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Fixed Point Theorems in Intuitionistic Fuzzy Graph Metric Space

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Abstract. In this paper, the contractive mapping on intuitionistic fuzzy graph metric space is discussed and the concepts of Lipschitz condition, ϵ −chain, contractive mapping, ϵ −contractive mapping on intuitionistic fuzzy graph metric space are studied. Theorems related to the above concepts are stated and proved.

Keywords: Intuitionistic fuzzy graph metric space; contractive mapping; ϵ − chain; ϵ −contractive mapping.

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1. Introduction

In 1965, Zadeh [14] introduced the concept of fuzzy set as a method of finding uncertainty. Rosenfeld [8] introduced the concept of fuzzy graphs in 1975. Yeh and Bang [13] also introduced fuzzy graphs independently. Sunitha and Mathew [10] studied the survey of fuzzy graph. Vaishnaw et al. [11] discussed some analogies results on fuzzy graphs. Atanassov [1] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graph. Parvathi and Karunambigai [6] gave a definition for intuitionistic fuzzy graph as a special case of intuitionistic fuzzy graphs defined by Atanassov and Shannon [9]. Parvathi and Thamizhendhi [5] were introduced cardinality of an intuitionistic fuzzy graph. The important graph theory approach to metric fixed point theory introduced so far is attributed to Jachymski [2]. Jha et al. [3] discussed OWC mapping in semi-metric space. In 1996, Kada et al. [4] defined the notion of w-distance in metric spaces. In 2018, Mohamed and Ali [12] introduced the notion of intuitionistic fuzzy graph metric space. In this paper, the contractive mapping on intuitionistic fuzzy graph metric space is introduced and studied the concepts of isometric function, Lipschitz condition, ϵ −chain, contractive mapping, ϵ −contractive mapping on intuitionistic fuzzy graph metric space.

2. Preliminaries

Definition 2.1. An ***intuitionistic fuzzy graph*** is of the form G = (V, E), where

i) $V = \{v_1, v_2, \dots v_n\}$ such that $\sigma_1: V \to [0,1]$ and $\sigma_2: V \to [0,1]$ denote the degree of membership and non membership of the element $v_i \in V$ respectively and $0 \le \sigma_1(v_i) + \sigma_2(v_i) \le 1$ for every $v_i \in V$ $(i = 1,2, \dots, n)$.

ii) $E \subset V \times V$ where $\mu_1 : V \times V \to [0,1]$ and $\mu_2 : V \times V \to [0,1]$ are defined by $\mu_1(v_i, v_j) \le \sigma_1(v_i) \wedge \sigma_1(v_j)$ and $\mu_2(v_i, v_j) \le \sigma_2(v_i) \vee \sigma_2(v_j)$ such that $0 \le \mu_1(v_i, v_j) + \mu_2(v_i, v_j) \le 1$, for every $(v_i, v_j) \in E$.

Definition 2.2. In an intuitionistic fuzzy graph $G = ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$, the weight of a vertex $u \in V$ is defined by

$$w(u) = \frac{1 + \sigma_1(u) - \sigma_2(u)}{2}$$

and also the weight of an edge $e = (u, v) \in E$ is defined by

$$w(e) = \frac{1 + \mu_1(u, v) - \mu_2(u, v)}{2}$$

Definition 2.3. Let $G: ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is an intuitionistic fuzzy graph, then the distance $d(u, v)$ between two of its vertices u and v is the length of shortest path between them.

$$d(u, v) = \wedge \left(\sum_{i,j} w(u_i, v_j) \right)$$

Definition 2.4. Let V be an arbitrary non-empty set and $IF(V)$ be the intuitionistic fuzzy subsets of X. $IF(V) = \{(\sigma_1(v), \sigma_2(v)) \in I^V \times I^V : 0 \le \sigma_1(v) + \sigma_2(v) \le 1, \forall\, v \in V\}$, where $I^V = [0,1]$.

Definition 2.5. [12] Let $G: ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph of the graph $G: (V, E)$, and then a function $d: IF(V) \times IF(V) \to [0, \infty)$ is said to be metric in intuitionistic fuzzy graph if it satisfied the following conditions: For all $u, v, w \in IF(V)$

1. $d(u, v) > 0$ if $\sigma_1(u) \neq \sigma_1(v); \sigma_2(u) \neq \sigma_2(v)$
2. $d(u, v) = 0$ iff $\sigma_1(u) = \sigma_1(v) = 0; \sigma_2(u) = \sigma_2(v) = 1$
3. $d(u, v) = d(v, u)$
4. $d(u, w) \le d(u, v) + d(v, w)$

Then d is called an intuitionistic fuzzy graph metric on V. The function $d(u, v)$ is the length of the shortest path between u and v. The pair $(IF(V), d)$ is called an intuitionistic fuzzy graph metric space.

Example 2.1. Consider a connected intuitionistic fuzzy graph of the graph $G: (V, E)$. The function "d" defined by

$$d\Big((\sigma_1(u), \sigma_2(u)), (\sigma_1(v), \sigma_2(v))\Big) = \begin{cases} 1 & if \;\; \sigma_1(u) \neq \sigma_1(v) \; and \; \sigma_2(u) \neq \sigma_2(v) \\ 0 & if \;\; \sigma_1(u) = \sigma_1(v) \; and \; \sigma_1(u) = \sigma_1(v) \end{cases}$$

is a metric in intuitionistic fuzzy graph and it is called discrete metric.

Example 2.2. Consider the intuitionistic fuzzy graph as given in Fig.1,

$IF(V) = \{a(0.4, 0.5), b(0.6, 0.2), c(0.5, 0.4), d(0.3, 0.2)\}$ $d(a, b) = 0.45 > 0$ (since $a \neq b$ is the shortest path) $d(a, b) = d(b, a)$, since shortest path between a and b = shortest path between b and a

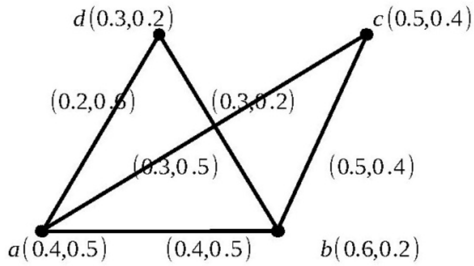

Figure 1:

$d(a,b) \leq d(a,c) + d(c,b) \Longrightarrow 0.45 \leq 0.4 + 0.55$, $d(a,b) \leq d(a,d) + d(d,b) \Longrightarrow 0.45 \leq 0.3 + 0.4.$ Hence, $(IF(V), d)$ is an intuitionistic fuzzy graph metric space.

Example 2.3. Let $G: ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be a connected intuitionistic fuzzy graph of the graph $G: (V, E)$ and d be a metric on intuitionistic fuzzy graph, and then $\bar{d}(u, v) = \min\{d(u, v), 1\}$ is also a metric on intuitionistic fuzzy graph.

Theorem 2.1. Let $G: ((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be a connected intuitionistic fuzzy graph then distance between vertices is metric in intuitionistic fuzzy graph.

Definition 2.6. Let $G: \big((\sigma_1, \sigma_2), (\mu_1, \mu_2)\big)$ be a connected intuitionistic fuzzy graph with $G: (V, E)$ and $d: IF(V) \times IF(V) \to [0, \infty)$ be a metric in intuitionistic fuzzy graph. The sequence (v_n) of vertices of $IF(V)$ is said to converge to a vertex $v \in IF(V)$, if for each $\epsilon > 0$, there exists a positive integer m such that $d(v_n, v) < \epsilon$ for all $n \geq m$.

$$i.e. \lim_{n \to \infty} d(v_n, v) \to 0$$

Example 2.4.

1. Let us consider the sequence $v_n = \big(\sigma_1(v_n), \sigma_2(v_n)\big)$ of vertices in intuitionistic fuzzy graph where $\sigma_1(v_n) = \left(\frac{1}{n}\right)$ and $\sigma_2(v_n) = \left(\frac{n-1}{n}\right)$. Then the sequence converges to $v = \big(\sigma_1(v), \sigma_2(v)\big)$ where $\sigma_1(v) = 0$ and $\sigma_2(v) = 1$ as $n \to \infty$.

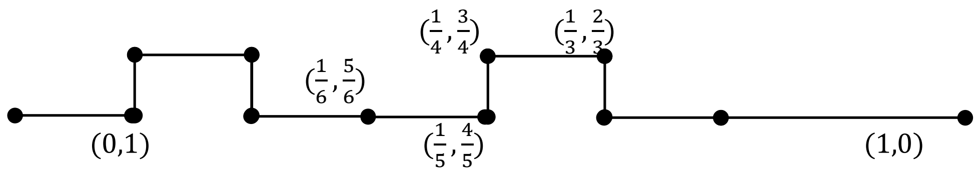

Figure 2:

Definition 2.7. A sequence $\{v_n\}$ of vertices of $IF(V)$ is said to be a Cauchy sequence if for each $\epsilon > 0$, there exists a positive integers N such that

$$d(v_n, v_m) < \epsilon \text{ for all } n, m \geq N,$$

i.e. $d(v_n, v_m) \to 0 \; as \; n \to \infty$.

Definition 2.8. An intuitionistic fuzzy graph metric space $((IF(V), d)$ is said to be complete intuitionistic fuzzy graph metric space if every Cauchy sequence is converges to a vertex of $IF(V)$.

Theorem 2.2. Every intuitionistic fuzzy graph metric space is always complete.

Definition 2.9. Let $(IF(V), d_1)$ and $(IF(V), d_2)$ be any two intuitionistic fuzzy graph metric space. A function $f: IF(V) \to IF(V)$ is said to be continuous at a vertex $u \in IF(V)$ if for given $\epsilon > 0$, there exists a $\delta > 0$ such that $d_2(f(u), f(v)) < \epsilon$ whenever $d_1(u, v) < \delta$, for all $u, v \in IF(V)$

Definition 2.10. Let $(IF(V), d_1)$ and $(IF(V), d_2)$ be any two intuitionistic fuzzy graph metric space. A function $f: IF(V) \to IF(V)$ is said to be uniformly continuous if for given $\epsilon > 0$, there exists a $\delta > 0$ such that $d_2(f(u), f(v)) < \epsilon$ whenever $d_1(u, v) < \delta$, $\forall \; u, v \in IF(V)$.

Definition 2.11. Let $(IF(V), d_1)$ and $(IF(V), d_2)$ be any two intuitionistic fuzzy graph metric space. A function $f: IF(V) \to IF(V)$ is said to be an isometric if $d_1(u, v) = d_2(f(u), f(v))$ for all $u, v \in IF(V)$.

3. Contraction, ϵ −contractive mapping and ϵ −chaniable on intuitionistic fuzzy graph metric space

Definition 3.1. A mapping T of an intuitionistic fuzzy graph metric space $(IF(V), d)$ into itself is said to satisfy a Lipschitz condition with Lipschitz constant K if $d\big(T(v), T(v)\big) \leq Kd(v, u)$ for all $u, v \in IF(V)$. If this condition is satisfied with a Lipschitz constant K such that $0 \leq K < 1$, then T is called contraction mapping.

Definition 3.2. Let $(IF(V), d)$ be an intuitionistic fuzzy graph metric space and $\epsilon > 0$. A finite sequence $v_0, v_1, \ldots, v_n$ of vertices of $IF(V)$ is called an ϵ −chain joining v_0 and v_n if $d(v_{i-1}, v_i) < \epsilon \quad (i = 1,2,3 \ldots, n)$. The intuitionistic fuzzy graph metric space $(IF(V), d)$ is said to be ϵ- chainable if for each pair $\{(v, u)\}$ of vertices, there exists an ϵ-chain joining v and u.

Definition 3.3. A mapping T of an intuitionistic fuzzy graph metric space $(IF(V), d)$ into itself is said to be contractive if $d\big(T(v), T(u)\big) < d(v, u), \sigma_1(u) \neq \sigma_2(u); \sigma_1(v) \neq \sigma_2(v)$; $\forall \; u, v \in IF(V)$ and is said to be ϵ contractive if

$$0 < d(v, u) < \epsilon \Longrightarrow d\big(T(v), T(u)\big) \leq d(v, u)$$

Definition 3.4. A mapping T of an intuitionistic fuzzy graph metric space $(IF(V), d_1)$ into an intuitionistic fuzzy graph metric space $(IF(V), d_2)$ is said to be continuous if for every convergence sequence $\{v_n\}$ of $IF(V)$, $\lim_{n\to\infty} T(v_n) = T \lim_{n\to\infty} (v_n)$.

4. Main results

Theorem 4.1. Let T be a contractive mapping of an intuitionistic fuzzy graph metric space $(IF(V), d)$ into itself. Then

a) T has a unique fixed point u in $IF(V)$.

b) If u_0 is an arbitrary vertex of $IF(V)$ and $\{u_n\}$ is defined inductively by $u_{n+1} = T(u_n)$ $(n = 0,1,2 \dots)$ then $\lim_{n\to\infty} u_n = u$ and $d(u_n, u) \le \frac{K^n}{1-K} d(u_1, u_0)$ where K is a Lipschitz constant for T.

Proof: Let K be a Lipschitz constant for T with $0 \le K < 1$.
Let $u_0 \in IF(V)$ and let u_n be the sequence defined by $u_{n+1} = T(u_n)$ $(n = 0,1,2,3, \dots)$

We have $d(u_{r+1}, u_{s+1}) = d\big(T(u_r), T(u_s)\big) \le K d(u_r, u_s)$ (1)

and so, $d(u_{r+1}, u_r) \le K^r d(u_1, u_0)$ (2)

Given p, q we have by Eqs. (1) and (2),

$$\begin{aligned} d(u_p, u_q) &\le K^q d(u_{p-q}, u_0) \\ &\le K^q [d(u_{p-q}, u_{p-q-1}) + d(u_{p-q-1}, u_{p-q-2}) + \\ &\qquad \dots + d(u_1, u_0)] \\ &\le K^q \{K^{p-q-1} + K^{p-q-2} + \dots + K + 1\}\, d(u_1, u_0) \\ &\le \frac{K^q}{1-K} d(u_1, u_0) \end{aligned}$$

Since the right-hand side tends to zero as $q \to \infty$, it follows that $\{u_n\}$ is a Cauchy sequence, and since $(IF(V), d)$ is an intuitionistic fuzzy graph metric space, it is complete,

$\therefore$ The sequence of vertices $\{u_n\}$ converges to a vertex u of $IF(V)$.

Since, $d\big(u_{n+1}, T(u)\big) \le K d(u_n, u) \to 0$ as $n \to \infty$.

$$T(u) = \lim_{n\to\infty} u_{n+1} = u.$$

Therefore,
$$\begin{aligned} d(u, u_n) &\le d(u, u_p) + d(u_p, u_n) \\ &\le d(u, u_p) + \frac{K^n}{1-K} d(u_1, u_0) \text{ for } n < p \quad \text{[by 3]} \end{aligned}$$

Letting $p \to \infty$, we obtain

$$d(u_n, u) \le \frac{K^n}{1-K} d(u_1, u_0).$$

Uniqueness:
Let v be another fixed vertex of T, Then $T(u) = u, T(v) = v$

$$\begin{aligned} d(u, v) &= d\big(T(u), T(v)\big) \\ &\le K d(u, v) \\ &\Rightarrow d(u, v) = 0,\ K < 1 \\ &\Rightarrow u = v \end{aligned}$$

Theorem 4.2. Let $(IF(V), d)$ be an intuitionistic fuzzy graph metric space and let T and $T_n (n = 1,2,3, \dots)$ be contractive condition mapping of $IF(V)$ into itself with the same Lipschitz constant $0 \le K < 1$ and with fixed vertices u and u_n respectively. Suppose that $\lim_{n\to\infty} T_n(v) = T(v)$ for every $v \in IF(V)$. Then, $\lim_{n\to\infty} u_n = u$.
By inequality in Theorem 4.1, we have for each $r = 1,2, \dots$

$$d(u_r, T_r^n(u_0)) \le \frac{K^n}{1-K} d(T_r(v_0), v_0), \quad v_0 \in IF(V)$$

Proof: Setting $n = 0$ and $v_0 = u$, we have,

$$d(u_r, u) \leq \frac{1}{1-K} d(T_r(u), u)$$
$$= \frac{1}{1-K} d(T_r(u), T(u))$$

But $d\big(T_r(u), T(u)\big) \to 0 \ as\ r \to \infty$

Hence, $\lim_{r\to\infty} d(u_r, u) = 0$.

Theorem 4.3. Let T be a continuous mapping of an intuitionistic fuzzy graph metric space $(IF(V), d)$ into itself such that T^k is a contraction mapping of $IV(F)$ for some positive integer k. Then T has a unique fixed vertex in $IF(V)$.

Proof: T^K has a unique fixed vertex u in $IF(V)$ and $u = \lim_{n\to\infty}\left(T^k\right)^n (x_0)$, $\quad x_0 \in IF(V)$ arbitrary.

Also $\lim_{n\to\infty}\left(T^k\right)^n T(x_0)) = u$.

Hence, $u = \lim_{n\to\infty}\left(T^k\right)^n T(x_0)$

$= \lim_{n\to\infty} T\left(T^k\right)^n (x_0)$

$= T\Big(\lim_{n\to\infty}\left(T^k\right)^n (x_0)\Big)$ (by continuity)

$= T(u)$

The uniqueness of the fixed vertex of T is obvious, since each fixed vertex of T is also fixed vertex of T^k.

Theorem 4.4. Let T be a mapping of a complete $\varepsilon -$ chainable intuitionistic fuzzy graph metric space $(IF(V), d)$ into itself and suppose that there is a real number, $0 \leq K < 1$ such that $d(x, y) < \varepsilon \ \Rightarrow d\{T(x), T(y)\} \leq Kd(x, y)$. Then T has a unique fixed vertex u in $IF(V)$, and $u = \lim_{n\to\infty}(T^n(x_0))$. where x_0 is an arbitrary vertex of $IF(V)$.

Proof: Let $(IF(V), d)$ be a $\varepsilon -$ chainable, we define for $x, y \in IF(V)$, $d_\varepsilon(x, y) = \inf \sum_{i=1}^{n} d(x_{i-1}, x_i)$ where the infimum is taken over all $\varepsilon -$chains $x_0, x_1, \dots \dots \dots \dots . x_n$ joining $x_0 = x$ and $x_n = y$.

Then, d is a distance function on $IF(V)$ satisfying

a) $d(x, y) \leq d_\varepsilon(x, y)$

b) $d(x, y) = d_\varepsilon(x, y)$ for $d(x, y) < \varepsilon$

From (b), it follows that a sequence $\{x_n\}$, where $x_n \in IF(V)$ is a Cauchy sequence with respect to d_ε if and only if it is a Cauchy sequence with respect to d and is convergent with respect to d_ε if and only if it converges with respect to d. Hence $(IF(V), d_\varepsilon)$ being complete, $(IF(V), d_\varepsilon)$ is also complete metric space. Moreover, T is contraction mapping w.r.t. d_ε .

Given $x, y \in IF(V)$ and any ε-chain $x_0, x_1, \dots \dots \dots x_n$ with $x_0 = x, x_n = y$, we have $d(x_{i-1}, x_i) < \varepsilon(n = 1,2,3, \dots \dots \dots . . , n)$

So that,

$$d\{T(x_i - 1), T(x_i)\} \leq Kd(x_{i-1}, x_i) < \varepsilon(i = 1,2,3, \dots \dots \dots . . n)$$

Hence $T(x_0), T(x_1), \dots \dots T(x_n)$ is an ε- chain joining $T(x)$ and $T(y)$ and

$$d_\varepsilon(T(x), T(y)) \leq \sum_{i=1}^{n} d(T(x_{i-1}), T(\ x_i)) \leq K \ \sum_{i=1}^{n} d(x_{i-1}, x_i)$$

$x_0, x_1, \dots \dots \dots x_n$ being an arbitrary ε-chain, we have
$d_\varepsilon(T(x), T(y)) \leq K d_\varepsilon(x, y)$ and T has a unique fixed vertex $u \in IF(V)$ given by
$\lim_{n\to\infty} d_\varepsilon(T^n(x_0), T(u)) = 0$ for $x_0 \in IF(V)$ arbitrary. (3)
Eq. (3) implies that $\lim_{n\to\infty} d_\varepsilon\big(T^n(x_0), T(u)\big) = 0$

Theorem 4.5. Let T be an ε −contractive mapping of an intuitionistic fuzzy graph metric space $(IF(V), d)$ into itself and let x_0 be a vertex of $IF(V)$ such that the sequence $\{T^n(x_0)\}$ a subsequence convergent to a vertex u of $IF(V)$. Then, u is a periodic vertex of T, i.e., there is a positive integer k such that $T^k(u) = u$.
Proof: Let $\{n_i\}$ be a strictly increasing sequence of positive integer such that
$\lim_{i\to\infty} T^{n_i}(x_0) = u$ and let $x_i = T^{n_i}(x_0)$.
There exist N such that $d(x_i, u) < \frac{\varepsilon}{4}$ for $i \geq N$.
Choose any $i \geq N$, $K = n_{i+1} - n_i$. Then,

$$\begin{aligned} d\big((x_{i+1}), T^K(u)\big) &= d\big(T^K(x_i), T^K(u)\big) \\ &\leq d(x_i, u) < \frac{\varepsilon}{4} \end{aligned}$$

and $d\big(T^k(u), u\big) \leq d\big(T^k(u), x_{i+1}\big) + d(x_{i+1}, u) < \frac{\varepsilon}{2}$
Suppose that $v = T^k(u) \neq u$. Then, T being ε-contractive

$$d\big(T(u), T(v)\big) < d(u, v)$$

or

$$\frac{d\big(T(u), T(v)\big)}{d(u,v)} < 1$$

The function $(x, y) \to \frac{\big(T(u), T(v)\big)}{d(u,v)}$ is continuous at (u, v). So there exists $\delta, K > 0$ with $0 < K < 1$ such that $d(x, u) < \delta, d(y, v) < \delta$.
Implies that $d\big(T(x), T(y)\big) \leq K d(x, y)$.
As $\lim_{r\to\infty} T^k(x_r) = T^k(u) = v$, there exists $N' \geq N$ such that
$d(x_r, u) < \delta, d(T(x_r), v) < \delta$ for $r \geq N'$ and so $d\left(T(x_r), TT^k(x_r)\right) \leq Kd\left(x_r, T^k(x_r)\right)$ (4)

$$\begin{aligned} d\{x_r, T^k(x_r)\} &\leq d(x, u) + d\left(u, T^k(u)\right) + d\left(T^k(u), T^k(x_r)\right) \\ &< \frac{\varepsilon}{4} + \frac{\varepsilon}{2} + \frac{\varepsilon}{4} = \varepsilon \text{ for } r \geq N' > N \end{aligned} \quad (5)$$

From Eqs. (4) and (5),
$d\left(T(x_r), TT^k(x_r)\right) \leq Kd\left(x_r, T^k(x_r)\right) < \varepsilon$ for $r \geq N'$ and so T being ε −contractive,
$d\left(T^p(x_r), T^pT^k(x_r)\right) < Kd\left(x_r, T^k(x_r)\right)\ for\ r \geq N', p > 0$ (6)
Setting $p = n_{r+1} - n_r$ in Eq. (6) $d\left(x_{r+1}, T^k(x_{r+1})\right) < Kd\left(x_r, T^k(x_r)\right)$ for any $r \geq N'$
Hence, $d\left(x_s, T^k(x_s)\right) < K^{s-r}\, d\left(x_r, T^k(x_r)\right) < K^{s-r}\varepsilon$
and $d(u, v) < d(u, x_s) + d\left(x_s, T^k(x_s)\right) + d\left\{T^k(\sigma(x_s)\right), \sigma(v)\} \to 0\ as\ n \to \infty$
This contradicts the assumption that $d(u, v) > 0$.
Thus, $u = v = T^k(u)$.

Theorem 4.6. Let T be a contractive mapping of an intuitionistic fuzzy graph metric space $(IF(V), d)$ into itself and let x_0 be a vertex of $IF(V)$ such that the sequence

$\{T^n(x_0)\}$ has a subsequence convergent to a vertex u of $IF(V)$, and then u is a fixed vertex of T and is unique.

Proof: By Theorem 4.5, there exist an integer $k > 0$ such that $T^k(u) = u$

Suppose that $v = T(u) \neq u$.

Then, $T^k(u) = u, T^k(v) = v$ and $d(u,v) = d\left(T^k(u), T^k(v)\right) < d(u,v)$

$T^k(v) < d(u,v),$ since T is contractive.

As this is impossible, $u = v$ is a fixed vertex. The uniqueness is obvious.

5. Conclusion

In this paper, the contractive mapping on intuitionistic fuzzy graph metric space is introduced and studied the concepts of isometric function, Lipschitz condition, ϵ −chain, contractive mapping, ϵ − contractive mapping on intuitionistic fuzzy graph metric space. In future, we will develop some other contraction like kannan-contraction, cyclic contraction, etc. on intuitionistic fuzzy graph metric space.

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