Modal Operator $F_{\alpha,\beta}$ in Intuitionistic Fuzzy Groups

P.K. Sharma

Department of Mathematics, D.A.V. College
Jalandhar-144001, Punjab, India
E-mail: pksharma@davjalandhar.com

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Abstract. In this paper, we study modal operator $F_{\alpha,\beta}$ in intuitionistic fuzzy subgroup of a group and derive some results.

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1. Introduction

The idea of intuitionistic fuzzy sets (IFSs) was given by [1, 2] to generalize the notion of fuzzy sets (FSs). Intuitionistic fuzzy modal operator was defined by Atanassov in [3]. The modal operators have been known to be important tools for IFSs where the operators are defined on the contrary to the FSs. Intuitionistic fuzzy operators and some properties of these operators were examined by several authors [4,5,6]. Recently modal operators in intuitionistic fuzzy matrices has been studied in [7]. Here in this paper, we study the impact of modal operator $F_{\alpha,\beta}$ on intuitionistic fuzzy groups.

2. Preliminaries

Here we recall some definitions and results which will be used later.

Definition 2.1. [3] Let X be a fixed non-empty set. An intuitionistic fuzzy set (IFS) A of X is an object of the following form $A = \{ < x , \mu_A(x) , \nu_A(x) > : x \in X \}$, where $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ respectively and for any $x \in X$, we have $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Remark 2.2. (i) When $\mu_A(x) + \nu_A(x) = 1$, i.e. when $\nu_A(x) = 1 - \mu_A(x) = \mu_A^c(x)$. Then A is called fuzzy set.
(ii) For convenience, we write the IFS $A = \{ < x , \mu_A(x) , \nu_A(x) > : x \in X \}$ by $A = (\mu_A , \nu_A)$.
(iii) The set of all IFS’s of X is denoted by IFS(X).

Definition 2.3. [3] Let A = $(\mu_A , \nu_A)$ and B = $(\mu_B , \nu_B)$ be any two IFS’s of X, then
(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$. 

Definition 2.4. [3] For any IFS $A=\{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ of $X$, if
$\pi_A(x)=1-\mu_A(x)-\nu_A(x)$, for all $x \in X$.
Then $\pi_A(x)$ is called the degree of indeterminacy of $x$ in $A$.

Definition 2.5. [3] For any IFS $A=\{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ of $X$ and $\alpha \in [0,1]$ the operators $\square\cdot IFS(X)\rightarrow IFS(X)$, $\Diamond\cdot IFS(X)\rightarrow IFS(X)$ and $D_\alpha:\cdot IFS(X)\rightarrow IFS(X)$ are defined as
(i) $\square A=\{< x, \mu_A(x), 1-\mu_A(x) > : x \in X\}$ is called Necessity Operator
(ii) $\Diamond A=\{< x, 1-\nu_A(x), \nu_A(x) > : x \in X\}$ is called Possibility Operator
(iii) $D_\alpha(A)=\{< x, \mu_A(x)+\alpha\pi_A(x), \nu_A(x)+(1-\alpha)\pi_A(x) > : x \in X\}$ is called $\alpha$-Modal operator.

Remark 2.6. (i) Clearly, $\square A \subseteq A \subseteq \Diamond A$ and the equality hold, when $A$ is a fuzzy set (ii) Notice that $D_0(A)=\square A$ and $D_1(A)=\Diamond A$. Thus $\alpha$-modal operator $D_\alpha(A)$ is an extension of necessary operator $\square A$ and possibility operator $\Diamond A$.

Definition 2.7. [3] For any IFS $A=\{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ of $X$ and for any $\alpha, \beta \in [0,1]$ such that $\alpha+\beta \leq 1$, the $(\alpha, \beta)$-modal operator $F_{\alpha,\beta}: IFS(X)\rightarrow IFS(X)$ is defined as $F_{\alpha,\beta}(A)=\{< x, \mu_{F_{\alpha,\beta}}(x), \nu_{F_{\alpha,\beta}}(x) > : x \in X\}$, where $\pi_{\alpha}(x)=1-\mu_A(x)-\nu_A(x)$ for all $x \in X$. Therefore, we can write $F_{\alpha,\beta}(A)$ as $F_{\alpha,\beta}(A)(x) = (\mu_{F_{\alpha,\beta}}(x), \nu_{F_{\alpha,\beta}}(x))$, where $\mu_{F_{\alpha,\beta}}(x) = \mu_{\alpha,\beta}(x) + \alpha \pi_{\alpha}(x)$ and $\nu_{F_{\alpha,\beta}}(x) = \nu_{\alpha,\beta}(x) + \beta \pi_{\alpha}(x)$

Remark 2.8. (i) Clearly, $F_{0,1}(A)=\square A$, $F_{1,0}(A)=\Diamond A$ and $F_{\alpha,1-\alpha}(A)=D_\alpha(A)$.

Definition 2.9. [8] Let $A$ be Intuitionistic fuzzy set of a universe set $X$. Then $(\alpha, \beta)$-cut of $A$ is a crisp subset $C_{\alpha,\beta}(A)$ of the IFS $A$ is given by $C_{\alpha,\beta}(A)=\{ x : x \in X \text{ such that } \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta \}$, where $\alpha, \beta \in [0,1]$ with $\alpha+\beta \leq 1$.

Definition 2.10. [3] Let $A=\{< x, \mu_A(x), \nu_A(x) > : x \in X\}$ be an IFS of a universe set $X$, then support of $A$ is denoted by $Supp_X(A)$ and is defined
$Supp_X(A) = \{ x \in X : \mu_A(x) > 0 \text{ and } \nu_A(x) < 1 \}$

Remark 2.11. Clearly,
$Supp_X(A) = \bigcup \{ C_{\alpha,\beta}(A) : \text{ for all } \alpha, \beta \in (0,1) \text{ such that } 0 < \alpha + \beta \leq 1 \}$
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Definition 2.12. [8] An IFS $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in G \}$ of a group $G$ is said to be intuitionistic fuzzy subgroup of $G$ (in short IFSG) of $G$ if
\[ \mu_A(xy^{-1}) \geq \min \{ \mu_A(x), \mu_A(y) \} \quad \text{and} \quad \nu_A(xy^{-1}) \leq \max \{ \nu_A(x), \nu_A(y) \} \quad \forall x, y \in G \]

Definition 2.13. [8] An IFSG $A = \{ < x, \mu_A(x), \nu_A(x) > : x \in G \}$ of a group $G$ said to be intuitionistic fuzzy normal subgroup of $G$ (in short IFNSG) if
\[ \mu_A(xy) = \mu_A(yx) \quad \text{and} \quad \nu_A(xy) = \nu_A(yx) \quad \forall x, y \in G \]

Or equivalently,
\[ \mu_A(xyx^{-1}) = \mu_A(y) \quad \text{and} \quad \nu_A(xyx^{-1}) = \nu_A(y) \quad \forall x, y \in G \]

Definition 2.14. [10, 11] An IFSG $A$ of a group $G$ is called an intuitionistic fuzzy abelian subgroup (IFASG) of $G$ if and only if $\text{Supp}_A$ is abelian subgroup of $G$.

Definition 2.15. [10, 11] An IFSG $A$ of a group $G$ is called an intuitionistic fuzzy cyclic subgroup (IFCSG) of $G$ if and only if $\text{Supp}_A$ is cyclic subgroup of $G$.

Proposition 2.16. [8] If $A$ be IFSG of a group $G$ and $e$ be the identity element of $G$, then
(i) $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$ for all $x \in G$
(ii) $\mu_A(e) \geq \mu_A(x)$ and $\nu_A(e) \leq \nu_A(x)$ for all $x \in G$
(iii) $\mu_A(xy^{-1}) = \mu_A(e)$ and $\nu_A(xy^{-1}) = \nu_A(e)$ then $\mu_A(x) = \mu_A(y)$ and $\nu_A(x) = \nu_A(y)$.

Definition 2.17. [9] Let $X$ and $Y$ be two non-empty sets and $f : X \rightarrow Y$ be a mapping. Let $A$ and $B$ be IFS's of $X$ and $Y$ respectively. Then the image of $A$ under the map $f$ is denoted by $f(A)$ and is defined as
\[ f(A)(y) = \left( \mu_{f(A)}(y), \nu_{f(A)}(y) \right), \]
\[ \mu_{f(A)}(y) = \begin{cases} \vee \{ \mu_A(x), x \in f^{-1}(y) \} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \]
\[ \nu_{f(A)}(y) = \begin{cases} \wedge \{ \nu_A(x), x \in f^{-1}(y) \} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{otherwise} \end{cases} \]

Also the pre-image of $B$ under $f$ is denoted by $f^{-1}(B)$ and is defined as
\[ f^{-1}(B)(x) = \left( \mu_{f^{-1}(B)}(x), \nu_{f^{-1}(B)}(x) \right) = \left( \mu_B(f(x)), \nu_B(f(x)) \right) \quad \forall x \in X \]

Remark (2.18) Note that $\mu_A(x) \leq \mu_{f(A)}(f(x))$ and $\nu_A(x) \geq \nu_{f(A)}(f(x)) \quad \forall x \in X$, however equality hold when the map $f$ is bijective.

3. Modal operator $F_{\alpha,\beta}$ in intuitionistic fuzzy groups

In this section, we study the relationship between the intuitionistic fuzzy subgroups and the modal operator on these intuitionistic fuzzy subgroups. We also establish a relationship between the support of IFS $A$ and the support of IFS under modal operator.

Theorem 3.1. If $A$ is IFSG of a group $G$, then $F_{\alpha,\beta}(A)$ is also IFSG of $G$.

Proof. Let $x, y$ be any element of $G$, then
\[ F_{\alpha,\beta}(A)(xy^{-1}) = \left( \mu_{F_{\alpha,\beta}}(xy^{-1}), \nu_{F_{\alpha,\beta}}(xy^{-1}) \right), \]
where $\mu_{F_{\alpha,\beta}}(xy^{-1}) = \mu_A(xy^{-1}) + \alpha \pi_A(xy^{-1})$ and $\nu_{F_{\alpha,\beta}}(xy^{-1}) = \nu_A(xy^{-1}) + \beta \pi_A(xy^{-1})$.
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Now, $\mu_{x,y}(xy^{-1}) = \mu_x(xy^{-1}) + \alpha \pi_x(xy^{-1}) = \mu_x(xy^{-1}) + \alpha[1 - \mu_x(xy^{-1}) - v_A(xy^{-1})]$

$$= \alpha + (1-\alpha)\mu_x(xy^{-1}) - \alpha v_A(xy^{-1})$$

$$\geq \alpha + (1-\alpha)\min[\mu_x(x),\mu_A(y)] - \alpha \max[v_A(x), v_A(y)]$$

$$= \alpha[1 - \max[v_A(x), v_A(y)] ] + (1-\alpha)\min[\mu_x(x),\mu_A(y)]$$

$$= \alpha[\min[1-v_A(x), 1-v_A(y))] + (1-\alpha)\min[\mu_x(x),\mu_A(y)]$$

$$= \alpha. \min[1-v_A(x), 1-v_A(y)] + (1-\alpha)\min[\mu_x(x),\mu_A(y)]$$

$$= \min\{\alpha(1-v_A(x)) + (1-\alpha)\mu_x(x), \alpha(1-v_A(y)) + (1-\alpha)\mu_A(y)\}$$

$$= \min\{\mu_x(x) + \alpha(1-\mu_x(x)-v_A(x)), \mu_A(y) + \alpha(1-\mu_A(y)-v_A(y))\}$$

$$= \min\{\mu_x(x) + \alpha \pi_x(x), \mu_A(y) + \alpha \pi_A(y)\}$$

$$= \min\{\mu_{x,y}(x), \mu_{x,y}(y)\}$$

Thus, $\mu_{x,y}(xy^{-1}) \geq \min\{\mu_{x,y}(x), \mu_{x,y}(y)\}$

Similarly, we can show that $v_{x,y}(xy^{-1}) \leq \max\{v_{x,y}(x), v_{x,y}(y)\}$

Hence $F_{x,y}(A)$ is IFSG of G.

Remark 3.2. The converse of the above theorem need not be true.

Example 3.3. Let G be the Klein 4-group $\{e, a, b, ab\}$, where $a^2 = b^2 = e$ and $ab = ba$.

Define $A = \{<e, 0.1, 0.1>, <a, 0.21, 0.39>, <b, 0.3, 0.4>, <ab, 0.2, 0.1>\}$ be IFS in G. It can be easily verified that A is not IFSG of G, for

$$\mu_a(ab) = 0.2 \leq \min\{0.21, 0.3\} = \min\{\mu_a(a), \mu_a(b)\}$$

Now take $\alpha = 0.2$, then it can be checked that

$$F_{x,y}(A) = \{<e, 0.26, 0.26>, <a, 0.29, 0.47>, <b, 0.36, 0.46>, <ab, 0.34, 0.24>\}$$

Clearly, we have

$$\mu_{F_{x,y}}(ab) = 0.34 \leq \min\{0.29, 0.36\} = \min\{\mu_{F_{x,y}}(a), \mu_{F_{x,y}}(b)\}$$

and

$$v_{F_{x,y}}(ab) = 0.24 \leq \max\{0.47, 0.46\} = \max\{v_{F_{x,y}}(a), v_{F_{x,y}}(b)\}.$$

Therefore, $F_{0.2,0.2}(A)$ is IFSG of G.

Theorem 3.4. If A is IFNSG of a group G, then $F_{x,y}(A)$ is also IFNSG of G.

Proof. Let $x, y$ be any element of G, then
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$\mu_{F_{\alpha, \beta}}(xy) = \mu_A(xy) + \alpha \pi_A(xy) = \mu_A(xy) + \alpha [1 - \mu_A(xy) - V_A(xy)] = \mu_A(xy) + \alpha [1 - \mu_A(xy) - V_A(xy)]$

$= \mu_A(xy) + \alpha \pi_A(xy) = \mu_{F_{\alpha, \beta}}(xy)$

Similarly, we can show that $V_{F_{\alpha, \beta}}(xy) = V_{F_{\alpha, \beta}}(yx)$.

Hence $F_{\alpha, \beta}(A)$ is IFNSG of $G$.

**Result 3.5.** If $A$ be IFSG of a group $G$ and $e$ be the identity element of $G$, then

(i) $\mu_{F_{\alpha, \beta}}(x^{-1}) = \mu_{F_{\alpha, \beta}}(x)$ and $V_{F_{\alpha, \beta}}(x^{-1}) = V_{F_{\alpha, \beta}}(x)$

(ii) $\mu_{F_{\alpha, \beta}}(e) \geq \mu_{F_{\alpha, \beta}}(x)$ and $V_{F_{\alpha, \beta}}(e) \leq V_{F_{\alpha, \beta}}(x)$, for all $x \in G$

Proof. (i) Now, $\mu_{F_{\alpha, \beta}}(x^{-1}) = \mu_A(x^{-1}) + \alpha \pi_A(x^{-1}) = \mu_A(x^{-1}) + \alpha [1 - \mu_A(x^{-1}) - V_A(x^{-1})]$

$= \mu_A(x) + \alpha [1 - \mu_A(x) - V_A(x)] = \mu_A(x) + \alpha \pi_A(x)$

$= \mu_{F_{\alpha, \beta}}(x)$

Similarly, we can show that $V_{F_{\alpha, \beta}}(x^{-1}) = V_{F_{\alpha, \beta}}(x)$

(ii) Now, $\mu_{F_{\alpha, \beta}}(e) = \mu_{F_{\alpha, \beta}}(xx^{-1}) \geq \min \{\mu_{F_{\alpha, \beta}}(x), \mu_{F_{\alpha, \beta}}(x^{-1})\} = \mu_{F_{\alpha, \beta}}(x)$ [using (i)]

Similarly, $V_{F_{\alpha, \beta}}(e) = V_{F_{\alpha, \beta}}(xx^{-1}) \leq \max \{V_{F_{\alpha, \beta}}(x), V_{F_{\alpha, \beta}}(x^{-1})\} = V_{F_{\alpha, \beta}}(x)$ [using (i)]

**Proposition 3.6.** Let $f : X \rightarrow Y$ be a mapping and $A \in IFS(X)$ and $B \in IFS(Y)$. Then

(i) $f^{-1}(F_{\alpha, \beta}(B)) = F_{\alpha, \beta}(f^{-1}(B))$ (ii) $f(F_{\alpha, \beta}(A)) \subseteq F_{\alpha, \beta}(f(A))$

Proof. (i) Now, $f^{-1}(F_{\alpha, \beta}(B))(x) = \left[\mu_{F_{\alpha, \beta}}(f^{-1}(B))(x), V_{F_{\alpha, \beta}}(f^{-1}(B))(x)\right] = \left[\mu_{F_{\alpha, \beta}}(f(x)), V_{F_{\alpha, \beta}}(f(x))\right]$.

But $\mu_{F_{\alpha, \beta}}(f(x)) = \mu_B(f(x)) + \alpha \pi_B(f(x)) = \mu_B(f(x)) + \alpha [1 - \mu_B(f(x)) - V_B(f(x))]$

$= \mu_{F_{\alpha, \beta}}(x) + \alpha [1 - \mu_{F_{\alpha, \beta}}(x) - V_{F_{\alpha, \beta}}(x)] = \mu_{F_{\alpha, \beta}}(x) + \alpha \pi_{F_{\alpha, \beta}}(x)$

Similarly, we can show that $V_{F_{\alpha, \beta}}(f(x)) = V_{F_{\alpha, \beta}}(f^{-1}(B))(x)$

Thus $f^{-1}(F_{\alpha, \beta}(B))(x) = \left[\mu_{F_{\alpha, \beta}}(f^{-1}(B))(x), V_{F_{\alpha, \beta}}(f^{-1}(B))(x)\right] = F_{\alpha, \beta}(f^{-1}(B))(x)$, for every $x \in X$

(ii) Now, $f(F_{\alpha, \beta}(A))(y) = \left[\mu_{F_{\alpha, \beta}}(f(A))(y), V_{F_{\alpha, \beta}}(f(A))(y)\right]$.

But $\mu_{F_{\alpha, \beta}}(f(A))(y) = \sup \{\mu_{F_{\alpha, \beta}}(A)(x) : f(x) = y\}$

$= \sup \{\mu_A(x) + \alpha \pi_A(x) : f(x) = y\}$

$= \sup \{\mu_A(x) + \alpha [1 - \mu_A(x) - V_A(x)] : f(x) = y\}$

$= \sup \{\alpha + (1-\alpha)\mu_A(x) - \alpha V_A(x) : f(x) = y\}$
Thus $\mu_{f(A)}(y) \leq \mu_{f(A)}(y)$, for all $y \in Y$

Similarly, we can show that $v_{f(A)}(y) \geq v_{f(A)}(y)$, for all $y \in Y$

Thus $f(A) = f(A)$, for all $y \in Y$

Hence proved.

**Corollary 3.7.** Let $f : X \to Y$ be a bijective mapping. Then $f(A) = f(A)$

**Proposition 3.8.** [10] Let $f : X \to Y$ be a mapping and $A$, $B$ are IFS of $X$ and $Y$ respectively. Then the following results hold

(i) $f(\text{Supp}_X(A)) \subseteq \text{Supp}_Y(f(A))$ and equality hold when the map $f$ is bijective

(ii) $f^{-1}(\text{Supp}_Y(B)) = \text{Supp}_X(f^{-1}(B))$

**Proof.** (i) Let $y \in f(\text{Supp}_X(A))$ be any element. Therefore, $\exists x \in \text{Supp}_X(A)$ such that $f(x) = y$. As $x \in \text{Supp}_X(A) \Rightarrow \mu_A(x) > 0$ and $v_A(x) < 1$.

But $\mu_A(x) \leq \mu_{f(A)}(x)$ and $v_A(x) \geq v_{f(A)}(x)$; for all $x \in X$

$\therefore \mu_{f(A)}(x) > 0$ and $v_{f(A)}(x) < 1 \Rightarrow y = f(x) \in \text{Supp}_Y(f(A))$

Hence $f(\text{Supp}_X(A)) \subseteq \text{Supp}_Y(f(A))$. The second part follow by Remark (2.18)

(ii) Let $x \in f^{-1}(\text{Supp}_Y(B))$ be any element $\iff x \in \text{Supp}_X(B)$

$\iff \mu_B(f(x)) > 0$ and $v_B(f(x)) < 1 \iff \mu_{f^{-1}(B)}(x) > 0$ and $v_{f^{-1}(B)}(x) < 1 \iff x \in \text{Supp}_X(f^{-1}(B))$

Hence $f^{-1}(\text{Supp}_Y(B)) = \text{Supp}_X(f^{-1}(B))$.

**Lemma 3.9.** If $A$ be any IFS of the universe set $X$, then $\text{Supp}_X(F_{A_{\alpha,\beta}}(A)) = \text{Supp}_X(A)$

**Proof.** Since $\text{Supp}_X(F_{A_{\alpha,\beta}}(A)) = \{ x \in X : \mu_{F_{A_{\alpha,\beta}}}(x) > 0 \text{ and } v_{F_{A_{\alpha,\beta}}}(x) < 1 \}$

But $\mu_{F_{A_{\alpha,\beta}}}(x) = \mu_A(x) + \alpha \pi_A(x) = \mu_A(x) + \alpha [1 - \mu_A(x) - v_A(x)]$

$= \alpha + (1 - \alpha) \mu_A(x) - \alpha v_A(x)$

and $v_{F_{A_{\alpha,\beta}}}(x) = v_A(x) + \beta \pi_A(x) = v_A(x) + \beta [1 - \mu_A(x) - v_A(x)]$

$= \beta - \beta \mu_A(x) + (1 - \beta) v_A(x)$

Now, $v_A(x) < 1 \iff - \alpha v_A(x) > - \alpha$. Also, $\mu_A(x) > 0 \iff (1 - \alpha) \mu_A(x) > 0$

Thus, $\alpha + (1 - \alpha) \mu_A(x) - \alpha v_A(x) > \alpha - \alpha = 0$

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i.e. $\mu_{F_{\alpha,\beta}}(x) > 0$. Similarly, we can show that $\beta - \beta \nu_{A}(x) + (1 - \beta)\nu_{A}(x) < 1$ i.e. $\nu_{F_{\alpha,\beta}}(x) < 1$. Therefore, $\text{Supp}_{x}(F_{\alpha,\beta}(A)) = \text{Supp}_{x}(A)$.

By using the Lemma (3.9) and the definition (2.14) and (2.15), we have the following theorem.

**Theorem 3.10.** If $A$ is IFSG of a group $G$, then $A$ is IFASG of $G$ if and only if $F_{\alpha,\beta}(A)$ is IFASG of $G$.

**Theorem 3.11.** If $A$ is IFSG of a group $G$, then $A$ is IFCSG of $G$ if and only if $F_{\alpha,\beta}(A)$ is IFCSG of $G$.

4. Homomorphisms of modal operator in intuitionistic fuzzy groups

**Theorem 4.1.** Let $f : G_{1} \rightarrow G_{2}$ be a group homomorphism and $F_{\alpha,\beta}(B)$ is IFSG of $G_{2}$. Then $f^{-1}(F_{\alpha,\beta}(B))$ is IFSG of $G_{1}$.

**Proof.** Let $F_{\alpha,\beta}(B)$ is IFSG of $G_{2}$. By Proposition (3.6)(i), it is enough to show that $F_{\alpha,\beta}(f^{-1}(B))$ is IFSG of $G_{1}$.

Let $x, y \in G_{1}$ be any element, then

$\mu_{F_{\alpha,\beta}(f^{-1}(B))}(xy^{-1}) = \mu_{F_{\alpha,\beta}(B)}(f(xy^{-1}))$

$= \mu_{F_{\alpha,\beta}(B)}(f(x)(f(y))^{-1})$

$\geq \text{Min}\{\mu_{F_{\alpha,\beta}(B)}(f(x)), \mu_{F_{\alpha,\beta}(B)}(f(y))\}$

$= \text{Min}\{\mu_{F_{\alpha,\beta}(f^{-1}(B))}(x), \mu_{F_{\alpha,\beta}(f^{-1}(B))}(y)\}$

Thus, $\mu_{F_{\alpha,\beta}(f^{-1}(B))}(xy^{-1}) \geq \text{Min}\{\mu_{F_{\alpha,\beta}(f^{-1}(B))}(x), \mu_{F_{\alpha,\beta}(f^{-1}(B))}(y)\}$

Similarly, we can show that $\nu_{F_{\alpha,\beta}(f^{-1}(B))}(xy^{-1}) \leq \text{Max}\{\nu_{F_{\alpha,\beta}(f^{-1}(B))}(x), \nu_{F_{\alpha,\beta}(f^{-1}(B))}(y)\}$

Thus $F_{\alpha,\beta}(f^{-1}(B))$ and hence $f^{-1}(F_{\alpha,\beta}(B))$ is IFSG of $G_{1}$.

**Theorem 4.2.** Let $f : G_{1} \rightarrow G_{2}$ be a group homomorphism and $F_{\alpha,\beta}(B)$ is IFNSG of $G_{2}$. Then $f^{-1}(F_{\alpha,\beta}(B))$ is IFNSG of $G_{1}$.

**Proof.** Let $F_{\alpha,\beta}(B)$ is IFNSG of $G_{2}$. By Proposition (3.6)(i), it is enough to show that $F_{\alpha,\beta}(f^{-1}(B))$ is IFNSG of $G_{1}$.

Let $x, y \in G_{1}$ be any element, then

$\mu_{F_{\alpha,\beta}(f^{-1}(B))}(xy) = \mu_{F_{\alpha,\beta}(B)}(f(xy)) = \mu_{F_{\alpha,\beta}(B)}(f(x)f(y)) = \mu_{F_{\alpha,\beta}(B)}(f(y)f(x)) = \mu_{F_{\alpha,\beta}(B)}(f(y)) = \mu_{F_{\alpha,\beta}(f^{-1}(B))}(xy)$ and
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\[
V_{\mu_{\alpha}\beta}(xy) = V_{\mu_{\alpha}\beta}(f(xy)) = V_{\mu_{\alpha}\beta}(f(x)f(y)) = V_{\mu_{\alpha}\beta}\left(f(\alpha)f(\beta)\right) = V_{\mu_{\alpha}\beta}(f(\alpha)(xy))
\]

Thus \( F_{\alpha}\beta(f^{-1}(B)) \) and hence \( f^{-1}(F_{\alpha}\beta(B)) \) is IFSG of \( G_1 \).

**Theorem 4.3.** Let \( f : G_1 \rightarrow G_2 \) be a group isomorphism and \( F_{\alpha}\beta(A) \) is IFSG of \( G_1 \). Then \( f(F_{\alpha}\beta(A)) \) is IFSG of \( G_2 \).

**Proof.** Let \( x_2, y_2 \in G_2 \) be any elements. As \( f \) is bijective, so let there be unique \( x_1, y_1 \in G_1 \) such that \( f(x_1) = x_2 \) and \( f(y_1) = y_2 \).

\[
f(F_{\alpha}\beta(A))(x_2y_2^{-1}) = \left[H_{f(F_{\alpha}\beta(A)}(x_2y_2^{-1})}, V_{f(F_{\alpha}\beta(A)}(x_2y_2^{-1})\right]
\]

But \( \mu_{f(F_{\alpha}\beta(A)}(x_2y_2^{-1}) = \mu_{f(F_{\alpha}\beta(A)}(t) \), where \( f(t) = x_1y_2^{-1} = f(x_1)f(y_1^{-1}) = f(x_1^{-1}) \) implies \( t = x_1y_1^{-1} \)

\[
\begin{align*}
&= \mu_{\alpha}(x_1)y_1^{-1} + \alpha \pi_{\alpha}(x_1) \\
&= \mu_{\alpha}(x_1,y_1^{-1}) + \alpha[1-\mu_{\alpha}(x_1,y_1^{-1})-V_{\alpha}(x_1,y_1^{-1})] \\
&= \alpha + (1-\alpha) \mu_{\alpha}(x_1,y_1^{-1}) - \alpha \pi_{\alpha}(x_1,y_1^{-1}) \\
&\geq \alpha + (1-\alpha) \text{Min}\{\mu_{\alpha}(x_1), \mu_{\alpha}(y_1)\} - \alpha \text{Max}\{\pi_{\alpha}(x_1), V_{\alpha}(y_1)\}
\end{align*}
\]

\[
= \alpha + (1-\alpha) \text{Max}\{\mu_{\alpha}(x_1), \mu_{\alpha}(y_1)\} - (1-\alpha) \text{Min}\{\mu_{\alpha}(x_1), \mu_{\alpha}(y_1)\}
\]

\[
= \text{Max}\{\mu_{\alpha}(x_1), \mu_{\alpha}(y_1)\} + (1-\alpha) \text{Min}\{\mu_{\alpha}(x_1), \mu_{\alpha}(y_1)\} - (1-\alpha) \text{Max}\{\mu_{\alpha}(x_1), \mu_{\alpha}(y_1)\}
\]

\[
\mu_{f(F_{\alpha}\beta(A)}(x_2) \leq \text{Min}\{\mu_{f(F_{\alpha}\beta(A)}(x_2), \mu_{f(F_{\alpha}\beta(A)}(y_2)\}
\]

**Theorem 4.4.** Let \( f : G_1 \rightarrow G_2 \) be a group isomorphism and \( F_{\alpha}\beta(A) \) is IFNSG of \( G_1 \). Then \( f(F_{\alpha}\beta(A)) \) is IFNSG of \( G_2 \).

**Proof.** Let \( x_2, y_2 \in G_2 \) be any elements. As \( f \) is bijective, so let there be unique \( x_1, y_1 \in G_1 \) such that \( f(x_1) = x_2 \) and \( f(y_1) = y_2 \). Now, \( f(F_{\alpha}\beta(A))(x_2y_2^{-1}) = \left[\mu_{f(F_{\alpha}\beta(A)}(x_2y_2^{-1}), V_{f(F_{\alpha}\beta(A)}(x_2y_2^{-1})\right] \]

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Modal Operator $F_{\alpha,\beta}$ in Intuitionistic Fuzzy Groups

But $\mu_{f(F_{\alpha,\beta}(A))}(x_2y_2) = \mu_{F_{\alpha,\beta}(A)}(t)$, where $f(t) = x_2y_2 = f(x_1)f(y_1) = f(x_1y_1)$ implies $t = x_1y_1$

$$= \mu_A(x_2y_2) + \alpha x_2 + \beta y_2$$

$$= \mu_A(x_2y_2) + \alpha [1 - \mu_A(x_1y_1) - V_A(x_1y_1) - V_A(x_1y_2)]$$

$$= \alpha + (1 - \alpha)\mu_A(x_1y_1) - \alpha V_A(x_1y_1)$$

$$= \alpha + (1 - \alpha)\mu_A(y_1x_2) - \alpha V_A(y_1x_2)$$

$$= \alpha + (1 - \alpha)\mu_A(f(y_1)x_2) - \alpha V_A(f(y_1)x_2)$$

$$= \alpha + (1 - \alpha)\mu_A(f(y_1)f(x_2) - \alpha V_A(f(y_1)f(x_2))$$

$$= \alpha + (1 - \alpha)\mu_A(f(y_1)f(x_2) - \alpha V_A(f(y_1)f(x_2))$$

$$= \mu_A(y_2x_2) + \alpha V_A(y_2x_2)$$

$$= \mu_{f(F_{\alpha,\beta}(A))}(y_2x_2)$$

Similarly, we can show that $\nu_{f(F_{\alpha,\beta}(A))}(x_2y_2) = \nu_{f(F_{\alpha,\beta}(A))}(y_2x_2)$

Hence $f(F_{\alpha,\beta}(A))$ is IFNSG of $G_2$.

**Theorem 4.5.** Let $f : G_1 \rightarrow G_2$ be a group homomorphism and $F_{\alpha,\beta}(B)$ is IFASG of $G_2$. Then $f^{-1}(F_{\alpha,\beta}(B))$ is IFASG of $G_1$.

**Proof.** Let $F_{\alpha,\beta}(B)$ is IFASG of $G_2 \Rightarrow \text{Supp}_{G_2}(F_{\alpha,\beta}(B))$ is abelian subgroup of $G_2$ [by definition (2.14)]

$\Rightarrow \text{Supp}_{G_1}(B)$ is abelian subgroup of $G_2$ [by Lemma (3.9)]

$\Rightarrow f^{-1}(\text{Supp}_{G_1}(B))$ is abelian subgroup of $G_1$

$\Rightarrow \text{Supp}_{G_1}(f^{-1}(B))$ is abelian subgroup of $G_1$ [by Proposition (3.8)(ii)]

$\Rightarrow \text{Supp}_{G_1}(F_{\alpha,\beta}(f^{-1}(B)))$ is abelian subgroup of $G_1$ [by Lemma (3.9)]

$\Rightarrow \text{Supp}_{G_1}(f^{-1}(F_{\alpha,\beta}(B)))$ is abelian subgroup of $G_1$ [by Proposition (3.6)]

$\Rightarrow f^{-1}(F_{\alpha,\beta}(B))$ is IFASG of $G_1$ [by definition (2.14)]

**Theorem 4.6.** Let $f : G_1 \rightarrow G_2$ be a group homomorphism and $F_{\alpha,\beta}(B)$ is IFCSG of $G_2$. Then $f^{-1}(F_{\alpha,\beta}(B))$ is IFCSG of $G_1$.

**Proof.** Follow similar to the proof of Theorem (4.5)

**Theorem 4.7.** Let $f : G_1 \rightarrow G_2$ be a group isomorphism and $F_{\alpha,\beta}(A)$ is IFSAG of $G_1$. Then $f(F_{\alpha,\beta}(A))$ is IFASG of $G_2$.

**Proof.** Since $F_{\alpha,\beta}(A)$ is IFASG of $G_1 \Rightarrow \text{Supp}_{G_1}(F_{\alpha,\beta}(A))$ is abelian subgroup of $G_1$ [by definition (2.14)]
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⇒ f(Supp_{G_1}(F_{α, β}(A))) is abelian subgroup of G_2
⇒ Supp_{G_1}(f(F_{α, β}(A))) is abelian subgroup of G_2 [by proposition (3.8)(i)]
⇒ f(F_{α, β}(A)) is IFASG of G_2.

Theorem 4.8. Let f : G_1 → G_2 be a group isomorphism and F_{α, β}(A) is IFCSG of G_1. Then f(F_{α, β}(A)) is IFCSG of G_2.

Proof. Since F_{α, β}(A) is IFCAG of G_1 ⇒ Supp_{G_1}(F_{α, β}(A)) is cyclic subgroup of G_1 [by definition (2.15)]
⇒ f(Supp_{G_1}(F_{α, β}(A))) is cyclic subgroup of G_2
⇒ Supp_{G_2}(f(F_{α, β}(A))) is cyclic subgroup of G_2 [by proposition (3.8)(i)]
⇒ f(F_{α, β}(A)) is IFCSG of G_2.

5. Conclusion
In this paper, we have studied the impact of modal operator F_{α, β} on intuitionistic fuzzy groups and proved that many properties of intuitionistic fuzzy subgroups like normality, commutatively (abelian-ness) and cyclic groups remain invariant under the modal operator. We have also obtained the impact of these operator under homomorphism. A similar type of impact of modal operator can be realized on other algebraic structure like intuitionistic fuzzy subring, intuitionistic fuzzy submodules etc. and this work is under progress and will be published shortly.

REFERENCES