x,y)$ of an arc (x,y) in a fuzzy graph G, Sunil Mathew and M.S.Sunitha [30] defined three different types of arcs.

Note that $CONN_{G-(x,y)}(x,y)$ is the the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the arc (x,y). An arc (x,y) in G is $\alpha-$ strong if $\mu(x,y) > CONN_{G-(x,y)}(x,y)$. An arc (x,y) in G is $\beta-$ strong if $\mu(x,y) = CONN_{G-(x,y)}(x,y)$. An arc (x,y) in G is $\delta-$ arc if $\mu(x,y) < CONN_{G-(x,y)}(x,y)$. Thus an arc $(x,y)$is a strong arc if it is either $\alpha-$ strong or $\beta-$ strong. A path P is called strong path if P contains only strong arcs.

Two nodes u and v in a fuzzy graph G are said to be adjacent if $\mu(u,v)>0$ and u and v are called neighbors. The set of all neighbors of u is denoted by $N(u)$. An arc (u,v) of a fuzzy graph is called an effective arc (M-strong arc) if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$. Then u and v are called effective neighbors. The set of all effective neighbors of u is called effective neighborhood of u and is denoted by $EN(u)$.

Also v is called strong neighbor of u if arc (u,v) is strong. The set of all strong neighbors of u is called the open strong neighborhood of u and is denoted by $N_s(u)$. The closed strong neighborhood $N_s[u]$ is defined as $N_s[u]=N_s(u)\cup\{u\}$.

A fuzzy graph G is said to be complete if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$, for all $u,v\in\sigma^*$. The order p and size q of a fuzzy graph $G\colon(\mathrm{V},\sigma,\mu)$ are defined to be $p=\sum_{x\in V}\sigma(x)$ and $q=\sum_{(x,y)\in V\times V}\mu(x,y)$. Let $G\colon(V,\sigma,\mu)$ be a fuzzy graph and $S\subseteq V$. Then the scalar cardinality of S is defined to be $\sum_{v\in S}\sigma(v)$ and it is denoted by $|S|_s$. Let p denotes the scalar cardinality of V, also called the order of G. The complement of a fuzzy graph G, denoted by $\overline{G}$ is defined to be $\overline{G}=(V,\sigma,\overline{\mu})$ where $\overline{\mu}(x,y)=\sigma(x)\wedge\sigma(y)-\mu(x,y)$ for all $x,y\in V$ [32]. A fuzzy graph G is said to be bipartite [29] if the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2\in V_1$ or $v_1,v_2\in V_2$. Further if $\mu(u,v)=\sigma(u)\wedge\sigma(v))$ for all $u\in V_1$ and $v\in V_2$ then G is called a complete bipartite graph and is denoted by K_{σ_1,σ_2}, where σ_1 and σ_2 are respectively the restrictions of σ to V_1 and V_2.

3. Strong total domination in fuzzy graphs

A precise notion of a dominating set, that is present in the current literature can be said to be given by Berge (1962) and Ore (1962). Since then a number of graph theorists, Allan and Laskar (1978) Allan et al., (1984); Cockayne and Hedetniemi (1977); Haynes and Slater (1998); Kulli and Sigarkant (1992) etc., have studied various domination parameters of graphs. For the terminology of total domination in crisp graphs we refer to [9].

In a graph a vertex u dominates a vertex v if either $u=v$ or v is a neighbor of u. But if we could restrict domination so that a vertex u is only permitted to dominate a vertex v if v is a neighbor of u. In this context, a vertex would not dominate itself. We refer to this type of domination as open or total domination. If $w\in N(v)$ then we say here that v openly dominates w. That is, a vertex v openly dominates the vertices in its open neighborhood $N(v)$.

A set S of vertices in a graph G is an open dominating set of G if every vertex of G is adjacent to atleast one vertex of S. Therefore, a graph G contains an open dominating set if and only if G can contain no isolated vertices and the subgraph $<S>$ induced by S contains no isolated vertices. The minimum cardinality of an open dominating set is the open domination number $\gamma_o(G)$ of G.

Somasundaram and Somasundaram [29] introduced the concept of total domination using effective arcs. These concepts motivated researchers to reformulate some of the concepts in total domination more effectively. This paper is our main motivation and we introduce total domination in fuzzy graphs using strong arcs. Here we restrict the domination so that a node u is only permitted to dominate only its strong neighbors and not to itself. That is a node u dominates openly in nodes of $N_s(u)$. This definition is required due to the fact that the parameter 'total domination number' defined by Somasundaram and Somasundaram is based on node weights instead of arc weights. Using the new definition of total domination number we reduce the value of old total domination number and extract classic results in a fuzzy graph.

According to Nagoorgani [15] a node v in a fuzzy graph G is said to strongly dominate itself and each of its strong neighbors, that is v strongly dominates the nodes in $N_s[v]$. A set D of nodes of G is a strong dominating set of G if every node of $V(G)-D$ is a strong neighbor of some node in D.

Definition 3.1. *[12] The weight of a strong dominating set D is defined as $W(D)=\sum_{u\in D}\mu(u,v)$,where $\mu(u,v)$ is the minimum of the weight of the strong arcs incident on u. The strong domination number of a fuzzy graph G is defined as the minimum weight of strong dominating sets of G and it is denoted by $\gamma_s(G)$ or simply γ_s. A minimum strong dominating set in a fuzzy graph G is a strong dominating set of minimum weight. Let $\gamma_s(\overline{G})$ or $\overline{\gamma_s}$ denote the strong domination number of the complement of a fuzzy graph G.*

Now we define total domination in fuzzy graphs using strong arcs as follows.

Definition 3.2. *A set D of nodes in a fuzzy graph $G:(V,\sigma,\mu)$ is a strong total(open) dominating set of G if every node of G is a strong neighbor of atleast one node of D.*

Remark 3.3. *Note that a fuzzy graph $G:(V,\sigma,\mu)$ contains a strong total dominating set if and only if G contains no isolated nodes and further the induced fuzzy subgraph $< D >$ contains no isolated nodes.*

Definition 3.4. *The weight of a strong total dominating set D is defined as $W(D)=\sum_{u\in D}\mu(u,v)$,where $\mu(u,v)$ is the minimum of the weight of the strong arcs incident on u. The strong total domination number of a fuzzy graph G is defined as the minimum weight of strong total dominating sets of G and it is denoted by $\gamma_{st}(G)$ or simply γ_{st}. A minimum strong total dominating set in a fuzzy graph G is a strong total dominating set of minimum weight. Let $\gamma_{st}(\overline{G})$ or $\overline{\gamma_{st}}$ denote the strong total domination number of the complement of a fuzzy graph G.*

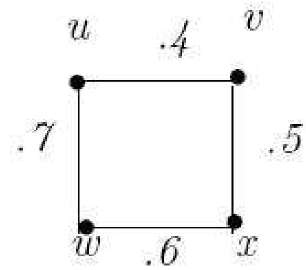

Figure 1: Illustration of strong total domination

Example 3.5. *In the fuzzy graph in Figure 1, $(u,w),(w,x),(x,v)$ are strong arcs and (u,v) is a δ arc. Hence $D=\{w,x\}$ is a minimum strong total dominating set and $W(D)=\sum_{u\in D}\mu(u,v)$ where $\mu(u,v)$ is the minimum weight of strong arcs incident on u. Hence $W(D)=\mu(w,x)+\mu(x,v)=.6+.5=1.1$. Therefore $\gamma_{st}=1.1$.*

Remark 3.6. *In a non trivial fuzzy graph $G:(V,\sigma,\mu)$ $\gamma_s\le\gamma_{st}$ always, since every strong* *total dominating set is a strong dominating set of* G.

Proposition 3.7. *If* $G:(V,\sigma,\mu)$ *is a complete fuzzy graph, then* $\gamma_{st}(G) = 2\mu(u,v)$ *where* $\mu(u,v)$ *is the weight of a weakest arc in* G.

Proof: Since G is a complete fuzzy graph, all arcs are strong [31] and each node is adjacent to all other nodes. Then the end nodes say $\{u,v\}$ of any weakest arc (u,v) in G form a strong total dominating set. Hence $\gamma_{st}(G) = \mu(u,v) + \mu(u,v) = 2\mu(u,v)$. Hence the proposition.

Proposition 3.8. *For a complete bipartite fuzzy graph* K_{σ_1,σ_2}, $\gamma_{st}(K_{\sigma_1,\sigma_2}) = 2\mu(u,v)$ *where* $\mu(u,v)$ *is the weight of a weakest arc in* K_{σ_1,σ_2}.

Proof: In K_{σ_1,σ_2}, all arcs are strong. Also each node in V_1 is adjacent with all nodes in V_2. Hence in K_{σ_1,σ_2}, the minimum strong total dominating set is any set containing 2 nodes, one in V_1 and other in V_2. Then the end nodes say $\{u,v\}$ of any weakest arc (u,v) in K_{σ_1,σ_2} form a strong total dominating set.

Hence $\gamma_{st}(K_{\sigma_1,\sigma_2}) = \mu(u,v) + \mu(u,v) = 2\mu(u,v)$. Hence the proposition.

Remark 3.9. *Note that* $min_{v\in V}\sigma(v) < \gamma_{st} \leq p$, *since every strong total dominating set contains atleast 2 nodes and in a connected fuzzy graph with distinct node weights,* $\gamma_{st} < p$, *always.*

Theorem 3.10. *In a non trivial fuzzy graph* $G:(V,\sigma,\mu)$ *of order* p, $\gamma_{st} = p$ *if and only if the following conditions hold.*

1. All nodes have same weight.
2. All arcs are M-strong arcs.
3. Each node of G has a unique strong neighbor.

Proof: If all nodes have same weight, all arcs are M-strong arcs and each node of G has a unique strong neighbor then obviously $\gamma_{st} = p$ since V is the only strong total dominating set.

Conversely suppose that $\gamma_{st} = p$. If any one of the conditions 1,2,3 violated then $\gamma_{st} < p$, a contradiction from the definition of γ_{st}.

Theorem 3.11. *In a non trivial fuzzy graph* $G:(V,\sigma,\mu)$, *if* $\gamma_{st} = p$ *then the number of nodes in* G *is even.*

Proof: Suppose $\gamma_{st} = p$ then by theorem 3.10 each node of G has a unique strong neighbor, all nodes have same weight and all arcs are M-strong arcs. If G contains an odd number of nodes say $2n+1$, then G contains atleast one node having two strong neighbors. But this is a contradiction since no node of G has two strong neighbors. Hence the theorem.

Definition 3.12. *A strong total dominating set* D *of a fuzzy graph* G *is said to be a minimal strong total dominating set if no proper subset of* D *is a strong total dominating set of* G.

Proposition 3.13. *Every minimum strong total dominating set of a fuzzy graph is a minimal strong total dominating set.*

Remark 3.14. *The converse of proposition 3.13 need not be true. For the fuzzy graph* G *given in Figure 2, the set* $\{u, w\}$ *is a minimal strong total dominating set with weight 0.9; but not a minimum strong total dominating set as* $\gamma_{st} = 0.8$.

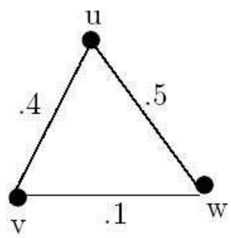

Figure 2: Illustration of minimal strong total dominating set

Remark 3.15. *In the case of strong domination if* D *is a minimal strong dominating set then* $V \backslash D$ *is a strong dominating set* [15]. *But this need not be true in the case of strong total domination as we see in the following example.*

Example 3.16. *Consider the fuzzy graph in Figure 3.*

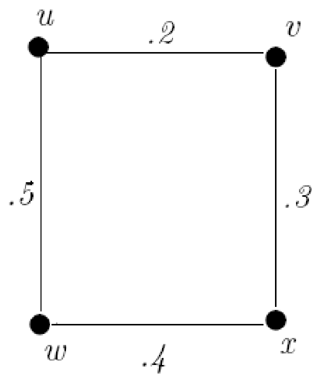

Figure 3: Illustration of Remark

In Figure 3, $D = \{u, w, x\}$ is a minimal strong total dominating set. But $V \backslash D = \{v\}$ is not a strong total dominating set.

Remark 3.17. *Let* γ *and* γ_t *be the domination number and total domination number of a fuzzy graph respectively defined by A Somasundaram and S.Somasundaram in [29]. They have proved that* $\gamma_t + \overline{\gamma_t} \leq 2p$ *and equality holds if and only if*

1. The number of vertices in G is even say $2n$
2. There is a set S_1 of n mutually disjoint effective edges in G.
3. There is a set S_2 of n mutually disjoint effective edges in $\overline{G}$ and
4. For any edge $xy \notin S_1 \cup S_2,\ 0 < \mu(x, y) < \sigma(x) \wedge \sigma(y)$

We observe that equality does not hold with total domination using strong arcs.

Theorem 3.18. *For any fuzzy graph* $G: (V, \sigma, \mu)$ *without isolated nodes* $\gamma_{st} + \overline{\gamma_{st}} < 2p$.
Proof: Since $\gamma_{st} \leq p$, $\overline{\gamma_{st}} \leq p$.

We have $\gamma_{st} + \overline{\gamma_{st}} \leq 2p$
Claim: $\gamma_{st} + \overline{\gamma_{st}} \neq 2p$
Suppose if possible $\gamma_{st} + \overline{\gamma_{st}} = 2p$. Then $\gamma_{st} = p$, $\overline{\gamma_{st}} = p$

Now $\gamma_{st} = p$ implies the number of nodes of G is even say $2n$, all nodes have same weight, every node of G has a unique strong neighbor and all arcs are M-strong [Theorem 3.10, 3.11].

If $|\sigma^*| = 2$ then $\gamma_{st} = p$ implies G is complete and all the nodes in $\overline{G}$ are isolated and this contradicts that $\overline{\gamma_{st}} = p$.

If $|\sigma^*| > 2$ then $|\sigma^*| = 2n$ for some $n > 1$. Since $\gamma_{st} = p$, each node in G has a unique strong neighbor and all arcs are M-strong. In this case $G^*$is disconnected and is a forest contains trees each of which is a K_2 , because otherwise some node will have more than one strong neighbor. Hence $\overline{G}$ is connected and $\overline{\gamma_{st}} < p$ [Remark 3.9], which is a contradiction. Similarly we get a contradiction if we start with $\overline{\gamma_{st}} = p$. Hence our claim is true.

Therefore $\gamma_{st} + \overline{\gamma_{st}} < 2p$.

Next we present the fuzzy analogue of a famous result regarding domination number and total domination number of a graph due to [9].

Theorem 3.19. *In any fuzzy graph* $G: (V, \sigma, \mu)$ *without isolated nodes*
$$\gamma_s \leq \gamma_{st} \leq 2\gamma_s.$$

Proof: Since every strong total dominating set is a strong dominating set we have $\gamma_s \leq \gamma_{st}$.
Next to prove $\gamma_{st} \leq 2\gamma_s$.

Let $D = \{v_1, v_2, \ldots, v_k\}$ be a minimum strong dominating set of G. The nodes of $V \backslash D$ are therefore openly strong dominated by the nodes of D. Since G contains no isolated nodes each open strong neighborhood $N_s(v_i)$ is non-empty. Now let $u_i \in N_s(v_i)$ $(1 \leq i \leq k)$ such that $\mu(u_i, v_i)$ is minimum among all strong arcs incident with v_i and let $D^1 = \{u_1, u_2, \ldots, u_k\}$. Thus the nodes of D are dominated by the nodes of D^1. Therefore $D \cup D^1$ is a strong open(total) dominating set of G. Hence $\gamma_{st} \leq W(D \cup D^1) \leq 2W(D) = 2\gamma_s$.

Therefore $\gamma_s \leq \gamma_{st} \leq 2\gamma_s$.

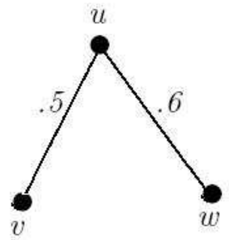

Figure 4:

Remark 3.20. *The two bounds given in Theorem 3.19 are sharp . For example, consider the fuzzy graph* G *in Figure 4.*

In this fuzzy graph $\gamma_s = 0.5,\ \gamma_{st} = 0.5 + 0.5 = 1,\ 2\gamma_s = 2 \times 0.5 = 1$.

Therefore $\gamma_{st} = 2\gamma_s$.

Next consider the fuzzy graph G in Figure 5.

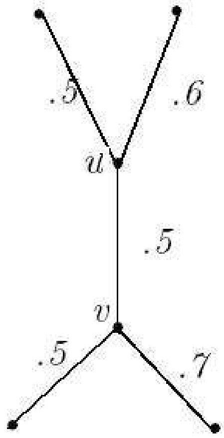

Figure 5: Illustration of theorem 3.20

In this fuzzy graph $D = \{u, v\}$ is both minimum strong dominating set and minimum strong total dominating set. Therefore

$\gamma_s = 0.5 + 0.5 = 1 = \gamma_{st}$. Also $2\gamma_s = 2$.

4. Strong total domination in fuzzy trees

A fuzzy subgraph $H: (\tau, \nu)$ spans the fuzzy graph $G: (V, \sigma, \mu)$ if $\tau = \sigma$ [14]. A connected fuzzy graph $G = (V, \sigma, \mu)$ is called a fuzzy tree(f-tree) if it has a fuzzy spanning subgraph $F: (\sigma, \nu)$, which is a tree (in crisp sense), where for all arcs (x, y) not in F there exists a path from x to y in F whose strength is more than $\mu(x, y)$ [14]. Note that here F is a tree which contains all nodes of G and hence is a spanning tree of G. Also note that F is the unique maximum spanning tree (MST) of G [14], where a maximum spanning tree of a connected fuzzy graph $G: (V, \sigma, \mu)$ is a fuzzy spanning subgraph $T: (\sigma, \nu)$, such that T^* is a tree in crisp sense, and for which $\sum_{u \neq v} \nu(u, v)$ is maximum [14].

Definition 4.1. [8,14] *An arc is called a fuzzy bridge(f-bridge) of a fuzzy graph $G: (V, \sigma, \mu)$ if its removal reduces the strength of connectedness between some pair of nodes in G. Similarly a fuzzy cut node(f-cut node) w is a node in G whose removal from G reduces the strength of connectedness between some pair of nodes other than w. A node z is called a fuzzy end node (f-end node) if it has exactly one strong neighbor in G.*

Remark 4.2. [7,8,14,30] *A non trivial fuzzy tree G contains at least two fuzzy end nodes and every node in G is either a fuzzy cut node or a fuzzy end node. In an f-tree G an arc is strong if and only if it is an arc of F where F is the associated unique maximum spanning tree of G. Note that these strong arcs are α-strong and there are no β-strong arcs in an f-tree. Also note that in an f-tree G an arc (x, y) is α-strong if and only if (x, y) is an f-bridge of G.*

Theorem 4.3. *[28] The strong arc incident with a fuzzy end node is a fuzzy bridge in any non trivial fuzzy graph* $G:(V,\sigma,\mu)$.

Corollary 4.4. *[28]In a non trivial fuzzy tree* $G:(V,\sigma,\mu)$ *except* K_2, *the strong neighbor of a fuzzy end node is a fuzzy cut node of* G.

Remark 4.5. *In a non trivial fuzzy tree* $G:(V,\sigma,\mu)$ *except* K_2, *the set of all fuzzy end nodes is never be a strong total dominating set since no two fuzzy end nodes are dominated by themselves by the corollary 4.4.*

Theorem 4.6. *In a non trivial fuzzy tree* $G:(V,\sigma,\mu)$ *except* K_2, *the set of all fuzzy cut nodes is a strong total dominating set if and only if every fuzzy cut node has at least one fuzzy cut node as strong neighbor.*

Proof: First assume that every fuzzy cut node has at least one fuzzy cut node as a strong neighbor in G. Let D be the set of all fuzzy cut nodes of G. Since D is a strong dominating set [11] every node in $V\setminus D$ is openly strong dominated by some node in D. By our assumption every fuzzy cut node has atleast one fuzzy cut node as strong neighbor. Hence every node in D is strongly dominated by some node in D. Hence D is a strong total dominating set of G.

Conversely assume that the set D of all fuzzy cut nodes is a strong total dominating set of G. Then every node of G is strongly dominated by atleast one node of D. To prove that every fuzzy cut node has atleast one fuzzy cut node as strong neighbor. Suppose on the contrary that there exists some fuzzy cut node u whose all strong neighbors are fuzzy end nodes. Since D is a strong total dominating set of G there exists some node v in D which dominates u. That is v is a fuzzy cut node. Hence the strong neighbor of u is a fuzzy cut node, a contradiction. Since u is arbitrary, every fuzzy cut node has at least one fuzzy cut node as a strong neighbor in G.

Theorem 4.7. *In a non trivial fuzzy tree* $G:(V,\sigma,\mu)$, *each node of a strong total dominating set is incident on a fuzzy bridge of* G.

Proof: Let D be a strong total dominating set of G. Let $u \in D$. Since D is a strong total dominating set, there exists $v \in D$ such that (u,v) is a strong arc. Then (u,v) is an arc of the unique MST F of G [7,14]. Hence (u,v) is an f-bridge of G [14]. Since u is arbitrary, this is true for every node of the strong total dominating set of G. This completes the proof.

5. Conclusion

The concept of domination in graph is very rich both in theoretical developments and applications. More than thirty domination parameters have been investigated by different authors, and in this paper we have introduced the concept of strong total domination in fuzzy graphs. We have proved the fuzzy analogue of a famous result regarding the domination number and total domination number of a graph. Also we have studied the strong total domination in fuzzy trees. Work on other strong domination parameters will be reported in forthcoming papers.

REFERENCES

1. Sk. Md. Abu Nayeem and M.Pal, Diameter constrained fuzzy minimum spanning tree problem, *International Journal of Computational Intelligence Systems*, 6(6) (2013) 1040-1051.
2. M. Akram, Bipolar fuzzy graphs, *Information Sciences*, 181 (2011) 5548-5564.
3. M. Akram, Interval-valued fuzzy line graphs, *Neural Computing and Applications,* 21(2012) 145-150.
4. M.Akram, Sheng-Gang Li and K.P.Shum, Antipodal bipolar fuzzy graphs, *Italian Journal of Pure and Applied Mathematics*, 31 (2013) 425-438.
5. Anjaly Kishore and M.S.Sunitha, Strong chromatic number of fuzzy graphs, *Annals of Pure and Applied Mathematics*, 7(2) (2014) 52-60.
6. K.R.Bhutani, On automorphisms of fuzzy graphs, *Pattern Recognition Letters*, 9 (1989) 159-162.
7. K.R.Bhutani and A.Rosenfeld, Strong arcs in fuzzy graphs, *Information Sciences*, 152 (2003) 319-322.
8. K.R.Bhutani and A. Rosenfeld, Fuzzy end nodes in fuzzy graphs, *Information Sciences,* 152 (2003), 323-326.
9. Gray Chartrand, Ping Zang, Introduction to Graph Theory, Tata McGraw-Hill Edition, 2006.
10. T.W.Haynes, S.T.Hedetniemi and P.J.Slater, Fundamentals of domination in graphs, Marcel Dekker, Inc., New York, 1998.
11. O.T.Manjusha and M.S.Sunitha, Notes on domination in fuzzy graphs, *Journal of Intelligent and Fuzzy Systems*, Article in Press. DOI:10.3233/IFS-141277.
12. O.T.Manjusha and M.S.Sunitha, Strong domination in fuzzy graphs, *Fuzzy Information and Engineering*, Communicated.
13. Mini Tom and M.S.Sunitha, Sum distance in fuzzy graphs, *Annals of Pure and Applied Mathematics*, 7(2) (2014) 73-89.
14. J.N.Mordeson and P.S.Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica – Verlag, 2000.
15. A.Nagoorgani and V.T.Chandrasekaran, Domination in Fuzzy graph, *Adv.in Fuzzy Sets and Systems,* 1(1) (2006) 17-26.
16. A.Nagoorgani and P.Vijayalakshmi, insensitive arc in domination of fuzzy graph, *Int.J.Contemp.Math.Sciences,* 6 (26) (2011) 1303-1309.
17. M.Pal and H.Rashmanlou, Irregular interval- valued fuzzy graphs, *Annals of Pure and Applied Mathematics,* 3(1) (2013) 56-66.
18. P.Debnath, Some results of domination and total domination in the direct product of two fuzzy graphs, arXiv:1312.3444v1 [cs.DM] (2013).
19. T.Pramanik, S.Samanta and M.Pal, Interval-valued fuzzy planar graphs, *Int. J. Mach. Learn. and Cyber.,* DOI 10.1007/s13042-014-0284-7
20. H.Rashmanlou and M.Pal, Antipodal interval-valued fuzzy graphs, *International journal of Applications of Fuzzy Sets and Artificial Intelligence*, 3 (2013) 107-130.
21. H.Rashmanlou and M.Pal, Isometry on interval-valued fuzzy graphs, *International Journal of Fuzzy Mathematical Archive*, 3 (2013) 28-35.
22. H.Rashmanlou, S.Samanta, M.Pal and R.A.Borzooei, A study on bipolar fuzzy graphs,

Journal of Intelligent and Fuzzy Systems, DOI: 10.3233/IFS-141333.
23. S.Samanta and M.Pal, Fuzzy tolerance graphs, *International Journal of Latest Trends in Mathematics*, 1(2) (2011) 57-67.
24. S.Samanta and M.Pal, Bipolar fuzzy hypergraphs, *International Journal of Fuzzy Logic Systems*, 2(1) (2012) 17-28.
25. S.Samanta and M.Pal, Fuzzy k-competition graphs and p-competition fuzzy graphs, *Fuzzy Inf. Eng*., 5(2) (2013) 191-204.
26. S.Samanta and M.Pal, Some more results on bipolar fuzzy sets and bipolar fuzzy intersection graphs, *The Journal of Fuzzy Mathematics*, 22(2) (2014) 253-262.
27. S.Samanta and M.Pal, Irregular bipolar fuzzy graphs, *International Journal of Applications of Fuzzy Sets*, 2 (2012) 91-102.
28. S.Kalathodi and M.S.Sunitha, Distance in Fuzzy Graphs, LAP LAMBERT Academic Publishing, 2012.
29. A.Somasundaram and S.Somasundaram, Domination in fuzzy graphs-I, *Pattern Recognition Letters*, 19 (1998) 787-791.
30. Sunil Mathew and M.S.Sunitha, Types of arcs in a fuzzy graph, *Information Sciences*, 179 (2009) 1760-1768.
31. Sunil Mathew and M.S.Sunitha, Node connectivity and arc connectivity of a fuzzy graph, *Information Sciences*, 180 (2010) 519-531.
32. M.S.Sunitha and A.Vijayakumar, Complement of a fuzzy graph, *Indian Journal of Pure and Applied Mathematics*, 33(9) (2002) 1451-1464.