Annals of Pure and Applied Mathematics
Vol. 6, No. 2, 2014, 150-159
ISSN: 2279-087X (P), 2279-0888(online)
Published on 26 June 2014
www.researchmathsci.org

A Note on Anti Q-Fuzzy R-Closed PS-ideals in PS-Algebras

***T.Priya*[1] and *T.Ramachandran*[2]**

[1]Department of Mathematics, PSNA College of Engineering and Technology
Dindigul-624 622, Tamilnadu, India
E-mail: tpriyasuriya@gmail.com

[2]Department of Mathematics, M.V.Muthiah Government Arts College for Women
Dindigul-624001, Tamilnadu, India
E-mail: yasrams@gmail.com

Received 16 June 2014; accepted 22 June 2014

Abstract. In this paper, we introduce the concept of Anti Q-fuzzy R-closed PS-ideals of PS-algebras, lower level cuts of a fuzzy set, R-closed PS-ideals and prove some results. We show that a Q-fuzzy set of a PS-algebra is a R- closed PS-ideal if and only if the complement of this Q-fuzzy set is an anti Q-fuzzy R-closed PS-ideal. Also we discussed few results of R-closed PS-ideals of PS-algebras in homomorphism and Cartesian product.

Keywords: PS-algebra, fuzzy PS- ideal, fuzzy R-closed PS- ideal, Anti Q-fuzzy R-closed PS-ideal, lower level cuts, homomorphism, Cartesian product.

***AMS Mathematics Subject Classification (2000)*:** 20N25, 03E72, 03F055, 06F35, 03G25

1. Introduction

The concept of fuzzy set was introduced by L.A.Zadeh in 1965 [19]. Since then these ideas have been applied to other algebraic structures such as groups, rings, modules, vector spaces and topologies. Iseki and Tanaka [2] introduced the concept of BCK-algebras in 1978 and Iseki [3] introduced the concept of BCI-algebras in 1980. It is known that the class of BCK –algebras is a proper subclass of the class of BCI algebras. Neggers and Kim [4] introduced a notion called d-algebra. Priya and Ramachandran [8,9] introduced a new notion, called PS-algebra, which is a generalization of BCK / BCI / d / KU algebras in 2014, and investigated some of its properties. Several related works have also been done in [6,7,10,11,12-15]. Biswas [1] introduced the concept of anti fuzzy subgroups of groups. Modifying his idea, in this paper we apply the idea in PS-algebras. In this paper, we introduce the notion of anti Q-fuzzy R-closed PS-ideals of PS-algebras and investigate some of its properties.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1.[6,7] A nonempty set X with a constant 0 and a binary operation ' $*$ ' is called PS – Algebra if it satisfies the following axioms.

1. $x * x = 0$
2. $x * 0 = 0$
3. $x * y = 0$ and $y * x = 0 \Rightarrow x = y$, $\forall$ $x, y \in X$.

Example 2.2. Let X = { 0,a,b } be the set with the following table.

*	0	a	b
0	0	a	b
a	0	0	0
b	0	b	0

Then (X , $*$, 0) is a PS – algebra.

Definition 2.3. [6-9] Let X be a PS-algebra and I be a subset of X, then I is called a PS-ideal of X if it satisfies the following conditions:

1. $0 \in I$
2. $y * x \in I$ and $y \in I \Rightarrow x \in I$.

Definition 2.4. [19] Let X be a non-empty set**.** A fuzzy subset μ of the set X is a mapping $\mu : X \to [0, 1]$.

Definition 2.5. [17,18] Let Q and G be any two sets.A mapping β: G x Q $\to$[0, 1] is called a Q –fuzzy set in G.

3. Anti Q-fuzzy R-closed PS-ideal of PS-algebras

Definition 3.1.[18] An ideal A of a PS-algebra X is said to be R-closed if $x * 0 \in A$ for all $x \in A$.

Definition 3.2. Let (X , * , 0) be a PS-algebra. A non empty subset I of X is called R-closed PS ideal of X if

(1) $x * 0 \in I$

(2) $y * x \in I$ and $y \in I \Rightarrow x \in I$ for all $x, y \in X$.

Remark: From Example 2.2, It is clear that $A_1 = \{0,a\}$ and $A_2 = \{0,a,b\}$ are R- closed PS-ideals of X.

Definition 3.3. A Q- fuzzy set μ in X is called a Q-fuzzy PS- ideal of X if

(i) $\mu(0,q) \geq \mu(x,q)$

(ii) $\mu(x,q) \geq \min\{\mu(y * x,q), \mu(y,q)\}$,for all $x, y \in X$ and $q \in Q$.

Definition 3.4. A Q-fuzzy set μ of a PS-algebra X is called an anti Q-fuzzy PS-ideal of X, if

(i) $\mu(0,q) \leq \mu(x,q)$

(ii) $\mu(x,q) \leq \max \{ \mu (y * x , q), \mu(y , q)\}$, for all $x,y \in X$ and $q \in Q$.

Definition 3.5. A Q-fuzzy set μ of a PS-algebra X is called an anti Q-fuzzy R- closed PS-ideal of X, if

(i) $\mu(x * 0, q) \leq \mu(x,q)$

(ii) $\mu(x,q) \leq \max\{\mu(y * x, q), \mu(y, q)\}$, for all $x,y \in X$ and $q \in Q$.

Theorem 3.1. Every Anti Q-fuzzy R-closed PS- ideal μ of a PS-algebra X is order preserving.

Proof: Let μ be an anti Q-fuzzy R-closed PS- ideal of a PS-algebra X and let $x, y \in X$ and $q \in Q$ be such that $x \leq y$, then $y * x = 0$

Then $\mu(x,q) \leq \max\{\mu((y * x), q), \mu(y,q)\}$

$= \max\{\mu(0,q), \mu(y,q)\}$

$= \max\{\mu(y*0, q), \mu(y,q)\}$

$= \mu(y,q)$

Hence $\mu(x,q) \leq \mu(y,q)$.

Theorem 3.2. μ is a Q-fuzzy R-closed PS-ideal of a PS-algebra X if and only if μ^c is an anti Q-fuzzy R-closed PS-ideal of X.

Proof: Let μ be a Q-fuzzy R-closed PS- ideal of X and let $x, y, z \in X$ and $q \in Q$.

(i) $\mu(x * 0, q) \geq \mu(x, q)$

$1 - \mu^c(x * 0, q) \geq 1 - \mu^c(x, q)$

$\mu^c(x*0, q) \leq \mu^c(x, q)$

That is $\mu^c(x * 0, q) \leq \mu^c(x, q)$

(ii) $\mu^c(x, q) = 1 - \mu(x, q)$

$\leq 1 - \min\{\mu(y * x, q), \mu(y, q)\}$

$= 1 - \min\{1 - \mu^c(y * x, q), 1 - \mu^c(y, q)\}$

$= \max\{\mu^c(y * x, q), \mu^c(y, q)\}$

That is $\mu^c(x * z, q) \leq \max\{\mu^c(y * x, q), \mu^c(y, q)\}$.

Thus μ^c is an anti Q-fuzzy R-closed PS-ideal of X. The converse also can be proved similarly.

Theorem 3.3. If μ is an anti Q-fuzzy R-closed PS-ideal of PS– algebra X, then for all $x,y \in X$ and $q \in Q$,

$\mu(x*(x*y), q) \leq \mu(y,q)$

Proof: Let $x,y \in X$ and $q \in Q$.

$\mu(x * (x * y), q) \leq \max\{\mu(y * (x * (x * y)), q), \mu(y, q)\}$

$= \max\{\mu(0, q), \mu(y, q)\}$

$= \max\{\mu(y * 0, q), \mu(y, q)\}$

$= \mu(y,q)$

$\therefore \mu(x * (x * y), q) \leq \mu(y, q)$.

Theorem 3.4 : Let X be a PS-algebra. For any anti Q- fuzzy R-closed PS-ideal μ of X, $X_\mu = \{x \in X \text{ and } q \in Q \,/\, \mu(x,q) = \mu(0,q)\}$ is a PS-ideal of X.

Proof: Let $y*x, y \in X_\mu$. Then $\mu(y*x,q) = \mu(y,q) = \mu(0,q)$

Since , μ is an anti Q-fuzzy R-closed PS-ideal of X ,

$\mu(x,q) \leq \max\{\mu(y * x,q), \mu(y,q)\}$

$= \max \{ \mu(0,q), \mu(0,q) \} = \mu(0,q)$

Hence, $x \in X_\mu$. Therefore X_μ is a PS-ideal of X.

Theorem 3.5. If λ and μ are anti Q-fuzzy R-closed PS ideals of a PS-algebra X, then $\lambda \cap \mu$ is also an anti Q-fuzzy R-closed PS-ideal of X.

Proof : Let $x, y \in X$ and $q \in Q$. Then

$$(\lambda\cap\mu)(0,q) = \min \{ \lambda(0,q), \mu(0,q) \} \leq \min \{ \lambda(x,q), \mu(x,q) \} = (\lambda\cap\mu)(x,q)$$

$$\begin{aligned}(\lambda\cap\mu)(x,q) &= \min \{ \lambda(x,q), \mu(x,q) \} \\ &\leq \min \{ \max \{\lambda(y*x,q), \lambda(y,q)\}, \max \{\mu(y*x,q), \mu(y,q)\}\} \\ &= \min \{ \max \{\lambda(y*x,q), \mu(y*x,q)\}, \max \{\lambda(y,q), \mu(y,q)\}\} \\ &\leq \max \{ \min \{\lambda(y*x,q), \mu(y*x,q)\}, \min \{\lambda(y,q), \mu(y,q)\}\} \\ &= \max \{(\lambda\cap\mu)(y*x,q), (\lambda\cap\mu)(y,q)\}.\end{aligned}$$

$\Rightarrow (\lambda\cap\mu)(x,q) \leq \max \{(\lambda\cap\mu)(y*x,q), (\lambda\cap\mu)(y,q)\}$.

Thus $(\lambda \cap \mu)$ is also an anti Q-fuzzy R-closed PS ideal of X.

Theorem 3.6. The union of any set of anti Q-fuzzy R-closed PS-ideals in PS-algebra X is also an anti Q-fuzzy R-closed PS-ideal.

Proof: Let $\{ \mu_i \}$ be a family of anti Q-fuzzy R-closed PS-ideals of PS-algebras X. Then for any $x, y \in X$ and $q \in Q$.

$$(\cup \mu_i)(0,q) = \sup (\mu_i(0,q)) \leq \sup (\mu_i(x,q)) = (\cup \mu_i)(x,q)$$

And

$$\begin{aligned}(\cup \mu_i)(x,q) &= \text{Sup} (\mu_i(x,q)) \\ &\leq \text{Sup} \{ \max \{ \mu_i(y*x,q), \mu_i(y,q)\}\} \\ &= \max \{ \text{Sup} (\mu_i(y*x,q)), \text{Sup} (\mu_i(y,q)) \} \\ &= \max \{ (\cup \mu_i)(y*x,q), (\cup \mu_i)(y,q) \}\end{aligned}$$

This completes the proof.

4. Lower level cuts in anti Q-fuzzy R-closed PS-ideals of PS-algebra

Definition 4.1.[7,8] Let μ be a Q-fuzzy set of X. For a fixed $t \in [0, 1]$, the set $\mu_t = \{x \in X \mid \mu(x,q) \leq t$ for all $q \in Q\}$ is called the lower level subset of μ. Clearly $\mu^t \cup \mu_t = X$ for $t \in [0,1]$ if $t_1 < t_2$, then $\mu_{t1} \subseteq \mu_{t2}$.

Theorem 4.1. If μ is an anti Q-fuzzy R-closed PS-ideal of PS-algebra X, then μ_t is a R-closed PS-ideal of X for every $t \in [0,1]$.

Proof: Let μ be an anti Q-fuzzy R-closed PS-ideal of PS-algebra X.

(i) Let $y \in \mu_t \Rightarrow \mu(y,q) \leq t$.

$$\begin{aligned}\mu(x*0,q) &\leq \max \{ \mu(y*(x*0)),q), \mu(y,q) \} \\ &= \max \{ \mu(y*0),q), \mu(y,q) \} \\ &= \mu(y,q) \leq t.\end{aligned}$$

$\Rightarrow x * 0 \in \mu_t$.

(ii) Let $y * x \in \mu_t$ and $y \in \mu_t$, for all $x, y \in X$ and $q \in Q$.

$\Rightarrow \mu(y * x, q) \leq t$ and $\mu(y, q) \leq t$.

$\mu(x, q) \leq \max\{\mu(y * x, q), \mu(y, q)\} \leq \max\{t, t\} = t$.

$\Rightarrow x \in \mu_t$.

Hence μ_t is an R-closed PS- ideal of X for every $t \in [0,1]$.

Theorem 4.2. Let μ be a Q-fuzzy set of PS- algebra X. If for each $t \in [0,1]$, the lower level cut μ_t is a R-closed PS-ideal of X, then μ is an anti Q- fuzzy R-closed PS-ideal of X.

Proof: Let μ_t be a R-closed PS-ideal of X.

If $\mu(x* 0,q) > \mu(x,q)$ for some $x \in X$ and $q \in Q$, then $\mu(x*0,q) > t_0 > \mu(x,q)$ by taking $t_0 = \frac{1}{2}\{\mu(x*0,q) + \mu(x,q)\}$.

Hence $x*0 \notin \mu_{t0}$ and $x \in \mu_{t0}$, which is a contradiction.

Therefore, $\mu(x*0,q) \leq \mu(x,q)$.

Let $x, y \in X$ and $q \in Q$ be such that $\mu(x, q) > \max\{\mu(y * x, q), \mu(y,q)\}$.

Taking $t_1 = \frac{1}{2}\{\mu(x, q) + \max\{\mu(y * x), q), \mu(y, q)\}\}$

$\Rightarrow \mu(x, q) > t_1 > \max\{\mu(y * x, q), \mu(y, q)\}$.

It follows that $(y * x), y \in \mu_{t1}$ and $x \notin \mu_{t1.}$ This is a contradiction.

Hence $\mu(x, q) \leq \max\{\mu(y * x, q), \mu(y, q)\}$

Therefore μ is an anti Q-fuzzy R-closed PS-ideal of X.

Definition 4.2. Let X be an PS- algebra and $a,b \in X$.We can define an set A(a,b) by $A(a,b) = \{ x \in X / a * (b * x) = 0 \}$. It is easy to see that $0,a, b \in A(a,b)$ for all $a,b \in X$.

Theorem 4.3. Let μ be a Q-fuzzy set in PS-algebra X. Then μ is an anti Q- fuzzy R-closed PS- ideal of X iff μ satisfies the following condition.

$(\forall a,b \in X), (\forall t \in [0,1])\ (a,b) \in \mu_t \Rightarrow A(a,b) \subseteq \mu_t$

Proof: Assume that μ is an anti Q-fuzzy R-closed PS- ideal of X.

Let $a,b \in \mu_t$. Then $\mu(a,q) \leq t$ and $\mu(b,q) \leq t$.

Let $x \in A(a,b)$. Then $a * (b * x) = 0$.

Now,

$$\begin{aligned}
\mu(x,q) &\leq \max\{\mu((b * x), q), \mu(b,q)\} \\
&\leq \max\{\max\{\mu(a * (b * x), q), \mu(a,q)\}, \mu(b,q)\} \\
&= \max\{\max\{\mu(0,q), \mu(a,q)\}, \mu(b,q)\} \\
&= \max\{\max\{\mu(a * 0,q), \mu(a,q)\}, \mu(b,q)\} \\
&= \max\{\mu(a,q)\}, \mu(b,q)\} \\
&\leq \max\{t, t\} = t
\end{aligned}$$

$\Rightarrow \mu(x,q) \leq t$.

$\Rightarrow x \in \mu_t$.

Therefore $A(a,b) \subseteq \mu_t$.

Conversely, suppose that $A(a,b) \subseteq \mu_t$.

Obviously $x*0 = 0 \in A(a, b) \subseteq \mu_t$ for all $a, b \in X$.

Let $x, y \in X$ be such that $(y * x) \in \mu_t$ and $y \in \mu_t$.

Since $(y * x) * (y * x) = 0$.

We have $x \in A(y * x, y) \subseteq \mu_t$.

$\therefore \mu_t$ is a R-closed PS- ideal of X.

Hence, by theorem 4.2, μ is an anti Q-fuzzy R-closed PS-ideal of X.

Theorem 4.4. Let μ be a Q-fuzzy set in PS-algebra X. If μ is an anti Q-fuzzy R-closed PS-ideal of X then

$$(\forall t \in [0,1])\ \mu_t \neq \emptyset \Rightarrow \mu_t = \bigcup_{a,b \in \mu_t} A(a,b).$$

Proof: Let t ∈ [0,1] be such that $\mu_t \neq \emptyset$. Since x * 0 = 0 ∈ μ_t , we have $\mu_t \subseteq \bigcup_{a \in \mu_t} A(a,0) \subseteq \bigcup_{a,b \in \mu_t} A(a,b)$.

Now, let x ∈ $\bigcup_{a,b \in \mu_t} A(a,b)$.

Then there exists (u, v) ∈ μ_t such that x ∈ A (u, v) ⊆ μ_t by theorem 4.3. Thus $\bigcup_{a,b \in \mu_t} A(a,b) \subseteq \mu_t$.

∴ $\mu_t = \bigcup_{a,b \in \mu_t} A(a,b)$.

5. Homomorphism and anti homomorphism on anti Q-fuzzy R-closed PS-algebras

In this section, we discussed about ideals in PS-algebra under homomorphism and anti homomorphism and some of its properties.

Definition 5.1.[6-11] Let (X,*,0) and (Y ,Δ ,0) be PS– algebras. A mapping f: X → Y is said to be a homomorphism if f(x * y) = f(x) * f(y) for all x,y ∈ X.

Definition 5.2. [17,18] Let (X,*,0) and (Y ,Δ ,0) be PS–algebras. A mapping f: X → Y is said to be an anti homomorphism if f(x * y) = f(y) Δ f(x) for all x,y ∈ X.

Definition 5.3. Let f: X → X be an endomorphism and μ be a fuzzy set in X. We define a new fuzzy set in X by μ_f in X as μ_f (x) = μ (f(x)) for all x in X.

Theorem 5.1. Let f be an endomorphism of a PS- algebra X. If μ is an anti Q- fuzzy R-closed PS-ideal of X, then so is μ_f .

Proof: Let μ be an anti Q-fuzzy R-closed PS-ideal of X.

Now, $\mu_f(x * 0,q) = \mu (f (x * 0,q))$

$\leq \mu (f(x,q)) = \mu_f(x,q)$, for all x,y ∈ X and q∈ Q.

Let x, y ∈ X and q∈ Q.

Then $\mu_f(x, q) = \mu (f(x, q))$

$\leq \max \{ \mu ((f(y, q) * f(x, q)), \mu(f (y, q)) \}$

$= \max \{ \mu (f(y * x), q), \mu(f (y, q))\}$

$= \max \{ \mu_f (y * x, q), \mu_f(y, q)\}$

∴ $\mu_f(x, q) \leq \max \{ \mu_f (y * x), q), \mu_f(y, q)\}$

Hence μ_f is an anti Q -fuzzy R-closed PS-ideal of X.

Theorem 5.2. Let f: X → Y be an epimorphism of PS- algebra. If μ_f is an anti Q-fuzzy R-closed PS-ideal of X, then μ is an anti Q-fuzzy R-closed PS-ideal of Y.

Proof: Let μ_f be an anti Q-fuzzy R-closed PS-ideal of X.

Let y ∈ Y and q∈ Q. Then there exists x ∈ X such that f(x, q) = (y, q).

Now,

$\mu(y * 0,q) = \mu ((y,q) * (0,q))$

$= \mu\ (f (x,q) * f(0,q))$
$= \mu\ (f ((x,q) * (0,q)))$
$= \mu_f ((x,q) * (0,q))$
$\leq \mu_f (x,q) = \mu\ (f(x,q)) = \mu\ (y,q)$
$\therefore\ \mu (y * 0,q) \leq \mu (y,q)$
Let $y_1, y_2 \in Y$ and $q \in Q$.
$\mu ((y_1,q)) = \mu\ (f (x_1,q))$
$= \mu_f (x_1, q)$
$\leq \max \{ \mu_f ((x_2,q) * (x_1,q)), \mu_f (x_2,q)\}$
$= \max \{ \mu [f ((x_2,q) * (x_1,q))], \mu\ (f(x_2,q))\}$
$= \max \{ \mu [f (x_2,q)) * f (x_1,q)] , \mu\ (f(x_2,q))\}$
$= \max \{ \mu [(y_2,q) * (y_1,q)] , \mu\ (y_2,q)\}$
$\therefore\ \mu\ (y_1,q) \leq \max \{ \mu [(y_2,q) * (y_1,q)] , \mu\ (y_2,q)\}$
$\Rightarrow$ μ is an anti Q-fuzzy R-closed PS-ideal of Y.

Theorem 5.3. Let f: $X \rightarrow Y$ be a homomorphism of PS- algebra. If μ is an anti Q-fuzzy R-closed PS-ideal of Y then μ_f is an anti Q-fuzzy R-closed PS-ideal of X.
Proof: Let μ be an anti Q-fuzzy R-closed PS-ideal of Y.
Let $x,y \in X$ and $q \in Q$.
$\mu_f (x*0,q) = \mu [f(x*0,q)]$
$\leq \mu [f(x,q)]$
$= \mu_f (x,q)$
$\Rightarrow\ \mu_f (x*0,q) \leq \mu_f (x,q)$.
$\mu_f\ (x , q) = \mu (f(x,q))$
$\leq \max\{ \mu\ [f(y, q) * f(x, q)] , \mu\ (f (y, q)) \}$
$= \max \{\mu\ [f(y * x, q)], \mu\ (f(y, q))\}$
$= \max \{\mu_f (y * x, q), \mu_f (y, q)\}$
$\therefore\ \mu_f (x, q) \leq \max \{\mu_f (y * x, q), \mu_f (y, q)\}$
Hence μ_f is an anti Q-fuzzy R-closed PS-ideal of X.

6. Cartesian product of anti Q-fuzzy PS-ideals of PS–algebras

In this section, we introduce the concept of Cartesian product of anti Q-fuzzy R-closed PS-ideals of PS-algebra.

Definition 6.1. [14,17] Let μ and δ be the fuzzy sets in X. The Cartesian product $\mu \times \delta$: $X \times X \rightarrow [0,1]$ is defined by $(\mu \times \delta) (x, y) = \min \{\mu(x),\delta(y)\}$, for all $x, y \in X$.

Definition 6.2. [18]
Let μ and δ be the anti fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by $(\mu \times \delta) (x, y) = \max \{\mu(x), \delta(y)\}$, for all $x, y \in X$.

Definition 6.3. [17,18] Let μ and δ be the anti Q-fuzzy sets in X. The Cartesian product $\mu \times \delta : X \times X \rightarrow [0,1]$ is defined by $(\mu \times \delta) ((x, y),q) = \max \{\mu(x, q), \delta(y, q)\}$, for all $x, y \in X$ and $q \in Q$.

Theorem 6.1. If μ and δ are anti Q-fuzzy R-closed PS-ideals in a PS– algebra X, then μ x δ is an anti Q-fuzzy R-closed KU-ideal in X x X.

Proof: Let $(x_1, x_2) \in X \times X$ and $q \in Q$.

$$(\mu \times \delta)((x_1 * 0, x_2 * 0), q) = \max\{\mu(x_1 * 0, q), \delta(x_2 * 0, q)\}$$
$$\leq \max\{\mu(x_1, q), \delta(x_2, q)\}$$
$$= (\mu \times \delta)((x_1, x_2), q)$$

$\therefore (\mu \times \delta)((x_1 * 0, x_2 * 0), q) \leq (\mu \times \delta)((x_1, x_2), q)$

Let $(x_1, x_2), (y_1, y_2) \in X \times X$ and $q \in Q$.

Now,

$$(\mu \times \delta)((x_1, x_2), q) = \max\{\mu(x_1, q), \delta(x_2, q)\}$$
$$\leq \max\{\max\{\mu(y_1 * x_1, q), \mu(y_1, q)\}, \max\{\delta(y_2 * x_2), q), \delta(y_2, q)\}\}$$
$$= \max\{\max\{\mu(y_1 * x_1), q), \delta(y_2 * x_2), q)\}, \max\{\mu(y_1, q), \delta(y_2, q)\}\}$$
$$= \max\{(\mu \times \delta)(((y_1, y_2), q) * ((x_1, x_2), q)), (\mu \times \delta)((y_1, y_2), q)\}$$

$\therefore (\mu \times \delta)((x_1, x_2), q) \leq \max\{(\mu \times \delta)(((y_1, y_2), q) * ((x_1, x_2), q)), (\mu \times \delta)((y_1, y_2), q)\}$.

Hence, μ x δ is an anti Q-fuzzy R-closed PS- ideal in X x X.

Theorem 6.2. Let μ and δ be fuzzy sets in a PS-algebra X such that μ x δ is an anti Q-fuzzy R-closed PS-ideal of X x X. Then

(i) Either $\mu(x * 0, q) \leq \mu(x, q)$ (or) $\delta(x * 0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$.

(ii) If $\mu(x * 0, q) \leq \mu(x, q)$ for all $x \in X$ and $q \in Q$, then either $\delta(x * 0, q) \leq \mu(x, q)$ (or) $\delta(x * 0, q) \leq \delta(x, q)$

(iii) If $\delta(x * 0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$, then either $\mu(x * 0, q) \leq \mu(x, q)$ (or) $\mu(x * 0, q) \leq \delta(x, q)$.

Proof: Straightforward.

Theorem 6.3. Let μ and δ be fuzzy sets in a PS-algebra X such that μ x δ is an anti Q-fuzzy R-closed PS-ideal of X x X. Then either μ or δ is an anti Q-fuzzy R-closed PS-ideal of X.

Proof: First we prove that δ is an anti Q- fuzzy R-closed PS-ideal of X.

Since by 6.2(i) either $\mu(x*0, q) \leq \mu(x, q)$ or $\delta(x*0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$.

Assume that $\delta(x*0, q) \leq \delta(x, q)$ for all $x \in X$ and $q \in Q$. It follows from 6.2(iii) that either $\mu(x*0, q) \leq \mu(x, q)$ (or) $\mu(x*0, q) \leq \delta(x, q)$.

If $\mu(x*0, q) \leq \delta(x, q)$, for any $x \in X$ and $q \in Q$,then

$\delta(x, q) = \max\{\mu(x*0, q), \delta(x, q)\} = \max\{\mu(0, q), \delta(x, q)\} = (\mu \times \delta)((0, x), q)$

$$\delta(x, q) = \max\{\mu(0, q), \delta(x, q)\}.$$
$$= (\mu \times \delta)((0, x), q)$$
$$\leq \max\{(\mu \times \delta)[((0, y), q) * ((0, x), q)], (\mu \times \delta)((0, y), q)\}$$
$$= \max\{(\mu \times \delta)[((0*0, y*x), q)], (\mu \times \delta)((0, y), q)\}$$
$$= \max\{(\mu \times \delta)[((0, (y*x)), q)], (\mu \times \delta)((0, y), q)\}$$
$$= \max\{\delta((y*x), q), \delta(y, q)\}$$

Hence, δ is an anti Q- fuzzy R-closed PS-ideal of X.

Similarly, we will prove that μ is an anti Q- fuzzy R-closed PS-ideal of X.

7. Conclusion

In this article we have discussed anti Q-fuzzy R-closed PS- ideals of PS-algebras and its lower level cuts in detail. In our aspect this R-closed definition and main results can be similarly extended to some other algebraic systems such as BG-algebras,TM-algebras etc. We hope that this work would other foundations for further study of the theory of PS-algebras. In our future study of fuzzy structure of PS-algebra, may be the topics , Intuitionistic fuzzy set, interval valued fuzzy sets, should be considered .

Acknowledgement

Authors wish to thank Dr. K.T.Nagalakshmi, Professor and Head, Department of Mathematics, K.L.N.College of Information and Technology, Pottapalayam, Sivagangai District, Tamilnadu, India, and Prof. P.M.Sithar Selvam, Professor and Head, Department of Mathematics, R.V.S. School of Engineering and Technology, Dindigul, Tamilnadu, India, for their help to make this paper as successful one.

Also, we greatful to the referees for their valuable comments to improve the presentation of the paper.

REFERENCES

1. R.Biswas, Fuzzy subgroups and anti fuzzy subgroups, *Fuzzy Sets and Systems*, 35 (1990) 121-124.
2. K.Iseki and S.Tanaka, An introduction to the theory of BCK–algebras, *Math Japonica*, 23(1978) 1- 20.
3. K.Iseki, On BCI-algebras, *Math. Seminar Notes*, 8 (1980) 125-130.
4. J.Neggers and H.S.Kim, On d-algebras, *Math. Slovaca*, 49 (1999) 19-26.
5. C.Prabpayak and U.Leerawat, On ideals and congurences in KU-algebras, *Scientia Magna J.,* 5(1) (2009) 54-57.
6. T.Priya and T.Ramachandran, Classification of PS-algebras, Communicated.
7. T.Priya and T.Ramachandran, Some properties of fuzzy dot PS-sub algebras of PS-algebras, *Annals of Pure and Applied Mathematics*, 6(1) (2014) 11-18**.**
8. T.Priya and T.Ramachandran, Some characterization of anti-fuzzy PS-ideals of PS-algebras in homomorphism and cartesian products, *International Journal of Fuzzy Mathematical Archive*, 4(2) (2014) 72-79.
9. T.Priya and T.Ramachandran, Anti fuzzy PS-ideals in PS-algebras and its level subsets, *International Journal of Mathematical Archive*, 5(4) (2014) 1-7.
10. T.Priya and T.Ramachandran, Normalisation of fuzzy PS-algebras, Communicated.
11. T.Priya and T.Ramachandran, On Pseudo fuzzy PS-algebras, Communicated.
12. T.Senapati, M.Bhowmik and M.Pal, Atanossov's intuitionistic fuzzy translation of intuitionistic fuzzy H-ideals in BCK/BCI algebras, *Notes on Intutionistic Fuzzy Sets,* 19(1) (2013) 32-47.
13. T.Senapati, M.Bhowmik and M. Pal, Fuzzy closed ideals of B-algebras with interval valued membership function, *Intern. J. Fuzzy Mathematical Archive*, 1 (2013) 79-91.
14. T.Senapati, M.Bhowmik and M. Pal, Fuzzy closed ideals of B-algebras, *International Journal of Computer Science Engineering and Technology*, 1(10) (2011) 669-673.

15. T.Senapati, M.Bhowmik and M. Pal, Interval-valued intuitionistic fuzzy closed ideals of BG-algebra and their products, *International Journal of Fuzzy Logic Systems*, 2 (2) (2012) 27-44.
16. P.M.Sithar Selvam, T.Priya and T.Ramachandran, Anti fuzzy subalgebras and Homomorphism of CI-algebras, *International Journal of Engineering Research and Technology*, 1(5) (2012) 1-6.
17. P.M.Sithar Selvam, T.Priya and T.Ramachandran, Anti Q- Fuzzy KU –Ideals in KU–algebras and its lower level cuts, *International journal of Engineering Research and Applications*, 2 (4) (2012) 1286-1289.
18. P.M.Sithar Selvam, T.Priya, T.Ramachandran and K.T.Nagalakshmi, Anti Q-fuzzy R –closed KU-ideals in KU-algebras and its lower level cuts, *International Journal of Fuzzy Mathematical Archive*, 4(2) (2014) 61-71.
19. L.A.Zadeh, Fuzzy sets, *Information and Control*, 8 (1965) 338-353.