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Traveling Wave Solutions of Nonlinear Klein-Gordon Equation by Extended (G'/G)-expansion Method

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Abstract. The extended (G'/G)-expansion method can be used to construct exact traveling wave solutions of non-linear evolution equations. In this paper, we explore new application of this method to non-linear Klein-Gordon equation, the balance numbers of which are both positive and negative. By using this method, we found some new traveling wave solutions of the above-mentioned equation.
Keywords: Extended (G'/G)-expansion method, Nonlinear Klein-Gordon, Traveling wave solutions

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1. Introduction

Nonlinear phenomena play a crucial role in applied mathematics and physics. Calculating exact and numerical solutions, in particular, traveling wave solutions, of nonlinear equations in mathematical physics plays an important role in nonlinear phenomena. Recently, it has become hot tropics and interesting that obtaining exact solutions of nonlinear partial differential equations through using symbolical computer programs such as Maple, Matlab, Mathematica that facilitate complex and tedious algebraical computations. It is important to find exact traveling wave solutions of nonlinear partial differential equations.

Looking for exact solutions of nonlinear partial differential equations has long been a major concern for both mathematicians and physicists. Various effective methods have been developed such as Backlund transformation method [1,2], Darboux Transformations [3], Riccati equation method [4], tanh-function method [5,6], Exp-function method [7], sine-cosine method [8], others some method [9-11] and so on. Wang et al. firstly proposed a $G'/G-$expansion method [12], then many diverse group of researchers extended this method by different names like extended, further extended , improved, Generalized and improve $G'/G-$ expansion method [12-24] with different auxiliary equations. A. Sousaraie [25] investigated traveling wave solutions for non-linear Klein-Gordon equation by using $G'/G-$ expansion method with help of Auxiliary equation $G''+\lambda G'+\mu G=0$. L. D. Chen [26] searched traveling wave solutions for generalized Klein-Gordon equation by using Modified $G'/G-$ expansion method with help of Auxiliary equation $G''+\lambda G'+\mu G=0$. In this article, following these method we obtain a new idea of searching traveling wave solutions of non-linear Klein-Gordon equation by extended $G'/G-$ expansion method in which $G=G(\xi)$ satisfying the differential equation $G''+\mu G=0$, where $\mu\neq 0$ and the balance numbers contain both positive and negative. Using this method we include few new results of traveling wave solutions for nonlinear evolution equations.

2. The Method

For given nonlinear evolution equations in two independent variables *x* and t, we consider the following form

$$F(u,u_t,u_x,u_{tt},u_{xx},u_{xt}\cdots\cdots)=0 \tag{1}$$

By using traveling wave transformation

$$u(x,t)=u(\xi),\quad \xi=x-Vt \tag{2}$$

where u is an unknown function depending on x and t, and is a polynomial F in $u=u(x,t)$ and its partial derivatives and V is a constant to be determined later. The existing steps of method are as follows:

Step 1. Using the Eq. (2) in Eq.(1), we can convert Eq. (1) to an ordinary differential equation

$$Q(u,-V\,u',u',V^2u'',u'',-Vu''\cdots\cdots)=0 \tag{3}$$

Step 2. Assume the solutions of Eq.(3) can be expressed in the form

$$u(\xi)=\sum_{i=-n}^{n}\left\{a_i(G'/G)^i+b_i(G'/G)^{i-1}\sqrt{\sigma\left[1+\frac{1}{\mu}(G'/G)^2\right]}\right\}, \tag{4}$$

with $G=G(\xi)$ satisfying the differential equation

$$G''+\mu G=0, \tag{5}$$

in which the value of σ must be $\pm 1,\mu\neq 0$, $a_i,b_i(i=-n,\cdots,n)$ and λ are constants to be determined later. We can evaluate n by balancing the highest-order derivative term with the nonlinear term in the reduced Eq. (3).

Step 3. Inserting Eq.(4) into Eq.(3) and making use of Eq.(5) and then extracting all terms of like powers of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left[1+(G'/G)^2/\mu\right]}$ together, then set each coefficient of them to zero yield a over-determined system of algebraic equations and then solving this system of algebraic equations for $a_i,b_i\ (i=-n,\cdots,n)$ and λ,V, we obtain several sets of solutions.

Step 4. For the general solutions of Eq. (5), we have

$$\begin{cases} \mu<0, & \dfrac{G'}{G}=\sqrt{-\mu}\left(\dfrac{A\sinh\left(\sqrt{-\mu}\xi\right)+B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right)+B\sinh\left(\sqrt{-\mu}\xi\right)}\right)=f_1(\xi) \\ \mu>0, & \dfrac{G'}{G}=\sqrt{\mu}\left(\dfrac{A\cos\left(\sqrt{\mu}\xi\right)-B\sin\left(\sqrt{\mu}\xi\right)}{A\sin\left(\sqrt{\mu}\xi\right)+B\cos\left(\sqrt{\mu}\xi\right)}\right)=f_2(\xi) \end{cases} \tag{6}$$

where A, B are arbitrary constants. At last, inserting the values of $a_i, b_i (i=-n,\cdots,n)$, λ, V and (6) into Eq. (4) and obtain required traveling wave solutions of Eq. (1).

3. Application of our Method

Let us consider the nonlinear Klein-Gordon equation

$$u_{tt}-u_{xx}+\alpha u+\beta u^3=0 \tag{7}$$

with auxiliary equation

$$G''+\mu G=0$$

where α, β, μ are constants and $\mu\neq 0$.

Under the traveling wave transformation with Eq. (2), Eq. (7) reduce to

$$(V^2-1)u''+\alpha u+\beta u^3=0 \tag{8}$$

By balancing the highest-order derivative term u'' and nonlinear term u^3 in Eq. (8) gives $n=1$, thus, we have the solutions of Eq. (8), according to Eq. (4) is

$$u(\xi)=a_0+a_1(G'/G)\ +a_{-1}(G'/G)^{-1}+(b_0(G'/G)^{-1}+b_1+b_{-1}(G'/G)^{-2})\sqrt{\sigma\left[1+\frac{1}{\mu}(G'/G)^2\right]} \tag{9}$$

where $G=G(\xi)$ satisfies Eq.(5). Substituting Eq. (9) and Eq. (5) into Eq.(8), collecting all terms with the like powers of $(G'/G)^j$ and $(G'/G)^j\sqrt{\sigma\left[1+(G'/G)^2/\mu\right]}$, and setting them to zero, we obtain a over-determined system that consists of twenty-five algebraic equations (we omitted these for convenience). Solving this over-determined system with the assist of Maple, we have the following results.

Case-1:

$$V=\pm\sqrt{\left(\frac{2\mu-\alpha}{2\mu}\right)},\ a_{-1}=\pm\sqrt{\left(\frac{\alpha\mu}{\beta}\right)},\ a_0=a_1=b_0=b_{-1}=b_1=0$$

Now when $\mu>0$ then using Eqs. (6) and (9), we have

$$u=\pm\sqrt{\left(\frac{\alpha\mu}{\beta}\right)}\frac{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)}{\sqrt{\mu}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))},\ \text{where } \xi=x-Vt,\ V=\pm\sqrt{\left(\frac{2\mu-\alpha}{2\mu}\right)}$$

and when $\mu<0$ then using Eqs. (6) and (9)

$$u=\pm\sqrt{\left(\frac{\alpha\mu}{\beta}\right)}\frac{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))},\ \text{where } \xi=x-Vt,$$

$$V = \pm\sqrt{(\frac{2\mu-\alpha}{2\mu})}$$

Case-2:

$$V = \pm\sqrt{(\frac{2\mu-\alpha}{2\mu})},\ a_1 = \pm\sqrt{(\frac{\alpha}{\beta\mu})},\ a_0 = a_{-1} = b_0 = b_{-1} = b_1 = 0$$

Now when $\mu > 0$ then using Eqs. (6) and (9), we have

$$u = \pm\sqrt{\frac{\alpha}{\beta\mu}}\ \frac{\sqrt{\mu}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)}$$, where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{2\mu-\alpha}{2\mu})}$

and when $\mu < 0$ then using Eqs. (6) and (9), we have

$$u = \pm\sqrt{\frac{\alpha}{\beta\mu}}\ \frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}$$, where $\xi = x - Vt$,

$V = \pm\sqrt{(\frac{2\mu-\alpha}{2\mu})}$

Case-3:

$$V = \pm\sqrt{(\frac{8\mu-\alpha}{8\mu})},\ a_{-1} = -1/2(\pm\sqrt{\frac{\alpha\mu}{\beta}}),\ a_1 = \pm\sqrt{(\frac{\alpha}{4\beta\mu})},$$

$a_0 = b_0 = b_1 = b_{-1} = 0$

Now when $\mu > 0$ then using Eqs. (6) and (9), we have

$$u = \sqrt{(\frac{\alpha}{4\beta\mu})}\ \frac{\sqrt{\mu}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)} - \frac{1}{2}(\sqrt{\frac{\alpha\mu}{\beta}})\frac{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)}{\sqrt{\mu}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{8\mu-\alpha}{8\mu})}$

and

$$u = -\sqrt{\frac{\alpha}{4\beta\mu}}\ \frac{\sqrt{\mu}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)} + \frac{1}{2}(\sqrt{\frac{\alpha\mu}{\beta}})\frac{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)}{\sqrt{\mu}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{8\mu-\alpha}{8\mu})}$

when $\mu < 0$ then using Eqs. (6) and (9), we have

$$u = \sqrt{\frac{\alpha}{4\beta\mu}}\ \frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)} - \frac{1}{2}\left(\sqrt{\frac{\alpha\mu}{\beta}}\right)\frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{8\mu-\alpha}{8\mu})}$

and

$$u = -\sqrt{\frac{\alpha}{4\beta\mu}}\ \frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)} +$$

$$\frac{1}{2}\left(\sqrt{\frac{\alpha\mu}{\beta}}\right)\frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{8\mu-\alpha}{8\mu})}$

Case-4:

$$V = \pm\sqrt{(\frac{4\mu+\alpha}{4\mu})},\ a_{-1} = -1/2(\pm(\frac{\alpha}{\beta\sqrt{(\frac{-\alpha}{2\beta\mu})}})),\ a_1 = \pm\sqrt{(\frac{-\alpha}{2\beta\mu})},$$

$$a_0 = b_1 = b_0 = b_{-1} = 0$$

Now when $\mu > 0$ then using Eqs. (6) and (9), we have

$$u = \sqrt{(\frac{-\alpha}{2\beta\mu})}\frac{\sqrt{(\mu)}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)} - \frac{1}{2}(\frac{\alpha}{\beta\sqrt{(\frac{-\alpha}{2\beta\mu})}})\frac{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)}{\sqrt{\mu}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{4\mu+\alpha}{4\mu})}$

and

$$u = -\sqrt{\frac{-\alpha}{2\beta\mu}}\frac{\sqrt{(\mu)}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)} + \frac{1}{2}(\frac{\alpha}{\beta\sqrt{(\frac{-\alpha}{2\beta\mu})}})\frac{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)}{\sqrt{\mu}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{4\mu+\alpha}{4\mu})}$

when $\mu < 0$ then using Eqs. (6) and (9), we have

$$u = \sqrt{\frac{-\alpha}{2\beta\mu}}\frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)} - \frac{1}{2}\left(\frac{\alpha}{\beta\sqrt{(\frac{-\alpha}{2\beta\mu})}}\right)\frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{4\mu+\alpha}{4\mu})}$

and

$$u = -\sqrt{\frac{-\alpha}{2\beta\mu}}\frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)} + \frac{1}{2}\left(\frac{\alpha}{\beta\sqrt{(\frac{-\alpha}{2\beta\mu})}}\right)\frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{4\mu+\alpha}{4\mu})}$

Case-5:

$$V = \pm\sqrt{(\frac{\mu+\alpha}{\mu})},\ b_1 = \pm\sqrt{(\frac{-2\alpha}{\beta\sigma})},\ a_0 = a_{-1} = a_1 = b_0 = b_{-1} = 0$$

Now when $\mu > 0$ then using Eqs. (6) and (9), we have

$$u = \pm\sqrt{\left(\frac{-2\alpha}{\beta\sigma}\right)}\sqrt{\sigma\left[1 + \frac{1}{\mu}(\frac{\sqrt{(\mu)}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)})^2\right]}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{\mu+\alpha}{\mu})}$

and when $\mu < 0$ then using Eqs. (6) and (9), we have

$$u = \pm\sqrt{\left(\frac{-2\alpha}{\beta\sigma}\right)}\sqrt{\sigma\left[1 + \frac{1}{\mu}(\frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)})^2\right]}$$

where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{\mu+\alpha}{\mu})}$

Case-6:

$$V = \pm\sqrt{(\frac{\mu-2\alpha}{\mu})},\ a_1 = \pm\sqrt{(\frac{\alpha}{\beta\mu})},\ b_1 = \pm\sqrt{(\frac{\alpha}{\beta\sigma})},\ a_0 = a_{-1} = b_0 = b_{-1} = 0$$

Now when $\mu > 0$ then using Eqs. (6) and (9), we have

$$u = \pm\sqrt{\frac{\alpha}{\beta\mu}}\frac{\sqrt{(\mu)}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)} \pm \sqrt{\frac{\alpha}{\beta\sigma}}\sqrt{\sigma\left[1 + \frac{1}{\mu}(\frac{\sqrt{(\mu)}(A\cos(\sqrt{\mu}\xi)-B\sin(\sqrt{\mu}\xi))}{A\sin(\sqrt{\mu}\xi)+B\cos(\sqrt{\mu}\xi)})^2\right]}$$

where $\xi = x - Vt$, where $\xi = x - Vt$, $V = \pm\sqrt{(\frac{\mu-2\alpha}{\mu})}$

and

when $\mu < 0$ then using Eqs. (6) and (9), we have

$$u = \pm\sqrt{\frac{\alpha}{\beta\mu}}\frac{\sqrt{-\mu}\left(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)\right)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}$$
$$\pm\sqrt{\frac{\alpha}{\beta\sigma}}\sqrt{\sigma\left[1+\frac{1}{\mu}\left(\frac{\sqrt{-\mu}(A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi))}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)^2\right]}$$

where $\xi = x - Vt$, where $\xi = x - Vt$, $V = \pm\sqrt{\left(\frac{\mu-2\alpha}{\mu}\right)}$

Remark: If $\lambda = 0$, all the solutions of [25] match with our solutions in the case 2.

4. Conclusion

The extended $G'/G-$expansion method has been applied to search exact travelling wave solutions for the non-linear Klien-Gordon equation. As a result, we obtained new plentiful exact solutions. The solutions are in the form of trigonometric and hyperbolic. It is shown that the performance of this method is productive, effective and well-built mathematical tool for solving nonlinear evolution equations.

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