Atanassov’s Intuitionistic Fuzzy Generalized Bi-ideals of $\Gamma$-Semigroups

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Abstract. In this paper we introduce the concept of Atanassov’s intuitionistic fuzzy generalized bi-ideals of $\Gamma$-semigroups in order to extend the concept of Atanassov’s intuitionistic fuzzy bi-ideal of a $\Gamma$-semigroup. Here we characterize regular $\Gamma$-semigroups in terms of Atanassov’s intuitionistic fuzzy generalized bi-ideals.

Keywords: $\Gamma$-semigroup, Regular $\Gamma$-semigroup, Atanassov’s intuitionistic fuzzy ideal, fuzzy ideal, fuzzy bi-ideal, fuzzy generalized bi-ideal.

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1. Introduction

Atanassov’s intuitionistic fuzzy sets[1,2] are intuitively straightforward extension of Zadeh’s[12] fuzzy sets; while a fuzzy set gives the degree of membership of an element in a given set, an Atanassov’s intuitionistic fuzzy set gives both a degree of membership and a degree of non-membership. Kuroki[3, 4, 5, 6] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by Kuroki[3, 4]. In [4], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. The notion of a $\Gamma$-semigroup was introduced by Sen and Saha[10] as a generalization of semigroups and ternary semigroups. S.K. Majumder and M. Mandal[7] studied fuzzy generalized bi-ideals in $\Gamma$-semigroups. We have initiated the study of $\Gamma$-semigroups in terms of Atanassov’s intuitionistic fuzzy subsets[8, 9]. The purpose of this paper is as mentioned in the abstract.

2. Preliminaries

Definition 2.1. [1] Let $X$ be a nonempty set. A mapping $A = (\mu_A, \nu_A) : X \rightarrow I \times I$ is called an intuitionistic fuzzy set in $X$ if $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$, where the mappings $\mu_A : X \rightarrow I$ and $\nu_A : X \rightarrow I$ denote respectively the degree of membership and the degree of non-membership of each $x \in X$ to $A$, $I$ is the unit interval $[0,1]$.
In this paper we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy subset $A = \{<x, \mu_A(x), \nu_A(x)> : x \in X\}$ of $X$.

**Definition 2.2.** [10] Let $S = \{x, y, z, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ be two non-empty sets. Then $S$ is called a $\Gamma$-semigroup if there exists a mapping $S \times \Gamma \times S \rightarrow S$ (images to be denoted by $a\alpha b$) satisfying

1. $x\gamma y \in S \quad \forall x, y \in S, \quad \gamma \in \Gamma$,
2. $(x\beta y)\zeta = x\beta(y\zeta)$, $\forall x, y, z \in S, \quad \forall \beta, \gamma \in \Gamma$.

**Definition 2.3.** [8] A non-empty intuitionistic fuzzy subsemigroup $A = (\mu_A, \nu_A)$ of a $\Gamma$-semigroup $S$ is called an intuitionistic fuzzy bi-ideal of $S$ if it satisfies:

1. $\mu_A(x\alpha \beta \gamma z) \geq \min\{\mu_A(x), \mu_A(z)\} \quad \forall x, y, z \in S \quad \forall \alpha, \beta \in \Gamma$,
2. $\nu_A(x\alpha \beta \gamma z) \leq \max\{\nu_A(x), \nu_A(z)\} \quad \forall x, y, z \in S \quad \forall \alpha, \beta \in \Gamma$.

For further preliminaries we refer the readers to [8, 11].

### 3. Intuitionistic fuzzy generalized bi-ideal

**Definition 3.1.** [7] Let $S$ be a $\Gamma$-semigroup. A non-empty subset $I$ of $S$ is called a generalized bi-ideal of $S$ if $I \subseteq I$. $\forall$ $\Gamma$ $\subseteq I$.

**Definition 3.2.** A non-empty intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of a $\Gamma$-semigroup $S$ is called an intuitionistic fuzzy generalized bi-ideal of $S$ if it satisfies:

1. $\mu_A(x\alpha \beta \gamma z) \geq \min\{\mu_A(x), \mu_A(z)\} \quad \forall x, y, z \in S \quad \forall \alpha, \beta \in \Gamma$,
2. $\nu_A(x\alpha \beta \gamma z) \leq \max\{\nu_A(x), \nu_A(z)\} \quad \forall x, y, z \in S \quad \forall \alpha, \beta \in \Gamma$.

**Remark 1.** It is clear that every intuitionistic fuzzy bi-ideal of $S$ is an intuitionistic fuzzy generalized bi-ideal of $S$. But in general the converse does not hold which will be clear from the following example. For a restricted converse we refer to Proposition 3.1.

**Example 1.** Let $S = \{x, y, z, r\}$ and $\Gamma = \{\gamma\}$, where $\gamma$ is defined on $S$ with the following Cayley table:

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$r$</th>
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<td>$x$</td>
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<td>$z$</td>
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<td>$y$</td>
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</tr>
<tr>
<td>$r$</td>
<td>$x$</td>
<td>$x$</td>
<td>$y$</td>
<td>$y$</td>
</tr>
</tbody>
</table>

Then $S$ is a $\Gamma$-semigroup. We define an intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ of $S$ as $\mu_A(x) = 0.5$, $\mu_A(y) = 0$, $\mu_A(z) = 0.2$, $\mu_A(r) = 0$. and $\nu_A(x) = 0.4$, $\nu_A(y) = 1$, $\nu_A(z) = 0.3$, $\nu_A(r) = 0.2$. $\forall x, y, z, r \in S$. $\forall \Gamma$. $\forall \alpha, \beta \in \Gamma$.

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Theorem 3.1. Suppose $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of a $\Gamma$-semigroup $S$. Then the upper and lower level cuts $U(\mu_A; t)$ and $L(\mu_A; t)$ are generalized bi-ideals of $S$, for every $t \in \text{Im}(\mu_A) \cap \text{Im}(\nu_A)$.

Theorem 3.2. Suppose $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subset of a $\Gamma$-semigroup $S$ such that the sets $U(\mu_A; t)$ and $L(\nu_A; t)$ are generalized bi-ideals of $S$ whenever $t \in [0,1]$ and the sets are nonempty. Then the intuitionistic fuzzy subset $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of $S$.

Theorem 3.3. If a non-empty subset $I$ of a $\Gamma$-semigroup $S$ is a generalized bi-ideal of $S$, then $(\chi_I, \chi_I^\beta)$ is an intuitionistic fuzzy generalized bi-ideal of $S$, where $\chi_I$ is the characteristic function of $I$.

Definition 3.4.[10] A $\Gamma$-semigroup $S$ is called regular if for each element $x \in S$, there exist $y \in S$ and $\alpha, \beta \in \Gamma$ such that $x = x\alpha y \beta x$.

Proposition 3.1. Let $S$ be a regular $\Gamma$-semigroup. Then every intuitionistic fuzzy generalized bi-ideal of $S$ is intuitionistic fuzzy bi-ideal of $S$.

Proof. Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy generalized bi-ideal of $S$. Let $a, b \in S$. Since $S$ is regular, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $b = b\alpha x \beta b$. Then for any $\gamma \in \Gamma$,

$$\mu_A(a \gamma b) \geq \min\{\mu_A(a), \mu_A(b)\} \text{ and } \nu_A(a \gamma b) \leq \max\{\nu_A(a), \nu_A(b)\}.$$ 

Hence $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy subsemigroup of $S$ and consequently $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy bi-ideal of $S$.

Remark 2. In view of above proposition and Remark 1 we can say that in a regular $\Gamma$-semigroup the concepts of intuitionistic fuzzy generalized bi-ideal and intuitionistic fuzzy bi-ideal coincide.
Definition 3.5. Let $S$ be a $\Gamma$-semigroup. Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two intuitionistic fuzzy subsets of a $\Gamma$-semigroup $S$. Then the product $A \circ B = (\mu_{A \circ B}, \nu_{A \circ B})$ of $A$ and $B$ is defined as

$$(\mu_{A \circ B})(x) = \begin{cases} \sup \{\min \{\mu_A(u), \mu_B(v)\} : u, v \in S; \gamma \in \Gamma\} \\ 0, \text{if for any } u, v \in S \text{ and for any } \gamma \in \Gamma, x \neq u \gamma v \end{cases}$$

and

$$(\nu_{A \circ B})(x) = \begin{cases} \inf \{\max \{\nu_A(u), \nu_B(v)\} : u, v \in S; \gamma \in \Gamma\} \\ 1, \text{if for any } u, v \in S \text{ and for any } \gamma \in \Gamma, x \neq u \gamma v \end{cases}$$

Lemma 3.1. Let $S$ be a $\Gamma$-semigroup and $A = (\mu_A, \nu_A)$ be a non-empty intuitionistic fuzzy subset of $S$. Then $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of $S$ if and only if $A \subseteq A^\circ S \subseteq A$, where $S = (\chi_S, \chi_S^c)$ and $\chi_S$ is the characteristic function of $S$.

Proof: Let $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy generalized bi-ideal of $S$. Then for all $x, y, p, q \in S$ and for all $\beta, \gamma \in \Gamma$,

$$\mu_A(p \beta q \gamma) \geq \min \{\mu_A(p), \mu_A(y)\} \quad \text{and} \quad \nu_A(p \beta q \gamma) \leq \max \{\nu_A(p), \nu_A(y)\}.$$

Hence for any $x, y \in S$ and for some $p, q \in S$ and for some $\beta, \gamma \in \Gamma$, then $(\mu_A \circ \chi_S \circ \mu_A)(a) \leq \mu_A(a)$ (by Lemma 1[7]) and

$$(\nu_A \circ \chi_S^c \circ \nu_A)(a) = \inf \{\max \{(\nu_A \circ \chi_S^c)(x), \nu_A(y)\} \}$$

$$= \inf \{\max \{\inf \{\max \{\nu_A(p), \chi_S^c(q)\}\}, \nu_A(y)\}\}$$

$$= \inf \{\max \{\inf \{\max \{\nu_A(p), 0\}\}, \nu_A(y)\}\}$$

$$= \inf \{\max \{\nu_A(p), \nu_A(y)\}\} \geq \nu_A(p \beta q \gamma) = \nu_A(x \gamma y) = \nu_A(a).$$

If for $a \in S$ no such $\beta, \gamma \in \Gamma$ exist then $(\mu_A \circ \chi_S \circ \mu_A)(a) = 0 \leq \mu_A(a)$ and $(\nu_A \circ \chi_S^c \circ \nu_A)(a) = 1 \geq \nu_A(a)$. Hence $A \circ S \subseteq A$. Conversely, let $A \circ S \subseteq A$. Then $\mu_A \circ \chi_S \circ \mu_A \subseteq \mu_A$ and $\nu_A \circ \chi_S^c \circ \nu_A \supseteq \nu_A$. Hence for $x, y, z \in S$, and $\beta, \gamma \in \Gamma$, we deduce by repeated use of Definition 3.5 $\mu_A(x \beta y \gamma z) \geq \min \{\mu_A(x), \mu_A(z)\}$ (by Lemma 1[7]) and

$$\nu_A(x \beta y \gamma z) \leq (\nu_A \circ \chi_S^c \circ \nu_A)(x \beta y \gamma z) \leq \max \{\max \{(\nu_A \circ \chi_S^c)(x \beta y), \nu_A(z)\}\}$$

$$\leq \max \{\max \{\nu_A(x), 0\}, \nu_A(z)\} = \max \{\nu_A(x), \nu_A(z)\}.$$}

Hence $A = (\mu_A, \nu_A)$ is an intuitionistic fuzzy generalized bi-ideal of $S$.

In view of the above lemma we obtain the following theorem by routine verification.
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**Theorem 3.4.** The product of any two intuitionistic fuzzy generalized bi-ideals of a $\Gamma$-semigroup $S$ is an intuitionistic fuzzy generalized bi-ideal of $S$.

**Theorem 3.5.** A $\Gamma$-semigroup $S$ is regular if and only if for every intuitionistic fuzzy generalized bi-ideal $A = (\mu_A, \nu_A)$ of $S$ and $a \in S$, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha \beta a$.

**Proof:** Suppose $S$ is regular. Then for an intuitionistic fuzzy generalized bi-ideal $A = (\mu_A, \nu_A)$ of $S$ and $a \in S$, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha \beta a$. Hence $\mu_A \circ \chi_s \circ \mu_A = \mu_A$ (by Theorem 3[7])

Again $(\nu_A \circ \chi_s \circ \nu_A)(a) \leq \max\{\nu_A(a), \chi_s(x), \nu_A(a)\}$

$= \max\{\nu_A(a), \chi_s(x), \nu_A(a)\}$

$= \max\{\nu_A(a), 0, \nu_A(a)\} = \nu_A(a)$.

So $\nu_A \supseteq \nu_A \circ \chi_s \circ \nu_A$. By Lemma 3.1 $\nu_A \circ \chi_s \circ \nu_A \supseteq \nu_A$. Consequently, $\nu_A \circ \chi_s \circ \nu_A = \nu_A$. Hence $A \circ S \circ A = A$.

Conversely suppose the given condition holds. Let $R$ be a generalized bi-ideal of $S$.

Then by Theorem 3.3, $(\chi_r, \chi'_r)$ is an intuitionistic fuzzy generalized bi-ideal of $S$.

Hence by given condition $\chi_r \circ \chi_s \circ \chi_r = \chi_r$ and $\chi'_r \circ \chi'_s \circ \chi'_r = \chi'_r$. Let $a \in R$. Then $\chi_r(a) = 1$ and $\chi'_r(a) = 0$. Hence $\sup_{a \in \chi_r} \{a \circ \chi_r(p), \chi'_r(c)\} = 1$. (By Theorem 3[7])

Also

$(\chi'_r \circ \chi'_s \circ \chi_r)(a) = 0$

i.e., $\inf_{a \in \chi_r} \{\max\{\chi'_r \circ \chi'_s\}(b), \chi_r(c)\} = 0$

i.e., $\inf_{a \in \chi_r} \{\max\{\inf_{b \in \chi_r} \{\chi'_r(p), \chi'_r(q)\}, \chi'_r(c)\}\} = 0$

i.e., $\inf_{a \in \chi_r} \{\max\{\inf_{b \in \chi_r} \{\chi'_r(p), 0\}, \chi'_r(c)\}\} = 0$

i.e., $\inf_{a \in \chi_r} \{\max\{\inf_{b \in \chi_r} \chi'_r(p), \chi'_r(c)\}\} = 0$.

Thus we get $p, c \in S$ such that $a = b \chi_r$ and $b = p \chi'_r$ with $\chi_r(p) = \chi'_r(c) = 1$ and $\chi_r(p) = \chi'_r(c) = 0$ whence $p, c \in S$. So $a = b \chi_r = p \chi'_r \in R^* \Gamma S^r \Gamma$. Consequently, $R \subseteq R^* \Gamma S^r \Gamma$. Since $R$ is a generalized bi-ideal of $S$ so $R^* \Gamma S^r \Gamma \subseteq R$. Hence $R = R^* \Gamma S^r \Gamma$ and so $S$ is regular.

Using Lemma 3.1, Theorem 3.16[8] and Theorem 3.5 we can have the following theorem.
Theorem 3.6. A Γ-semigroup $S$ is regular if and only if for each intuitionistic fuzzy generalized bi-ideal $A = (\mu_A, \nu_A)$ of $S$ and each intuitionistic fuzzy ideal $B = (\mu_B, \nu_B)$ of $S$, $A \cap B = A \circ B \circ A$.

To conclude the paper we obtain the following result that characterizes regular Γ-semigroups in terms of intuitionistic fuzzy generalized bi-ideals.

Theorem 3.7. Let $S$ be a Γ-semigroup. then the following are equivalent:

1. $S$ is regular,
2. $A \cap B \subseteq A \circ B$ for each intuitionistic fuzzy bi-ideal $A = (\mu_A, \nu_A)$ of $S$ and for each intuitionistic fuzzy left ideal $B = (\mu_B, \nu_B)$ of $S$,
3. $A \cap B \subseteq A \circ B$ for each intuitionistic fuzzy generalized bi-ideal $A = (\mu_A, \nu_A)$ of $S$ and for each intuitionistic fuzzy left ideal $B = (\mu_B, \nu_B)$ of $S$,
4. $C \cap A \cap B \subseteq C \circ A \circ B$ for each intuitionistic fuzzy bi-ideal $A = (\mu_A, \nu_A)$ of $S$, for each intuitionistic fuzzy left ideal $B = (\mu_B, \nu_B)$ of $S$, and for each intuitionistic fuzzy right ideal $C = (\mu_C, \nu_C)$ of $S$,
5. $C \cap A \cap B \subseteq C \circ A \circ B$ for each intuitionistic fuzzy generalized bi-ideal $A = (\mu_A, \nu_A)$ of $S$, for each intuitionistic fuzzy left ideal $B = (\mu_B, \nu_B)$ of $S$, and for each intuitionistic fuzzy right ideal $C = (\mu_C, \nu_C)$ of $S$.

Proof: (1) $\Rightarrow$ (2): Let $S$ be regular, $A = (\mu_A, \nu_A)$ be an intuitionistic fuzzy bi-ideal of $S$ and $B = (\mu_B, \nu_B)$ be an intuitionistic fuzzy left ideal of $S$. Let $a \in S$. Then there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = a \alpha \beta a = a \alpha \beta a \alpha \beta a$. Then $\mu_A \circ \mu_B \supseteq \mu_A \cap \mu_B$ (cf. Theorem 6[7]). Again since $A$ is a intuitionistic fuzzy bi-ideal and $B$ is a intuitionistic fuzzy left ideal,

$$(\nu_A \circ \nu_B)(a) = \inf \left\{ \max\{\nu_A(y), \nu_B(z)\}; y = a \alpha \beta a \alpha \beta a \right\}$$

$\leq \max\{\nu_A(a \alpha \beta a), \nu_B(x \beta a)\} (a = a \alpha \beta a \alpha \beta a)$

$\leq \max\{\nu_A(a), \nu_B(a)\} = (\nu_A \cup \nu_B)(a)$.

So $\nu_A \circ \nu_B \subseteq \nu_A \cup \nu_B$. Hence $A \cap B \subseteq A \circ B$.

Similarly we can prove that (1) implies (3).

(2) $\Rightarrow$ (1): Let (2) hold. Let $A$ be an intuitionistic fuzzy right ideal and $B$ be an intuitionistic fuzzy left ideal of $S$. Then since every intuitionistic fuzzy right ideal of $S$ is intuitionistic fuzzy quasi ideal of $S$ and every intuitionistic fuzzy quasi ideal of $S$ is intuitionistic fuzzy bi-ideal of $S$, so $A$ is an intuitionistic fuzzy bi-ideal of $S$. Hence by (2), $A \cap B \subseteq A \circ B$. Also $A \circ B \subseteq A \cap B$ always holds. Hence $A \circ B = A \cap B$ and consequently, by Theorem 3.20 [8], $S$ is regular.

(3) $\Rightarrow$ (1): Suppose (3) holds. Let $T$ be a generalized bi-ideal of $S$, $L$ be a left ideal of $S$ and $a \in T \cap L$. Then $a \in T$ and $a \in L$. Since $T$ is a generalized bi-ideal
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of \( S \), so by Theorem 3.3, \((\chi_T, \chi'_T)\) is an intuitionistic fuzzy generalized bi-ideal of \( S \).

By Corollary 3.13 [8], \((\chi_L, \chi'_L)\) is an intuitionistic fuzzy left ideal of \( S \). Hence by (3),
\( \chi_T \cap \chi_L \subseteq \chi_T \circ \chi_L \) and \( \chi'_T \cup \chi'_L \supseteq \chi'_T \circ \chi'_L \). Then
\( (\chi_T \circ \chi_L)(a) = (\chi_T \cap \chi_L)(a) = \min\{\chi_T(a), \chi_L(a)\}\).

and 
\( (\chi'_T \circ \chi'_L)(a) = (\chi'_T \cup \chi'_L)(a) = \max\{\chi'_T(a), \chi'_L(a)\}\).

Hence \( \chi_{T,L}(a) = 1 \) and \( \chi'_{T,L}(a) = 0 \).

Hence in view of Definition 3.5, there exist \( b, c \in S \) and \( \delta \in \Gamma \) such that
\( a = b \delta c \) and \( \chi_T(b) = \chi'_L(c) = 1 \) and \( \chi'_T(b) = \chi_L(c) = 0 \), whence, \( b \in T \) and \( c \in L \).

Hence \( a = b \delta c \in T \cap L \). Thus \( T \cap L \subseteq T \cap L \). Hence by Theorem 5[7], \( S \) is regular.

(1) \( \Rightarrow \) (4): Let \( S \) be regular. Let \( A = (\mu_A, \nu_A) \) be an intuitionistic fuzzy bi-
ideal, \( B = (\mu_B, \nu_B) \) be an intuitionistic fuzzy left ideal and \( C = (\mu_C, \nu_C) \) be an
intuitionistic fuzzy right ideal of \( S \) respectively. Let \( a \in S \). Then there exist \( x \in S \) and
\( \alpha, \beta \in \Gamma \) such that
\( a = a \alpha \beta a = a \alpha \beta a \alpha \beta a = a \alpha \beta a \alpha \beta a \alpha \beta a \alpha \beta a \). Then
\( \mu_C \cap \mu_A \cap \mu_B \subseteq \mu_C \circ \mu_A \circ \mu_B \) (cf. Theorem 6[7]). Again
\( (\nu_C \circ \nu_A \circ \nu_B)(a) \leq \max\{\nu_C(a), (\nu_A \circ \nu_B)(a)\}\).

(since \( C \) is an intuitionistic fuzzy right ideal of \( S \))
\( \leq \max\{\nu_C(a), \max\{\nu_A(a), \nu_B(x)\}\}\)
\( \leq \max\{\nu_C(a), \max\{\nu_A(a), \nu_B(a)\}\}\)

(since \( A \) is an intuitionistic fuzzy bi-ideal of \( S \) and \( B \) is an intuitionistic fuzzy left ideal)
\( \leq \max\{\nu_C(a), \nu_A(a), \nu_B(a)\}\). Hence \( C \cap A \cap B \subseteq C \circ A \circ B \).

Similarly we can prove that (1) implies (5).

(4) \( \Rightarrow \) (1): Let (4) hold. Let \( B = (\mu_B, \nu_B) \) and \( C = (\mu_C, \nu_C) \) be any
intuitionistic fuzzy left ideal and intuitionistic fuzzy right ideal of \( S \). Since
\( S = (\chi_S, \chi'_S) \) itself is an intuitionistic fuzzy bi-ideal of \( S \), by (4), we have
\( C \cap B = C \cap S \cap B \subseteq C \circ S \circ B \subseteq C \circ B \). Also \( C \circ B \subseteq C \cap B \).

Therefore \( C \circ B = C \cap B \). Hence by Theorem 3.20 [8], \( S \) is regular.

(5) \( \Rightarrow \) (1): Suppose (5) holds. Let \( T \) be a generalized bi-ideal of \( S \), \( L \) be a
left ideal of \( S \), \( R \) be a right ideal of \( S \) and \( a \in R \cap L \). Then \( a \in R \), \( a \in A \) and
\( a \in L \). Since \( T \) is a generalized bi-ideal of \( S \), so by Theorem 3.3, \((\chi_T, \chi'_T)\) is an
intuitionistic fuzzy generalized bi-ideal of \( S \), by Theorem 3.13 [8], \((\chi_L, \chi'_L)\) is an
intuitionistic fuzzy left ideal of \( S \) and \((\chi_R, \chi'_R)\) is an intuitionistic fuzzy right ideal of

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S. Hence by (5), \( \mathcal{X}_R \cap \mathcal{X}_L \cap \mathcal{X}_T \subseteq \mathcal{X}_R \circ \mathcal{X}_L \circ \mathcal{X}_T \) and \( \mathcal{X}_R^c \cup \mathcal{X}_L^c \cup \mathcal{X}_T^c \supseteq \mathcal{X}_R^c \circ \mathcal{X}_L^c \circ \mathcal{X}_T^c \). Then \((\mathcal{X}_R \circ \mathcal{X}_T \circ \mathcal{X}_L)(a) \geq (\mathcal{X}_R \cap \mathcal{X}_T \cap \mathcal{X}_L)(a) = \min \{\mathcal{X}_R(a), \mathcal{X}_T(a), \mathcal{X}_L(a)\} = 1\). and \((\mathcal{X}_R^c \circ \mathcal{X}_T^c \circ \mathcal{X}_L^c)(a) \leq (\mathcal{X}_R^c \cup \mathcal{X}_T^c \cup \mathcal{X}_L^c)(a) = \max \{\mathcal{X}_R(a), \mathcal{X}_T(a), \mathcal{X}_L(a)\} = 0\).

Hence \( \mathcal{X}_{(R \cap T) \cap L}(a) = 1 \) and \( \mathcal{X}_{(R \cap T) \cap L}(a) = 0 \).

Hence in view of Definition 3.5, there exist \( b, c \in S \) and \( \delta \in \Gamma \) such that \( a = b \delta c \) and \( (\mathcal{X}_R \circ \mathcal{X}_T)(b) = \mathcal{X}_L(c) = 1 \) and \( (\mathcal{X}_R^c \circ \mathcal{X}_T^c)(b) = \mathcal{X}_L^c(c) = 0 \). Hence by applying similar argument as above we see that there exist \( d, e \in S \) and \( \theta \in \Gamma \) such that \( b = d \theta e \) and \( \mathcal{X}_R(d) = \mathcal{X}_T(e) = 1 \) and \( \mathcal{X}_R^c(d) = \mathcal{X}_T^c(e) = 0 \). Thus \( c \in L \), \( d \in R \) and \( e \in T \), with \( a = b \delta c = d \theta e \delta \in RTTL \). Hence \( R \cap T \cap L \subseteq RTTL \). Consequently, by Theorem 5 [7], \( S \) is regular.

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