The Middle Edge Dominating Graph of Prime Cycles

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Abstract. The middle edge dominating graph \(M_{ed}(G)\) of a graph \(G=(V,E)\) is a graph with the vertex set \(E \cup S\) where \(S\) is the set of all minimal edge dominating set \(G\) and with two vertices \(u, v \in E \cup S\) adjacent if \(u \in E\) and \(V=F\) is a minimal edge dominating set of \(G\) containing \(u\) or \(u, v\) are not disjoint minimal edge dominating sets in \(S\). In this paper we discuss about the middle edge dominating graph of Prime cycles.

Keywords: Graph, Cycle, Edge dominating graph, Middle edge dominating graph

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1. Introduction

All graphs considered here are finite, undirected without loops, isolated vertices or multiple edges. Any undefined term in this paper may be found in Kulli [4]. Let \(G=(V,E)\) be a graph with \(|V|\geq 2\) and \(|E|=q\). A set \(D \subseteq V\) is a dominating set if every vertex in \(V-D\) is adjacent to some vertex in \(D\). The domination number \(\gamma(G)\) is the minimum cardinality of a dominating set of \(G\). A set \(F \subseteq E\) of edges in \(E-F\) is adjacent to at least one edge in \(F\). An edge dominating set \(F\) of \(G\) is a minimal edge dominating set if for every \(e\) in \(F\), \(F-e\) is not an edge dominating set of \(G\). The edge domination number \(\gamma'(G)\) of \(G\) is the minimum cardinality of an edge dominating set of \(G\).

Let \(A\) be a finite set. Let \(F=\{A_1, A_2, \ldots, A_n\}\) be a partition of \(A\). Then the intersection graph \(\Omega(G)\) of \(F\) is the graph whose vertices are the subsets in \(F\) and in which two vertices \(A_i\) and \(A_j\) are adjacent if and only if \(A_i \cap A_j = \emptyset\).

The minimal edge dominating graph \(MD_e(G)\) of a graph \(G\) is the intersection graph defined on the family of all minimal edge dominating sets of \(G\). This concept was introduced in [2]. Some other dominating graphs are studied, for example, in [1] [3] [5] [6].

The edge dominating graph \(D_e(G)\) of a graph \(G\) is the graph with the vertex set \(E \cup S\) where \(S\) is the set of all minimal edge dominating sets of \(G\) and with two vertices \(u, v\) in \(E \cup S\) adjacent if \(u\in E\) and \(v\in F\) is a minimal edge dominating set of \(G\) containing \(u\). This concept was introduced by Kulli [5].

The middle edge dominating graph \(M_{ed}(G)\) of a graph \(G=(V,E)\) is the graph with the vertex set \(E \cup S\) where \(S\) is the set of all minimal edge dominating sets of \(G\) and with
two vertices \( u, v \in E \cup S \) adjacent if \( u \in E \) and \( v \in F \) is a minimal edge dominating set of \( G \) containing \( u \) or \( u, v \) are not disjoint minimal edge dominating sets in \( G \).

In Figure 1, a graph \( G \) and its middle edge dominating graph \( \text{med}(G) \) are shown.

We note that the middle edge dominating graph \( \text{med}(G) \) is defined only if \( G \) has not isolated vertices.

The degree of an edge \( uv \) is defined to be \( \text{deg}u + \text{deg}v - 2 \). An edge called an isolated edge if \( \text{deg}uv = 0 \). Let \( \Delta_1(G) \) denote the maximum degree among the edges of \( G \).

In this paper, we discuss about the middle edge dominating graph of prime cycles.

**2. Results of Middle Edge Dominating Graph**

**Theorem 2.1.** Let \( G \) be a graph without isolated vertices and with at least two edges. The edge dominating graph \( D_e(G) \) is connected if and only if \( \Delta_1(G) < q - 1 \).

**Remark 1.** For any graph \( G \) without isolated vertices, \( D_e(G) \) is a subgraph of \( \text{med}(G) \).

**Remark 2.** For any graph \( G \) without isolated vertices, \( D_e(G) \) and \( MD_e(G) \) are edge disjoint subgraphs of \( \text{med}(G) \).

**Theorem 2.2.** \( \text{med}(G) = k_{1,p} \) if and only if \( G = pk_2, p \geq 1 \).

**Proof:** Suppose \( \text{med}(G) = k_{1,p} \). Assume \( G \neq pk_2 \). Then there exist at least two minimal edge dominating sets. Thus \( |V(\text{med}(G))| \geq p + 2 \) which is a contradiction. Thus \( G = pk_2 \).

Conversely, suppose \( G = pk_2 \). Then there exists exactly one minimal edge dominating set containing all the edges of \( G \). From the definition of \( \text{med}(G) \), the result follows.
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Theorem 2.3. The middle edge dominating graph $M_{ed}(G)$ of $G$ is complete bipartite if and only if $G = pk_2$, $p \geq 1$.

Theorem 2.4. $M_{ed}(G) = pk_2$ if and only if $G = k_{1,p}$, $p \geq 1$.

Proof: Suppose $M_{ed}(G) = pk_2$, $p \geq 1$. Assume $G \neq k_{1,p}$ or $k_3$. Then there exists at least one minimal edge dominating set $S$ containing two or more edges of $G$. By definition, $S$ will form a subgraph $P_n$ in $M_{ed}(G)$, which is a contradiction.

Conversely suppose $G = k_{1,p}$ or $k_3$. Then each edge $e_i$ of $G$ forms a minimal edge dominating set $\{e_i\}$. Thus $e$ and $\{e_i\}$ are adjacent vertices in $M_{ed}(G)$. Since each minimal edge dominating set $\{e_i\}$ contains only one edge no two vertices of $G$ are adjacent in $M_{ed}(G)$ and no two corresponding vertices of minimal edge dominating sets are adjacent in $M_{ed}(G)$. Thus $M_{ed}(G) = pk_2$ or $k_3$.

3. Main results

In this section we study about the middle edge dominating graph of prime cycles and one edge union of prime cycles is shown below.

Definition 3.1. A one edge union $C_n^k$ of $K$ copies of cycles is the graph obtained by taking $e$ as a common edge such that any two cycles $C_n^1$ and $C_n^j$ are edge disjoint and do not have any vertex in common except $v_i$ and $v_j$.

Theorem 3.1. $M_{ed}(G) = pK_{1,(n+1)}$ for $n=1,2,3,...$ if and only if $G = C_p$ for all $p=5,7,11,...$.

Proof: Suppose $M_{ed}(G) = pK_{1,(n+1)}$ for $n=1,2,3,...$. Assume $G \neq C_p$, $p=5,7,11,...$. Then there exists at least one minimal edge dominating set $S$ containing two or more edges of $G$. By definition, $S$ will form a subgraph $P_n$ in $M_{ed}(G)$, which is a contradiction.

Conversely suppose $G = C_p$, $p=5,7,11,...$. Then each edge $e_i$ of $G$ forms a minimal edge dominating set $\{e_i\}$. Thus $e$ and $\{e_i\}$ are adjacent vertices in $M_{ed}(G)$. Since each minimal edge dominating set $\{e_i\}$ contains only one edge no two vertices of $G$ are
adjacent in $M_{ed}(G)$ and no two corresponding vertices of minimal edge dominating sets are adjacent in $M_{ed}(G)$. Thus $M_{ed}(G)=pK_{1,(n+1)}$.

**Example**

![Diagram](image)

**Theorem 3.2.** $M_{ed}(G)=K_{1,(n+1)}$ for $n=1,2,3,...$ if and only if $G=C_p$ where $p=5,7,11,...$ with one edge is common.

**Proof:** Suppose $M_{ed}(G)=K_{1,(n+1)}$ for $n=1,2,3,...$ Assume $G\neq C_p$; $p=5,7,11,...$ with one edge is common. Then there exists at least one minimal edge dominating set $S$ containing two or more edges of $G$. By definition, $S$ will form a subgraph $P_n$ in $M_{ed}(G)$, which is a contradiction.

Conversely suppose $G= C_p$; $p=5,7,11,...$ with one edge is common and take that edge as one of the dominating edge and remaining edge $e_i$ of $G$ forms a minimal edge dominating set $\{e_i\}$. Thus $e$ and $\{e_i\}$ are adjacent vertices in $M_{ed}(G)$. Since each minimal edge dominating set $\{e_i\}$ contains only one edge no two vertices of $G$ are adjacent in $M_{ed}(G)$, and no two corresponding vertices of minimal edge dominating sets are adjacent in $M_{ed}(G)$. Thus $M_{ed}(G)=K_{1,(n+1)}$ for $n=1,2,3,...$

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