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Prime Labeling to Drums Graphs

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Abstract. In this paper, we introduce the new graph namely drums graph D_n. We investigate prime labeling for some graphs related to drums graph. We discuss prime labeling in the context of some graph operations namely duplication, fusion, switching in drums graph D_n. We prove that the duplication of any vertex of drums graph are prime graph. We prove that the identifying any vertex of drums graph are prime graph. We prove that the switching of an apex vertex and other vertex of drums graphs are prime graph and also apply coloring of the graph operations of the drums graph is satisfying coloring condition.

Keywords: Prime Labeling, Prime Graph, Drums Graph, Duplication, Fusion, Switching, coloring.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

We consider only simple, finite, undirected and non – trivial graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. For notations and terminology we refer to Bondy and Murthy [8]. Many researchers have studied prime graph for example in F [7] have proved that the path P_n on n vertices is a prime graph. In Deretsky [11] have proved that the Cycle C_n on n vertices is a prime graph. For latest survey on graph labeling we refer to [9]. In Edward Samuel and Kalaivani [1] have proved the Prime labeling for some octopus related graphs. In Edward Samuel and Kalaivani [2] have proved the Prime labeling for some planter related graphs. In Edward Samuel and Kalaivani [3] have proved the Prime labeling for some vanessa related graphs. In Edward Samuel and Kalaivani [4] have proved the square sum labeling for some lilly related graphs. Sugumaran and Mohan [5] have discussed further results on prime cordial labeling. Sunoj and Mathew Varkey [6] have investigated the ADCSS-labeling for some middle graphs. Rajesh Kumar and Mathew Varkey [12] have discussed Gaussian neighborhood prime labeling of some classes of graphs and cycles.

2. Preliminary definitions

Definition 2.1. [11] Let $G = (V(G), E(G))$ be a graph with p vertices. A bijection $f : V(G) \to \{1, 2, \dots, p\}$ is called a *prime labeling* if for each edge $e = uv, \gcd\{f(u), f(v)\} = 1$. A graph which admits prime labeling is called a *prime graph*.

Definition 2.2. [2] *Duplication* of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex $v_k{}'$ with $N(v_k{}') = N(v_k)$. In other words a vertex $v_k{}'$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_k{}'$ also.

Definition 2.3. [4] Let *u* and *v* be two distinct vertices of a graph G. A new graph G_1 is constructed by *identifying(fusing)* two vertices *u* and *v* by a single vertex *x* is such that every edge which was incident with either *u* or *v* in G is now incident with *x* in G_1.

Definition 2.4. [4] A *vertex switching* G_v of a graph G is obtained by taking a vertex *v* of G, removing all the entire edges incident with *v* and adding edges joining *v* to every vertex which are not adjacent to *v* in G.

Definition 2.5. [10] A k – coloring of a graph $G = (V, E)$ is a function $c : V \to C$, where $|c| = k$. (Most often we use $c = [k]$). Vertices of the same color form a color class. A coloring is *proper* if adjacent vertices have different colors. A graph is *k – colorable* if there is a proper *k* – coloring. The chromatic number $\chi(G)$ of a graph G is the minimum *k* such that G is *k* – colorable.

3. Prime labeling to drums graphs

Drums graph 3.1. The *Drums graph* $D_n, n \geq 3$ can be constructed by two cycle graphs $2C_n, n \geq 3$ joining two path graphs $2P_n, n \geq 2$ with sharing a common vertex. i.e., $D_n = 2C_n + 2P_n$.

Example 3.2.

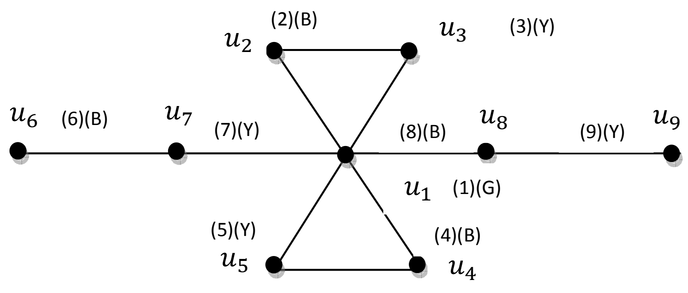

Figure 3.1: Drums graph D_3.

Theorem 3.3. The Drums graph $D_n, n \geq 3$ is a prime graph, where *n* is any positive integer.

Proof: Let D_n be the Drums graph with vertices $\{u_1, u_2, \dots, u_{4n-3}\}$. Here $|V(D_n)| = 4n - 3$, where *n* is any positive integer.

Define a labeling $f : V(D_n) \to \{1, 2, \dots, 4n - 3\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \le i \le 4n-3$$

Clearly vertex labels are distinct. Then for any edge $e = u_1 u_i \in D_n$, $\gcd\big(f(u_1), f(u_i)\big) = \gcd\big(1, f(u_i)\big) = 1$ and for any edge $e = u_i u_{i+1} \in D_n$, $\gcd\big(f(u_i), f(u_{i+1})\big) = 1$ for $2 \le i \le 4n-3$ Since it is consecutive positive integers. Thus labeling defined above gives a prime labeling for a graph D_n. Thus D_n is a prime graph.

Example 3.4.

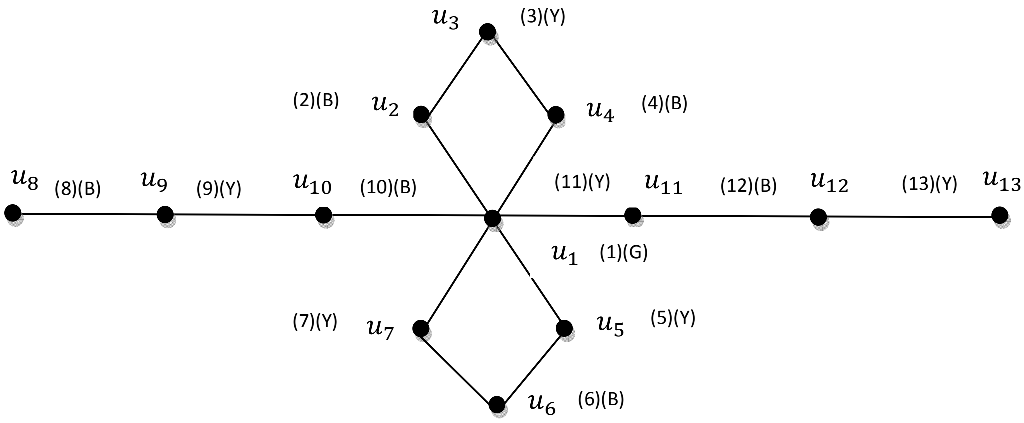

Figure 3.2: Prime labeling for D_4.

Theorem 3.5. The graph obtained by duplication of any vertex u_k to $u_k{}'$ of drums graph $D_n, n \ge 3$ is a prime graph, where n is any positive integer.

Proof: Let G be the graph of drums graph $D_n, n \ge 3$. Let u_k be the vertex of the drums graph $D_n, n \ge 3$ and $u_k{}'$ be its duplicated vertex and G_k be the graph resulted due to duplication of the vertex u_k in $D_n, n \ge 3$, where n is any positive integer. Let $V(D_n) = \{u_1, u_2, \dots, u_{4n-3}\}$. Here $|V(G_k)| = 4n-2$.

We define a labeling $f : V(G_k) \to \{1, 2, \dots, 4n-2\}$ as follows.

$$f(u_i) = i \quad \text{for } 1 \le i \le 2n$$

$$f(u_k{}') = 11, \quad f(u_i) = i+1 \quad \text{for } 2n+1 \le i \le 4n-3$$

Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_k is a prime graph.

Example 3.6.

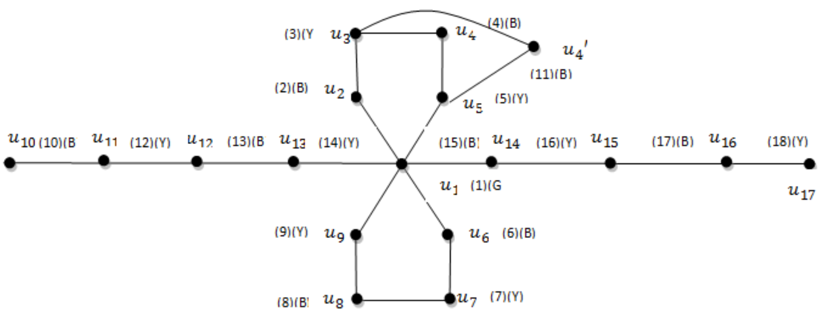

Figure 3.3: Duplication of u_4 in D_5.

Theorem 3.7. The graph obtained by identifying any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of drums graph $D_n, n \geq 3$ is a prime graph, where n is any positive integer.

Proof: Let $D_n, n \geq 3$ be the drums graph with vertices $\{u_1, u_2, \ldots, u_{4n-3}\}$ and the vertex u_i be the fused with u_k. Denote the resultant graph as G_k. Here we note that $|V(G_k)| = 4n - 4$.

Define a labeling $f: V(G_k) \to \{1,2, \ldots, 4n-4\}$ as follows

$$f(u_i) = i \qquad \text{for } 1 \leq i \leq 4n-5$$

$$f(u_{20} = u_{21}) = 4n - 4$$

Then f admits prime labeling. According to this pattern the vertices are labeled such that for any edge $e = u_i u_k \in G_k$, $\gcd\big(f(u_i), f(u_k)\big) = 1$.Clearly vertex labels are distinct. Thus we proved that the graph under consideration admits prime labeling. That is, the graph obtained by fusing (identifying) any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of drums graph $D_n, n \geq 3$ is a prime graph.

Example 3.8.

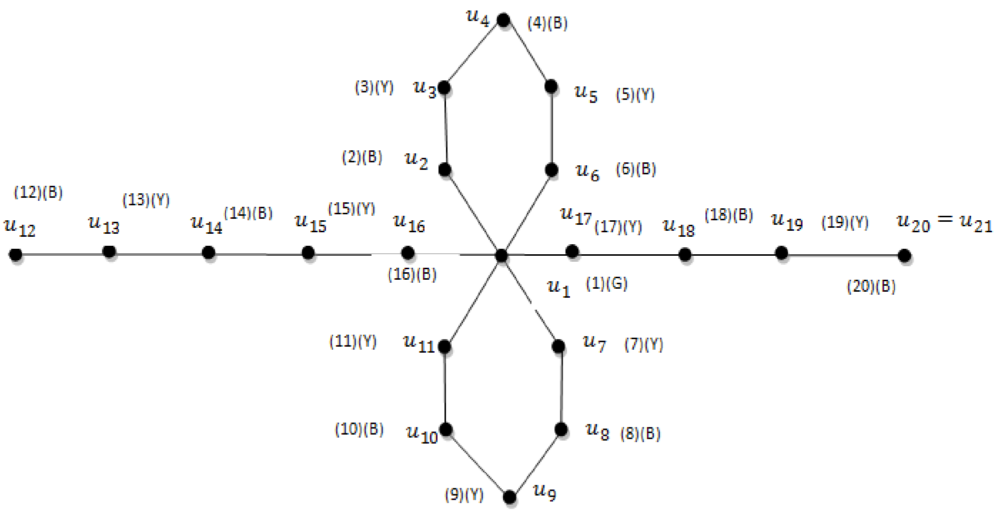

Figure 3.4: Fusion of u_{20} and u_{21} in D_6.

Theorem 3.9. The switching of any vertex u_k in a drums graph $D_n, n \geq 3$ produces a Prime graph, where n is any positive integer.

Proof: Let $G = D_n$ and $\{u_1, u_2, \ldots, u_{4n-3}\}$ be the successive vertices of drums graph $D_n, n \geq 3$ and G_u denotes the graph obtained by a vertex switching of G with respect to the vertex u. It is obvious that $|V(G_u)| = 4n - 3$.

Define a labeling $f: V(G_u) \to \{1,2, \ldots, 4n-3\}$ as follows

$$f(u_i) = i \quad \text{for } 1 \leq i \leq n+1$$

$$f(u_i) = i + 1 \text{ for } n+2 \leq i \leq 4n-4$$

$$f(u_9) = 5$$

Then for any edge $e = u_i u_{i+1} \in G_u$, $\gcd\big(f(u_i), f(u_{i+1})\big) = 1$ and for any edge $e = u_1 u_i \in G_u$, $\gcd\big(f(u_1), f(u_i)\big) = \gcd\big(1, f(u_i)\big) = 1$. Clearly vertex labels are distinct. Then f admits prime labeling. Thus G_u is a prime graph. That is, the switching of any vertex u_k in a drums graph $D_n, n \geq 3$ produces a Prime graph, where n is any positive integer.

Example 3.10.

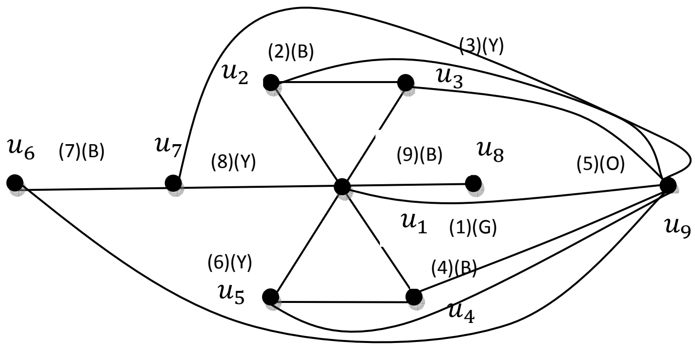

Figure 3.5: Switching the vertex u_9 in D_3.

Theorem 3.11. The switching of an apex vertex u_1 in a drums graph $D_n, n \geq 3$ produces a Prime graph, where n is any positive integer.

Proof: Let $G = D_n$ and $\{u_1, u_2, \dots, u_{4n-3}\}$ be the successive vertices of drums graph $D_n, n \geq 3$ and G_u denotes the graph obtained by an apex vertex switching of G with respect to the vertex u_1. It is obvious that $|V(G_u)| = 4n - 3$. Without loss of generality, we initiate the labeling from u_1 and proceed in the clock – wise direction.

Define a labeling $f: V(G_u) \to \{1,2, \dots, 4n - 3\}$ as follows

$$f(u_i) = i \quad \text{for} \quad 1 \leq i \leq 4n - 3$$

Then for any edge $e = u_i u_{i+1} \in G_u$, $\gcd(f(u_i), f(u_{i+1})) = 1$ and for any edge $e = u_1 u_i \in G_u$, $\gcd(f(u_1), f(u_i)) = \gcd(1, f(u_i)) = 1$. Clearly vertex labels are distinct. Then f is a prime labeling and consequently G_u is a prime graph. That is, the switching of an apex vertex u_1 in a drums graph $D_n, n \geq 3$ produces a Prime graph and it is a disconnected graph.

Example 3.12.

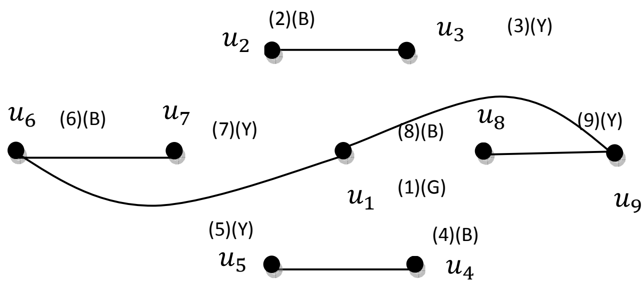

Figure 3.6: Switching an apex vertex u_1 in D_3

4. Applications

The field of graph theory plays a vital role in various fields. One of the important areas in graph theory is graph labeling which is used in many applications like coding theory, radar, astronomy, circuit design, missile guidance, communication network addressing, x-ray crystallography, data base management. Graph labeling is most useful to computer science like data mining, image processing, cryptography, software testing, information security, communication networks etc….

5. Conclusion

In this paper, we proved that the Drums graph D_n, duplication of the Drums graph D_n, fusing of the Drums graph D_n, switching of the Drums graph D_n are prime graphs and also applied coloring condition to Drums graph D_n.

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