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# A Study of Similarity Solution of Unsteady Combined Free and Force Convective Laminar Boundary Layer Flow About a Vertical Porous Surface with Suction and Blowing

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***Abstract.*** In the study of similarity solution of unsteady convective laminar boundary layer flow above a vertical porous plate, four different similarity cases arise of which we will present one of them. As usual the governing non-dimensional boundary layer partial differential equations are simplified first by using Boussinesq approximation. Secondly, similarity transformations are introduced on the basis of detailed analysis in order to transform the simplified coupled partial differential equations into a set of ordinary differential equations. The transformed complete similarity equations are then solved numerically by using Nachtsheim-Swigert shooting iteration technique along with sixth order Runga-Kutta method. The flow phenomenon has been characterized with the help of obtained flow controlling parameters on the velocity and temperature fields across the boundary layer are investigated. Numerical results for the velocity and temperature distributions are presented graphically. It is found that a small suction or blowing can play a significant role on the patterns of flow and temperature fields.

***Keywords:*** Similarity solution; combined convection; porous surface; suction and blowing

***AMS Mathematics Subject Classification (2010):*** 76Dxx, 76D99, 76E09

## 1. Introduction

Mixed convection flows or combined forced and free convection flows, arise in many transport processes in engineering devices and in nature. These flows are characterized by the buoyancy parameter (measure of the influence of the free convection in comparison with that of forced convection on the fluid flow) which depends on the flow configuration and the surface heating conditions. The problem of free mixed and forced convection over a horizontal porous plate has been attracted the interest of many investigators (Viz. Clark and Riley [1]. Schneider [2] and Merkin and Ingham [3] among several others) in view of its application in many engineering and geophysical problems. Ramanaiah *et al.* [4] considered the problem of mixed convection over a horizontal plate subjected to a temperature or surface heat flux varying as a power of $x$.

The problem of mixed convection due to a heated or cooled vertical flat plate provides one of the most basic scenarios for heat transfer theory and thus is of considerable theoretical and practical interest and has been extensively studied by Sparrow et al. [5], Wilks [6], Afzal & Banthiya [7] Hunt & Wilks [8], Lin & chen [9]. Hussain & Afzal [10], Merkin et al. [11] etc. However, the problem of forced, free and mized convection flows past a heated or cooled body with porous wall is of interest in realtion to the boundary layer control on airfoil, lubrication of ceramic machine parts and food processing. Watanabe [12] has considered the mixed convection boundary layer flow past an isothermal vertical porous flat plate plate with uniform suction or injection. Sattar [13] made analytical studies on the combined forced and free convection flow in a porous medium. Further, a vast literature of similarity solution has appeared in the area of fluid mechanics, heat transfer, and mass transfer, etc as it is one of the important means for the reduction of a number of independent variables with simplifying assumptions. It is revealed that the similarity solution, which being attained for some suitable values of different parameters, might be thought of being the solution of the convective boundary layer context either near the leading edge or far away in the downstream. Deswita et al. [14] obtained a similarity solution for the steady laminar free convection boundary layer flow on a horizontal plate with variable wall temperature hossain and Mojumder [15] presented the similarity solution for the steady laminar free convection boundary layer lfow generated above a heated horizontal rectangular surface. Furthermore the study of compete similarity solutions of the unsteady laminar natural convection boundary layer flow above a heated horizontal semi-infinite porous plate have been considered by hossain et al. [16.17].

The similarity solutions in the contest of mixed convection boundary layer flow of steady viscous incompressible fluid over an impermeable vertical flat plate were discussed by ishak et al [18]. Ramanaiah et al [19] studied the similarity solutions of free, mixed and forced convection problems in a saturated porous media. Recently, Hossain at al.[20] Presented Similarity solution of unsteady combined free and force convective laminar boundary layer flow about a vertical porous surface with suction and blowing. But in their analysis, they considered the first similarity case out of the four cases. In the present study we will consider another case for the complete similarity solution of the unsteady laminar combined free and forced convection boundary layer flow about a heated vertical porous plate in viscous incompressible fluid and be attempted to investigate the effects of several involved parameters on the velocity and temperature fields and other flow parameters like skin friction, heat transfer coefficients across the boundary layer. We are also tried to predict the role of small suction or blowing velocity on these parameters as well.

## 2. Basic equations of the flow and mathematical analysis

A semi- infinite flat-plate extending vertically upwards and which is fixed with its leading edge horizontal is placed in an unsteady free stream. The plate is heated to a certain unsteady temperature above the ambient temperature $T_e$. Heat is supplied by diffusion from the plate. The density of the fluid near the plate is reduced so that the fluid there is buoyant compared with the fluid in the free stream at a large distance from the plate. Consequently layers of the fluid close to the plate begin to rise. It is supposed that the maximum velocity created in this buoyant layer at a distance *L* from the bottom of the plate is *U*. If the Reynolds number based on this velocity *U* is sufficiently large, buoyant flow is amenable to Prandtl's boundary layer analysis.

Considering the flow direction along the *x*-axis. Then the simplified form of the basic boundary layer equations of mass, momentum and energy for a viscous and heat conducting fluid of variable properties subject to a body force are as follows:

$$\frac{D\rho}{Dt}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0 \tag{1}$$

$$\rho\frac{Du}{Dt}=(\rho-\rho_e)g_x+\rho_e\left(\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho C_p\left[\frac{D\theta}{Dt}+\theta\left\{\frac{\partial}{\partial t}(\log\Delta T)+u\frac{\partial}{\partial x}(\log\Delta T)\right\}\right]=\frac{\partial}{\partial y}\left(k\frac{\partial\theta}{\partial y}\right)-\frac{\rho C_p}{\Delta T}(u-u_e)\frac{\partial T_e}{\partial x} \tag{3}$$

Since at a particular station $(x,t)$ the pressure *p* does not very with y through the boundary layer, we have written $p=p_e$, $u\to u_e$, $\rho\to\rho_e$, $T\to T_e$, and $\frac{\partial}{\partial y}\to 0$. Also we have

written, $$\frac{T-T_e}{T_w-T_e}=\theta,\ \Delta T=T_w-T_e \tag{4}$$

where $\Delta T$ and $T_e$ are functions of *x* and *t*. $T_e=T_0$ (=constant) is one of the solutions of (4).

### 3. Similar solutions for the Boussinesq approximation

In this section we will simplify the above boundary layer equations (1)-(3) using the usual Boussinesq approximation Thus the elimination of the first term $\frac{D\rho}{Dt}$ in the continuity equation(1) will be found to lead to great simplifications in the boundary layer equations. Since the fluid property variations other than density variation in the buoyancy term of the momentum equation are ignored completely in this approximation, it is also assumed here that the fluid temperature outside the boundary layer $T_e$, is constant. Hence we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5}$$

$$\frac{Du}{Dt}=-g_x\beta_T\Delta T\theta+\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2} \tag{6}$$

$$\frac{D\theta}{Dt}+\theta\left\{\frac{\partial}{\partial\theta}\left(\log\Delta T\right)+u\frac{\partial}{\partial x}\left(\log\Delta T\right)\right\}=\frac{\upsilon}{P_r}\frac{\partial^2 u}{\partial y^2} \tag{7}$$

where, $\rho\approx\rho_r,\mu\approx\mu_r,k\approx k_r,C_p\approx C_{p_r}$ and $\upsilon=\frac{\mu_r}{\rho_r}$ , $\Pr=\frac{\mu_r C_{p_r}}{k_r}$ are the kinematic viscosity and Pr is the Prandtl number of the fluid respectively.

**4. Equations governing similar solutions**

To reduce the above system equations (5)-(6) into suitable forms we adopt the method of similarity solutions. Hence the following substitutions are introduced-

$$\xi=x,\ \varphi=\frac{y}{\gamma(x,t)},\ \ \tau=t,\ -v=\left(\gamma UF\right)_\xi-\varphi\gamma_x UF_\varphi-v_w$$

The equation of continuity (5) permits us to write $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$, where $\psi(x,y,t)$ is the stream function at any point (*x*, *y*, *t*). With the traditional substitution $\int_0^\varphi\frac{u}{U(x,t)}d\varphi=F(\xi,\varphi,\tau)$, we have

$$\psi(\xi,\varphi,\tau)=\gamma(\xi,\tau)U(\xi,\tau)F(\xi,\varphi,\tau)+\psi(\xi,0,\tau),$$

where the velocity components $u$ and $v$ are found to be

$$u=UF,\ v_w=-\frac{\partial\psi(\tau,\xi,0)}{\partial\xi}\ \text{and}\ -v=\left(\gamma UF\right)_\xi-\varphi\gamma_x UF_\varphi-v_w.$$

Here $v_w=-\frac{\partial\psi(\xi,0,\tau)}{\partial\xi}$ represents the non-zero wall velocity called suction or blowing velocity normal to the porous surface, so that fluid can either be sucked or blown through it. Physically, $v_w<0$ and $v_w>0$ represent the suction and blowing velocity through the porous surface respectively. For uniform suction (or blowing) $v_w$=constant. However, $v_w=0$ implies that the surface is impermeable to the fluid. In view of the above transformation, equations (5) to (7) become

$$\upsilon F_{\varphi\varphi\varphi}+\left(a_0\varphi+a_3\right)F_{\varphi\varphi}+\frac{1}{2}\left(a_1+a_2\right)FF_{\varphi\varphi}-a_2F_\varphi^2-a_4F_\varphi+a_5\vartheta+a_6=0 \tag{8}$$

$$\frac{\upsilon}{\Pr}\vartheta_{\varphi\varphi}+\left(a_0\varphi+a_3\right)\vartheta_\varphi+\frac{1}{2}\left(a_1+a_2\right)F\vartheta_\varphi-\left(a_7+a_8F_\varphi\right)\vartheta=0 \tag{9}$$

Where $F(\xi,\phi,\tau)$ and $\vartheta(\xi,\phi,\tau)$ are assumed t this stage to be function of $\phi$ alone and the $a's$ are given by (i) $\gamma\gamma_\tau=a_0$, (ii) $\left(\gamma^2u_e\right)_\xi=a_1$, (iii) $\gamma^2\left(u_e\right)_\xi=a_2$, (vi) $-\gamma v_w=a_3$, (v) $\frac{\gamma^2(u_e)_\tau}{u_e}=a_4$,

(vi) $-\dfrac{\gamma^2}{u_e}\Delta T\beta_T g_x = a_5$, (vii) $\dfrac{\gamma^2}{u_e}\left\{(u_e)_\tau + u_e (u_e)_\xi\right\} = a_6$, (viii) $\gamma^2\left(\log\Delta T\right)_\tau = a_7$,

(ix) $\gamma^2 u_e\left(\log\Delta T\right)_\xi = a_8$ (10)

The boundary conditions which are imposed in order to determine the solutions of the transformed boundary layer equations (8)-(9) are given by: $F(0) = F_\varphi(0) = 0$

$F_\varphi(\infty) = \vartheta(0) = 1,\ \vartheta(\infty) = 0$ (11)

The relations in equation (10) furnish us with the conditions under which similarity solutions are obtained provided that all $a's$ must be constants and thus the equations (8)-(9) will become non-linear ordinary differential equations.

In view of the conditions (ii) and (i) stated in equation (10), we have $\gamma^2 u_e = a_1\xi + A(\tau)$ and

$\gamma^2 = 2a_0\tau + B(\xi)$, where $A(\tau)$ is either a function of $\tau$ or constant and $B(\xi)$ is either a function of $\xi$ or constant. The above two relations yield

$$\frac{dA(\tau)}{d\tau}.\frac{dB(\xi)}{d\xi} = (a_1 - a_0)(a_4 + 2a_0) \qquad (12)$$

Therefore, the forms of the similarity equations, the scale factors $u_e(\xi,\tau)$ and $\gamma(\xi,\tau)$ depends wholly on the equation (12) and this situation leads to the following four possibilities:

(A) both $\dfrac{dA(\tau)}{d\tau}$ and $\dfrac{dB(\xi)}{d\xi}$ are finite constants, (B) both $\dfrac{dA(\tau)}{d\tau}$ and $\dfrac{dB(\xi)}{d\xi}$ are zero,

(B) $\dfrac{dA(\tau)}{d\tau} \neq 0$ but $\dfrac{dB(\xi)}{d\xi} = 0$, (D) $\dfrac{dA(\tau)}{d\tau} = 0$ but $\dfrac{dB(\xi)}{d\xi} \neq 0$.

**4.1. Similarity case to be considered**

Of these four similarity cases, only the Case (B) for which both $\left(\dfrac{dA}{d\tau}\right)$ and $\left(\dfrac{dB}{d\xi}\right)$ are zero has been studied here for the sake of brevity. Thus we have $\gamma^2 = 2a_0\tau + B(\xi)$ and $u_e = \dfrac{a_1\xi + A(\tau)}{2a_0 + B(\xi)}$. Substituting these in the conditions (i) to (ix) of the equation (10) yields the relations between the constants as follows: $a_0$, $a_1$ are arbitrary and

$a_4 = -2a_0$, $a_1 = a_2$, $a_3 = \sqrt{2a_0(\tau + \tau_0)}\,\nu_w$, $a_5 = -\beta_T\Delta T g_x.\dfrac{(2a_0\tau + B)^2}{(a_1\xi + A)}$,

$a_6 = a_1 - 2a_0$, $a_7 = -4a_0$, $a_8 = a_1$. Substituting the constants and choosing $F = \alpha_1 f$ and $\varphi = \alpha_1\eta$ the above equations (8) to (9) reduce to:

$$f_{\eta\eta\eta} + (\eta + f_w) f_{\eta\eta} + 2\beta\left( ff_{\eta\eta} - f_\eta^2 + 1\right) + 2\left( f_\eta - 1\right) + \frac{U_F^2}{u_e^2}\vartheta = 0 \qquad (13)$$

$$\mathrm{Pr}^{-1}\vartheta_{\eta\eta} + (\eta + f_w)\vartheta_\eta + 2\beta f \vartheta_\eta + \left(4 - 2\beta f_\eta\right)\vartheta = 0 \qquad (14)$$

Subject to the transformed boundary conditions

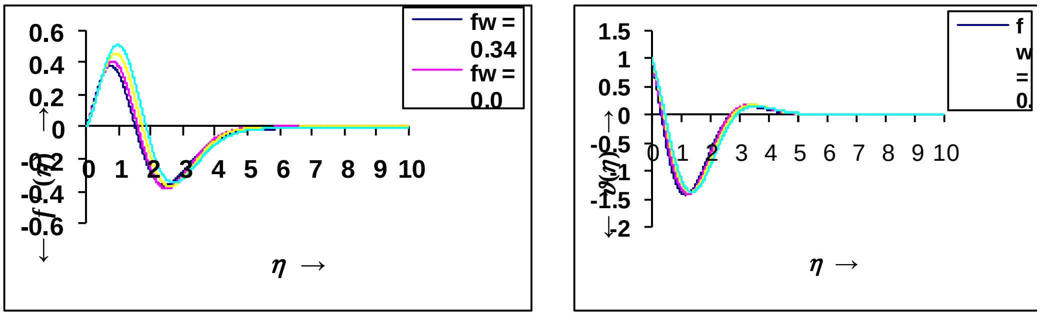
$$f(0) = f_\eta(0) = 0; \quad f_\eta(\infty) = 1; \quad \vartheta(0) = 1; \quad \vartheta(\infty) = 0 \qquad (15)$$

where is also chosen that $\alpha_1 = \alpha_2$, $\frac{a_0\,\alpha_1^2}{\nu} = 1$, $\frac{a_1}{a_0} = 2\beta$, $v_w = \sqrt{\frac{\upsilon}{L_c}}\, f_w$, $\frac{a_5}{a_0} = \frac{U_F^2}{u_0^2}$ with $U_F^2 = -g_x \beta_T \Delta T L_c$ and $L_c = 2u_e(\tau + \tau_0)$ is the local characteristic length. The terms $\frac{a_5}{a_0}\vartheta = \frac{U_F^2}{u_0^2}\vartheta$ in the momentum equation indicates how important buoyancy effects are compared with the forced flow effects. The flow is said to be aided when $U_F^2 / u_0^2$ is greater than zero and called an opposing when this parameter is less than zero. When $U_F^2 << u_0^2$ the flow becomes a forced flow, whereas for $U_F^2 << u_0^2$ the flow becomes a free convection flow. The skin friction and heat transfer coefficients $\tau_w$ and $q_w$ associated with the equations (13) and (14) are: $\tau_w = \mu \frac{u_e}{\sqrt{2\beta}} \left\{ \frac{u_e}{\nu(x + x_0)} \right\}^{1/2} f_{\eta\eta}(0)$ and $q_w = -\frac{k\,\Delta T}{\sqrt{2\beta}} \left\{ \frac{u_e}{\nu(x + x_0)} \right\}^{1/2} \vartheta_\eta(0)$. The $\Delta T$ - variation for this is $\Delta T \propto \frac{x + x_0}{t + t_0}$.

## 5. Numerical solution and discussions

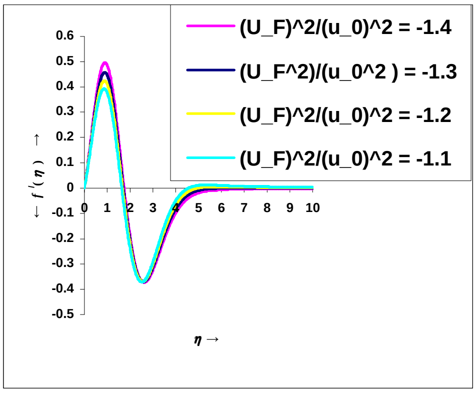
To obtain the solution of differential equations (13)–(14) with the boundary conditions (15), a numerical procedure based on Nachtsheim-Swigert shooting iteration technique (guessing the missing value) (Nachtsheim & Swigert (1965)) together with Runge-Kutta sixth order integration scheme is implemented. The effects of various pameters on the flow and temperature fields have been determined for different values of the suction/blowing parameter $f_w$, the driving parameter $\beta$ (the ratio between the changes of local boundary-layer thickness with regard to position and time), the buoyancy parameter $U_F^2 / u_0^2$ (the square of the ratio between the fluid velocity caused by buoyancy effects and external velocity for the forced flow) and the prandtl number $\mathrm{Pr}$. Since there are four parameters of interest in the present problem which can be varied, to observe the effect of one, the other three parameters are kept as constants. Under these conditions the solutions to the problem thus obtained finally by employing the above mentioned numerical technique are plotted and tabulated in terms of the similarity variables.

The effect of $f_w$ on the velocity and temperature profiles are plotted in the Fig. 1(a) and (b) respectively. From Fig.1 (a), we observe that the velocity is increasing for the decreasing value of $f_w$ in the region $\eta \le 1.02$. The maximum velocity appears at $\eta = 1.0$. Then the velocity profiles start decreasing and become negative when $\eta > 1.59$ again the velocities take the reverse direction and finally become zero at about $\eta = 5.1$. The magnitude of the velocities reaches the highest value when $\eta \approx 2.53$. Further we conclude that the velocity profiles increase with the decreasing value of $f_w$ in the region $(0 \le \eta \le 2.53)$ and increasing with the increasing of $f_w$ in the region $(2.53 \le \eta \le 5.1)$ for both suction and blowing. From the Fig. 1(b), we observe the effect of $f_w$ on the temperature profiles. From the figure it is observed that the wall lost its temperature to the fluid and after sometimes it receives the temperature from the fluid. In the region very close to the surface, the temperature falls sharply and decreases with the increase in $f_w$. When $\eta \approx 1.22$, the temperature profiles take the reverse direction and increase with increasing $f_w$. Here the temperature again decrease with the increase of $f_w$ when $\eta > 3.5$ and finally approaches to zero when $\eta > 5.06$.

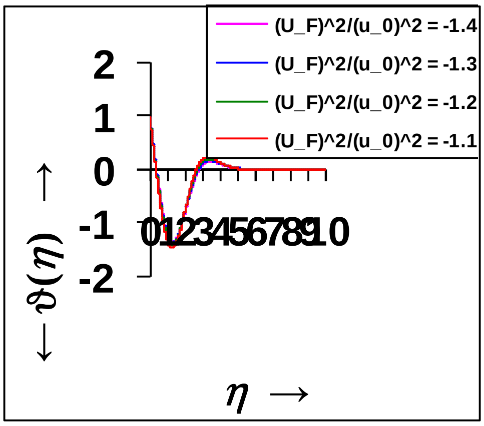



(a) (b)

**Figure 1:** (a) Velocity profiles and (b) Temperature profiles for different values of $f_w$ (with fixed values of $U_F^2 / u_0^2 = -1.3$, $\beta = 1.0$ and $\Pr = 0.72$).



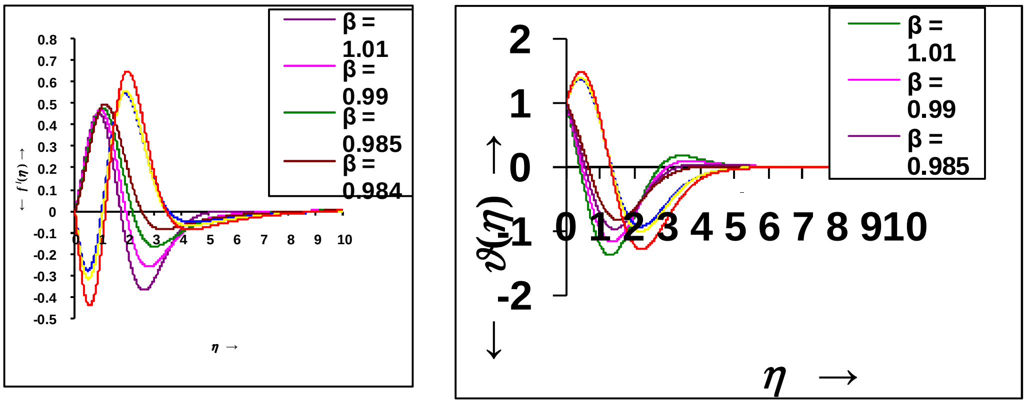

(a)




(b):

**Figure 2:** (a) Velocity profiles and (b) for different values of $U_F^2/u_0^2$ (with fixed values of $\beta = 1.0$, $f_w = -0.3$ and $P_r = 0.72$).

Fig. 2(a) and Fig. 2 (b) show the effects of $U_F^2/u_0^2$ on the velocity and temperature profiles. we observe from Fig.2(a) that the velocity profiles are increasing near the surface with the decreasing values of $U_F^2/u_0^2$ and obtained maximum value at $\eta \approx 0.99$. Then the velocity profiles change their directions and obtained negative values at $\eta > 1.65$ and finally become zero at $\eta = 6.5$. The magnitude of velocity obtained its highest value when $\eta \approx 2.65$ and after that a reverse characteristic is found. Here the magnitude of velocity is decreased with the increases of magnitude of $U_F^2/u_0^2$.Again the effects of $U_F^2/u_0^2$ on the temperature profiles show that, very close to the wall the temperature falls sharply in the region $0 \le \eta < 1.35$. The unusual shape of the temperature profiles in Fig. 2(b) indicates that the wall rejects more and more heat to the fluid as the buoyancy parameter $U_F^2/u_0^2$ decreases. This is due to the plate possessing an infinite source of heat at the leading edge, that is, $\Delta T \propto \frac{1}{t}$ at $x = 0$, hence $T_w \to \infty$ as $t \to 0$ at the leading edge.



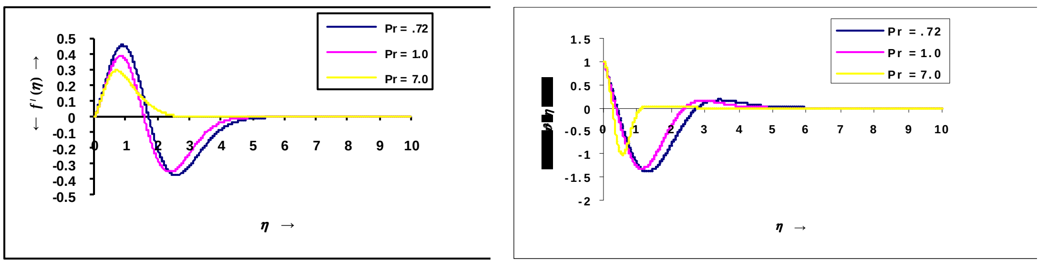

(b)

(a)

**Figure 3:** (a) Velocity profiles and Tempeture for different values of $\beta$ (with fixed values of $U_F^2/u_0^2 = -1.3$, $f_w = -0.3$ and $\Pr = 0.72$).

From Fig. 3(a), we see that the velocity profile becomes positive and a maximum appears at $\eta \approx 2.35$ for minimum value of $\beta = 0.967$. After that the velocity profiles again change their directions and become negative values when $\eta \geq 3.25$ and asymptotically approaches zero far away from the plate surface. Here the velocity profiles decreases with increasing $\beta$. Fig. 3(b) we see temperature first increases with decreasing $\beta$ when $\eta > 2.3$ and asymptotically approach to zero for all values of $\beta$ for far away and then they are again increase with increasing $\beta$.

From the Fig. 4(a) we observe that the velocity profiles decreases with the increase in $\Pr$. The velocity is positive in the region $0 \leq \eta \leq 1.58$ and become maximum at $\eta = 0.97$. After that the velocity profiles changes their directions and become negative in the region $1.58 < \eta \leq 5.1$.




(a) (b)

**Figure 4:** (a) Velocity profiles and (b) for different values of $\Pr$ (with fixed values of $U_F^2/u_0^2 = -1.3, \beta = 1.0$ and $f_w = -0.3$).

Finally reduced to zero asymptotically except for $\Pr = 7.0$. From the figure 4(b) we observe that the temperature profiles decreasing more with the decreasing in Pr close to the wall. For relatively higher values of $\eta$ they changes there direction and become positive. Before reaching zero finally, the temperature becomes positive and never negative again.

The values proportional to the coefficients of skin friction ($f^{//}(0)$) and heat transfer $\left(-\vartheta^{/}(0)\right)$ are tabulated in Table (4.1)–(4.4). From the table it is seen that with the increase in $f_w$, both the coefficients of skin friction and heat transfer increase. The coefficient of skin friction decreases whereas and coefficient of heat transfer increases with increasing $U_F^2/u_0^2$. Two different situations are observed for $\beta$ variation. In the range of $0.967 \le \beta < 0.984$, the skin friction decrease but the coefficient of heat transfer increases whereas in the range of $0.984 \le \beta \le 1.01$ both the skin friction and heat transfer coefficients increase for the increase in $\beta$. Again both the skin friction ($f^{//}(0)$) and heat transfer $\left(-\vartheta^{/}(0)\right)$ coefficients reduces with the increase in Pr. Unfortunately no experimental data is available to us to correspond our numerical results.

**Table 1:** Variation of the coefficients of skin friction and heat transfer with $f_w$, $U_F^2/u_0^2$, $\beta$, Pr.

| Values proportional to the coefficients of skin friction ($f^{//}(0)$) and heat transfer ($-\vartheta^{/}(0)$) with | | |
|---|---|---|
| the variation of suction parameter for fixed $U_F^2/u_0^2 = -1.3$, $\beta = 1.0$ and $\Pr = 0.72$ | | |
| $f_w$ | $f^{//}(0)$ | $-\vartheta^{/}(0)$ |
| 0.34 | 0.64352 | 3.07080 |
| 0.30 | 0.63844 | 3.01001 |
| 0.00 | 0.545924 | 2.50976 |
| - 0.30 | 0.63844 | 1.99617 |
| - 0.50 | 0.446546 | 1.69337 |
| Buoyancy parameter $U_F^2/u_0^2$ for fixed $f_w = -0.3, \beta = 1.0$ and $\Pr = 0.72$ | | |
| $U_F^2/u_0^2$ | $f^{//}(0)$ | $-\vartheta^{/}(0)$ |
| -1.1 | 0.438147 | 2.1618 |
| -1.2 | 0.454657 | 2.0588 |
| -1.3 | 0.477470 | 1.99617 |
| -1.4 | 0.509787 | 1.96947 |
| driving parameter $\beta$ for fixed $f_w = -0.3, U_F^2/u_0^2 = -1.3$ and $\Pr = 0.72$. | | |

| $\beta$ | $f^{//}(0)$ | $-\vartheta^{/}(0)$ |
|---|---|---|
| 0.967 | - 1.568036 | -1.992520 |
| 0.98 | - 1.08877 | -1.674788 |
| 0.983 | -1.006547 | -1.621930 |
| 0.984 | 0.268703 | 0.827137 |
| 0.985 | 0.331071 | 1.17268 |
| 0.99 | 0.411649 | 1.588687 |
| 1.01 | 0.528188 | 2.457179 |

Prandtl's number Pr , for fixed $f_w = -0.3$, $\beta = 1.0$ and $U_F^2 / u_0^2 = -1.3$.

| Pr | $f^{//}(0)$ | $-\vartheta^{/}(0)$ |
|---|---|---|
| 0.72 | 0.47747 | 1.99617 |
| 1.00 | 0.422092 | 2.09212 |
| 7.00 | 0.4 | 0.3 |

**6. Conclusion**

Similarity solution for the two-dimensional unsteady laminar combined free and forced convection boundary layer flow over a semi-infinite heated vertical porous plate with the similarity case $\frac{dA}{d\tau}$ and $\frac{dB(\xi)}{d\xi}$ =0 has been studied in this paper. On the basis of the findings the following conclusions can be drawn:

(i) Velocity increase with the decrease of suction/blowing and increasing with the increasing for both suction and blowing.. In the region very close to the surface, the temperature falls sharply and decreases with the increase suction and blowing. Here the temperature again decreases with the increase of suction and blowing and finally approaches to zero.

(ii) The velocity increase near the surface with the decreasing values of buoyancy parameter. The unusual shape of the temperature indicates that the wall rejects more and more heat to the fluid as the buoyancy parameter decreases.

(iii) The velocity decreases with increasing control parameter and temperature first increases with decreasing control parameter and asymptotically approach to zero for all values of control parameter for far away and then they are again increase with increasing control parameter

(iv) With the increase in $P_r$ both the velocity and temperature decrease. Also both the values proportional to the coefficients of skin-friction and heat transfer decrease with the increase in the prandtl's number.

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