Annals of Pure and Applied Mathematics
Vol. 16, No. 2, 2018, 295-304
ISSN: 2279-087X (P), 2279-0888(online)
Published on 17 February 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v16n2a5

Annals of
Pure and Applied Mathematics

A Note on Fuzzy Bi-Ideals in Ternary Semigroups

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Received 2 January 2018; accepted 26 January 2018

Abstract. The prime fuzzy bi-ideals of semigroups are introduced by Shabir, Jun and Bano. They characterized those semigroups for which each fuzzy bi-ideal is semiprime and also characterized those semigroups for which each fuzzy bi-ideal is strongly prime. In this paper, we prove some properties of fuzzy bi-ideals in ternary semigroups and the relation between fuzzy quasi ideals and fuzzy bi- ideals is considered.

Keywords: Ternary semigroup, fuzzy set, fuzzy ternary semigroup, fuzzy ideal, fuzzy quasi ideal, fuzzy bi-ideal, characteristic function.

AMS Mathematics Subject Classification (2010): 47D03

1. Introduction

Lehmer introduced the ternary algebraic system in 1932, and after such structures were studied by Kasner. In 1965, ideal theory in ternary semigroups studied by Sioson [16]. After the introduction of fuzzy sets by Zadeh [9] reconsideration of the concept of classical mathematics began. Fuzzy set has an important impact over the field of mathematical research in both theory and application. It has found manifold applications in mathematics and related areas. Kuroki introduced and studied the notion of fuzzy semigroups. He also studied the concept of fuzzy bi-ideals [6] (1979) and fuzzy quasi-ideals (1982) of semigroups. Since then many papers have been published in the field of fuzzy algebra [1-5,7,8,10-15,17,18]. Many researchers conducted the researches on the generalizations of the notions of fuzzy sets with huge applications in computer, logics and many branches of pure and applied mathematics.

2. Basic definitions and preliminaries

Definition 2.1. A non-empty set T is said to be ternary semigroup if there exists a ternary operation $\cdot : T\times T\times T \to T$ written as $(a,b,c) \to a.b.c$ satisfies the following identity $(abc)de = a(bcd)e = ab(cde)$ for all $a, b, c, d, e \in T$.

Example 1. Let $T = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ then T is a ternary semigroup under usual multiplication.

Definition 2.2. A non-empty subset A of a ternary semigroup T is called a ternary subsemigroup of T if $AAA \subseteq A$.

Definition 2.3. A non-empty subset A of a ternary semigroup T is called a quasi ideal in T if
$(ATT) \cap (TAT) \cap (TTA) \subseteq A$ and $(ATT) \cap (TTATT) \cap (TTA) \subseteq A$.

Definition 2.4. A ternary sub semigroup A of a ternary semigroup T is said to be a bi-ideal in T if $ATATA \subseteq A$.

Definition 2.5. Let T be a non-empty set. A fuzzy subset of a ternary semigroup T is a function $\mu : T \to [0,1]$.

Definition 2.6. Let μ be a fuzzy subset of a non-empty set T for any $t \in [0,1]$, the subset $\mu_t = \{ x \in T : \mu(x) \geq t \}$ of T is called a level set of μ.

Definition 2.7. For any two fuzzy subsets μ_1 and μ_2 of a non-empty set T, the union and the intersection of μ_1 and μ_2 denoted by $\mu_1 \cup \mu_2$ and $\mu_1 \cap \mu_2$ are fuzzy subsets of T and defined as
$(\mu_1 \cup \mu_2)(x) = \max\{\mu_1(x), \mu_2(x)\} = \mu_1(x) \vee \mu_2(x)$
and $(\mu_1 \cap \mu_2)(x) = \min\{\mu_1(x), \mu_2(x)\} = \mu_1(x) \wedge \mu_2(x)$ for all $x \in T$.
Where $\vee$ denotes maximum or supremum and $\wedge$ denotes minimum or infimum.

Definition 2.8. Let μ_1, μ_2 and μ_3 are any three fuzzy sets of a ternary semigroup T. Then their fuzzy product $\mu_1 \circ \mu_2 \circ \mu_3$ is defined by

$$(\mu_1 \circ \mu_2 \circ \mu_3)(a) = \begin{cases} \bigvee_{a=xyz} \{\mu_1(x) \wedge \mu_2(y) \wedge \mu_3(z)\} & if\ a\ is\ \text{expressible}\ as\ a=xyz \\ & for\ all\ x,y,z \in T \\ 0 & otherwise \end{cases}$$

Definition 2.9. A fuzzy set μ of a ternary semigroup T is called a fuzzy ternary subsemigroup of T if $\mu(xyz) \geq \{\mu(x) \wedge \mu(y) \wedge \mu(z)\}$ for all $x, y, z \in T$.

Definition 2.10. A fuzzy ternary sub semigroup μ of a ternary semigroup T is called a fuzzy bi-ideal in T if $\mu(xmynz) \geq \{\mu(x) \wedge \mu(y) \wedge \mu(z)\}$ for all $x, y, z\ m, n \in T$.

Definition 2.11. A fuzzy set μ of a ternary semigroup T is called a fuzzy left (right, lateral) ideal in T if $\mu(xyz) \geq \mu(z)$, ($\mu(xyz) \geq \mu(x)$, $\mu(xyz) \geq \mu(y)$) for all $x, y, z \in T$..

Definition 2.12. A fuzzy set μ of a ternary semigroup T is a fuzzy ideal in T if it is fuzzy left, right and lateral ideal in T.

Definition 2.13. Let A be a non-subset of a ternary semigroup T. Then the characteristic function of A is defined by $C_A(x) = \begin{cases} 1 & if \quad x \in A \\ 0 & if \quad x \notin A \end{cases}$

We denote the characteristic function C_T *of* T . i.e., $T = C_T$ thus $T(x) = 1$ for all $x \in T$.

Definition 2.14. A fuzzy set μ of a ternary semigroup T is called a fuzzy quasi ideal of T if $(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T) \subseteq \mu$ and

$$(T \circ T \circ \mu) \cap (T \circ T \circ \mu \circ T \circ T) \cap (\mu \circ T \circ T) \subseteq \mu.$$

i.e., $[(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T)](a) \leq \mu(a)$ and

$$[(T \circ T \circ \mu) \cap (T \circ T \circ \mu \circ T \circ T) \cap (\mu \circ T \circ T)](a) \leq \mu(a)$$

3. Main results

Theorem 3.1. A non-empty subset A of a ternary semigroup T is a bi- ideal in T if and only if characteristic function C_A of A is a fuzzy bi - ideal in T.

Proof: Let C_A is a characteristic function of A in ternary semigroup T.

Assume that A is a bi-ideal in T .Then we have $ATATA \subseteq A$.

Now consider $C_A \circ T \circ C_A \circ T \circ C_A = C_A \circ C_T \circ C_A \circ C_T \circ C_A$

$$= C_{ATATA} \subseteq C_A$$

$$C_A \circ T \circ C_A \circ T \circ C_A \subseteq C_A$$

Therefore C_A is a fuzzy bi ideal in T.

Conversely, suppose C_A is fuzzy bi- ideal in T.

Then $C_A \circ T \circ C_A \circ T \circ C_A \subseteq C_A$

Let $x \in ATATA$. Then $C_A(x) \geq (C_A \circ T \circ C_A \circ T \circ C_A)(x)$

$$= (C_A \circ C_T \circ C_A \circ C_T \circ C_A)(x)$$

$$= C_{ATATA}(x)$$

$$\geq 1 \, (\because x \in (ATATA)$$

$$C_A(x) \geq 1 \Rightarrow x \in A$$

Therefore $ATATA \subseteq A$.

Hence A is a bi-ideal in a ternary semi group T.

Theorem 3.2. If μ be a fuzzy bi-ideal in a ternary semigroup T, then the level set μ_t is a bi -ideal in T for every $t \in [0,1]$.

Proof: Let μ be a fuzzy subset of a ternary semigroup T and let $x \in T$. For $t \in [0,1]$, let $u \in \mu_t T \mu_t T \mu_t$, where μ_t be the level set of μ. Then there exists $x, y, z \in \mu_t$, $m, n \in T$ and such that $u = xmynz$.

Consider $[\mu \circ T \circ (\mu \circ T \circ \mu)](u) = \vee_{u=abc} \{\mu(a) \wedge \mu(b) \wedge \mu(c)\}$

$$= \vee_{u=abc} \{\mu(a) \wedge T(b) \wedge \{\vee_{c=def} \{\mu(d) \wedge T(e) \wedge \mu(f)\}\}\}$$

$$= \bigvee_{u=abdef} \{\mu(a) \wedge 1 \wedge \mu(d) \wedge 1 \wedge \mu(f)\}$$

$$= \bigvee_{xmynz=abdef} \{\mu(a) \wedge \mu(d) \wedge \mu(f)\}$$

$$= \mu(x) \wedge \mu(y) \wedge \mu(z)$$

$$\geq t \wedge t \wedge t = t$$

$$[\mu \circ T \circ (\mu \circ T \circ \mu)](u) \geq t$$

Since μ is a fuzzy bi-ideal in $T \Rightarrow \mu(u) \geq t$ for all $u \in T$

$\Rightarrow u \in \mu_t \Rightarrow \mu_t T \mu_t T \mu_t \subseteq \mu_t$.

Hence μ_t is a bi- ideal in T.

Theorem 3.3. Every fuzzy quasi ideal in a ternary semigroup T is a fuzzy bi -ideal in T.

Proof: Suppose μ is fuzzy quasi ideal in a ternary semigroup T. Then we have $(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T) \subseteq \mu$ and

$(T \circ T \circ \mu) \cap (T \circ T \circ \mu \circ T \circ T) \cap (\mu \circ T \circ T) \subseteq \mu$.

Consider $\mu(xyz) \geq [(T \circ T \circ \mu) \cap (T \circ \mu \circ T) \cap (\mu \circ T \circ T)](xyz)$

$\geq (T \circ T \circ \mu)(xyz) \cap (T \circ \mu \circ T)(xyz) \cap (\mu \circ T \circ T)(xyz)$

$$\geq [\bigvee_{xyz=abc} \{T(a) \wedge T(b) \wedge \mu(c)\}] \wedge [\bigvee_{xyz=uvw} \{T(u) \wedge T(v) \wedge \mu(w)\}]$$

$$\wedge [\bigvee_{xyz=rst} \{\mu(r) \wedge T(s) \wedge T(t)\}]$$

$\geq [T(x) \wedge T(y) \wedge \mu(z)] \wedge [T(x) \wedge \mu(y) \wedge T(z)] \wedge [\mu(x) \wedge T(y) \wedge T(z)]$

$\geq [1 \wedge 1 \wedge \mu(z)] \wedge [1 \wedge \mu(y) \wedge 1] \wedge [\mu(x) \wedge 1 \wedge 1]$

$\geq \mu(z) \wedge \mu(y) \wedge \mu(x)$

$\geq \mu(x) \wedge \mu(y) \wedge \mu(z)$

$\mu(xyz) \geq [\mu(x) \wedge \mu(y) \wedge \mu(z)]$

Therefore μ is a fuzzy ternary sub semigroup of T.

Again consider

$\mu(xmynz) \geq [(T \circ T \circ \mu) \cap (T \circ T \circ \mu \circ T \circ T) \cap (\mu \circ T \circ T)](xmynz)$

$\geq [(T \circ T \circ \mu)(xmynz) \cap (T \circ T \circ \mu \circ T \circ T)(xmynz) \cap (\mu \circ T \circ T)(xmynz)]$

$\geq [\bigvee_{xmynz=abcde} \{T(a) \wedge T(bdc) \wedge \mu(e)\}] \wedge$

$[\bigvee_{(xmynz)=pqrst} \{T(p) \wedge T(q) \wedge \mu(r) \wedge T(r) \wedge T(t)\}]$

$\wedge [\bigvee_{xmynz=ijklo} \{\mu(i) \wedge T(jkl) \wedge T(o)\}]$

$\geq [T(x) \wedge T(myn) \wedge \mu(z)] \wedge [T(x) \wedge T(m) \wedge \mu(y) \wedge T(n) \wedge T(z)]$

$\wedge [\mu(x) \wedge T(myn) \wedge T(z)]$

$\geq [1 \wedge 1 \wedge \mu(z)] \wedge [1 \wedge 1 \wedge \mu(y) \wedge 1 \wedge 1] \wedge [\mu(x) \wedge 1 \wedge 1]$

$\geq \mu(z) \wedge \mu(y) \wedge \mu(x)$

$\mu(xmynz) \geq \mu(x) \wedge \mu(y) \wedge \mu(z)$

Therefore μ is a fuzzy bi - ideal in T.

Theorem 3.4. Let μ be a fuzzy bi-ideal in a ternary semigroup T. Then the fuzzy subset μ^* defined by $\mu^* = \mu(x) + 1 - \mu(0)$ for all $x \in T$ is also a fuzzy bi- ideal of T.

Proof: Given that T be a ternary semigroup and μ be a fuzzy bi-ideal in T. Then μ is a fuzzy ternary sub semigroup of T. That is for all $x, y, z \in T$,

$\mu(xyz) \geq \mu(x) \wedge \mu(y) \wedge \mu(z)$

or $\mu(xyz) \geq \min \{\mu(x), \mu(y), \mu(z)\}$.

and $\mu(amynz) \geq \min \{\mu(a), \mu(b), \mu(c)\}$ for all $a, b, c, m, n \in T$.

Let μ^* be a fuzzy subset of T where $\mu^* = \mu(x) + 1 - \mu(0)$ for all $x \in T$.

we have to prove that μ^* is a fuzzy bi – ideal of T.

(i) Let $x, y, z \in T$

We have $\mu^*(xyz) = \mu(xyz) + 1 - \mu(0)$

$\geq \min \{\mu(x), \mu(y), \mu(z)\} + 1 - \mu(0)$

$= \min\{\mu(x) + 1 - \mu(0), \mu(y) + 1 - \mu(0), \mu(z) + 1 - \mu(0)\}$

$= \min \{\mu^*(x), \mu^*(y), \mu^*(z)\}$

$\mu^*(xyz) \geq \min \{\mu^*(x), \mu^*(y), \mu^*(z)\}$

Therefore μ^* is a fuzzy ternary sub semigroup of T.

(ii) Let $a, b, c, m, n \in T$

We have $\mu^*(ambnc) = \mu(ambnc) + 1 - \mu(0)$

$\geq \min \{\mu(a), \mu(b), \mu(c)\} + 1 - \mu(0)$

$= \min\{\mu(a) + 1 - \mu(0), \mu(b) + 1 - \mu(0), \mu(c) + 1 - \mu(0)\}$

$= \min \{\mu^*(a), \mu^*(b), \mu^*(c)\}$

$\mu^*(ambnc) \geq \min\{\mu^*(a), \mu^*(b), \mu^*(c)\}$

Therefore μ^* is a fuzzy bi – ideal in T.

Theorem 3.5. Let T be a left zero ternary semigroup and μ be a fuzzy left ideal in T. Then $\mu(x) = \mu(z)$ for all $x, y, z \in T$.

Proof: Let T be a left zero ternary semigroup.

i.e., $x, y, z \in T \Rightarrow xyz = x$ and $zyx = z$.

since μ is fuzzy left ideal in T.

Consider $\mu(x) = \mu(xyz) \geq \mu(z)$

$$\mu(x) \geq \mu(z) \tag{1}$$

and we have $\mu(x) = \mu(zyx) \geq \mu(x)$

$$\mu(z) \geq \mu(x) \tag{2}$$

from (1) and (2), We have $\mu(x) = \mu(z)$ for all $x, y, z \in T$.

Theorem 3.6. A non-empty fuzzy subset μ of a ternary semigroup T is a fuzzy ternary sub semigroup of T if and only if $\mu \circ \mu \circ \mu \subseteq \mu$.

Proof: Let μ be a non- empty fuzzy subset of a ternary semigroup T.

Suppose $\mu \circ \mu \circ \mu \subseteq \mu$

we have to prove that μ is a fuzzy ternary sub semigroup of T.

$\mu(xyz) \geq \mu(x) \wedge \mu(y) \wedge \mu(z)$ *for all* $x, y, z \in T$.

Let $x, y, z \in T$, such that $p = xyz$

Consider $\mu(xyz) \geq (\mu \circ \mu \circ \mu)(xyz)$

$$= \vee_{p=abc} \{\mu(a) \wedge \mu(b) \wedge \mu(c)\}$$

$$= \vee_{xyz=abc} \{\mu(a) \wedge \mu(b) \wedge \mu(c)\}$$

$$= \mu(x) \wedge \mu(y) \wedge \mu(z)$$

$\mu(xyz) \geq \{\mu(x) \wedge \mu(y) \wedge \mu(z)\}$.

Conversely, assume that μ is a fuzzy ternary sub semigroup of T.

i.e., $\mu(xyz) \geq \{\mu(x) \wedge \mu(y) \wedge \mu(z)\}$.

We have to prove that $\mu \circ \mu \circ \mu \subseteq \mu$.

i.e., $(\mu \circ \mu \circ \mu)(a) \leq \mu(a)$ for all $a \in T$

Let $a = xyz$ for all $x, y, z \in T$.

Consider $(\mu \circ \mu \circ \mu)(a) = \vee_{a=pqr} \{\mu(p) \wedge \mu(q) \wedge \mu(r)\}$

$$= \vee_{xyz=pqr} \{\mu(p) \wedge \mu(q) \wedge \mu(r)\}$$

$$= \{\mu(x) \wedge \mu(y) \wedge \mu(z)\}$$

$$\leq \mu(xyz)$$

$= \mu(a)$

$(\mu \circ \mu \circ \mu)(a) \leq \mu(a) \quad \Rightarrow \quad \mu \circ \mu \circ \mu \subseteq \mu$.

If $a \neq xyz$ then $(\mu \circ \mu \circ \mu)(a) = o \leq \mu(a) \quad \Rightarrow \quad (\mu \circ \mu \circ \mu)(a) \leq \mu(a)$.

Therefore $\mu \circ \mu \circ \mu \subseteq \mu$.

Theorem 3.7. For any nonempty fuzzy subset μ of a ternary semigroup T, the following conditions are equivalent.

(i) μ is a fuzzy bi-ideal in T .

(ii) $\mu \circ \mu \circ \mu \subseteq \mu$ and $\mu \circ C_T \circ \mu \circ C_T \circ \mu \subseteq \mu$ where C_T is the characteristic function of T .

Proof: Let T be a ternary semigroup and let μ be a fuzzy subset of T .

First to prove that (i) $\Rightarrow$ (ii):

Let μ is a fuzzy bi-ideal in T then μ is a fuzzy ternary sub semigroup of T . i.e., $\mu(xyz) \geq \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \}$ *for all* $x, y, z \in T$.

and $\mu(amynz) \geq \{ \mu(a) \wedge \mu(b) \wedge \mu(c) \}$ *for all* $a, b, c, m, n \in T$..

From the theorem 3.6, we have $\mu \circ \mu \circ \mu \subseteq \mu$.

To prove only $\mu \circ C_T \circ \mu \circ C_T \circ \mu \subseteq \mu$

i.e., $(\mu \circ C_T \circ \mu \circ C_T \circ \mu)(a) \leq \mu(a)$ for all $a \in T$.

Let $a = xmynz$ for all $x, y, z, m, n, p, q, r \in T$.

Consider

$$
\begin{aligned}
\mu \circ C_T \circ \mu \circ C_T \circ \mu)(a) &= \bigvee_{xmynz = puqvr} \{ \mu(p) \wedge C_T(u) \wedge \mu(q) \wedge C_T(v) \wedge \mu)(r) \} \\
&= \{ \mu(x) \wedge C_T(m) \wedge \mu(y) \wedge C_T(n) \wedge \mu(z) \} \\
&= \{ \mu(x) \wedge 1 \wedge \mu(y) \wedge 1 \wedge \mu(z) \} \\
&= \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \} \\
&\leq \mu(xmynz) \\
&= \mu(a)
\end{aligned}
$$

$\Rightarrow (\mu \circ C_T \circ \mu \circ C_T \circ \mu)(a) \leq \mu(a)$.

Therefore $\mu \circ C_T \circ \mu \circ C_T \circ \mu \subseteq \mu$

Next to prove that (ii) $\Rightarrow$(i):

We assume that $\mu \circ \mu \circ \mu \subseteq \mu \rightarrow (1)$ and $\mu \circ C_T \circ \mu \circ C_T \circ \mu \subseteq \mu \rightarrow (2)$.

From theorem 3.6 and by equation (1), we have μ is a fuzzy ternary sub semigroup of T .

It is enough to prove that $\mu(amynz) \geq \min \{ \mu(a), \mu(b), \mu(c) \}$ . .

Consider $\mu(xmynz) \geq \{ \mu \circ C_T \circ \mu \circ C_T \circ \mu \}(xmynz)$.

$$= \bigvee_{xmynz = puqvr} \{ \mu(p) \wedge C_T(u) \wedge \mu(q) \wedge C_T(v) \wedge \mu(r) \}.$$
$$= \{ \mu(x) \wedge C_T(m) \wedge \mu(y) \wedge C_T(n) \wedge \mu(z) \}.$$
$$= \{ \mu(x) \wedge 1 \wedge \mu(y) \wedge 1 \wedge \mu(z) \}.$$
$$= \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \}.$$

$\mu(xmynz) \geq \{ \mu(x) \wedge \mu(y) \wedge \mu(z) \}$ *for all* $a, b, c, m, n \in T$.

Therefore μ is a fuzzy bi – ideal in T .

Definition 3.8 . Let T_1 and T_2 be two ternary semigroups. A mapping $f : (T_1, *_1) \to (T_2, *_2)$ is called a ternary homomorphism if $f(x *_1 y *_1 z) = (f(x) *_2 f(y) *_2 f(z))$ for all $x, y, z \in T$.

Definition 3.9. Let f be a mapping from a set X to Y and μ be a fuzzy subset of Y, then the pre image of μ under f , denoted by $f^{-1}(\mu)$, is defined as

$$f^{-1}(\mu)(x) = \mu(f(x)) \text{ for all } x \in X.$$

Theorem 3.10. Let $f : T_1 \to T_2$ be a homomorphism of ternary semigroups. If μ is a fuzzy bi- ideal in T_2 then the pre image $f^{-1}(\mu)$ is a fuzzy bi – ideal in T_1.

Proof: Let T_1 and T_2 be two ternary semigroups and given that $f : T_1 \to T_2$ is a homomorphism.

Then $f(xyz) = (f(x) f(y) f(z))$ for all $x, y, z \in T_1$.

Let μ be a fuzzy bi – ideal in T_2 .

We have to prove that $f^{-1}(\mu)$ is a fuzzy bi-ideal in T_1 .

Consider $f^{-1}(\mu(xyz)) = \mu(f(xyz))$.

$$= \mu(f(x) f(y) f(z)).$$
$$\geq \min\{ \mu(f(x)), \mu(f(y)) \ f(\mu(z))\}.$$
$$= \min\{ f^{-1}(\mu(x)), f^{-1}(\mu(y)) \ f^{-1}(\mu(z))\}.$$
$$f^{-1}(\mu(xyz)) \geq \min\{ f^{-1}(\mu(x)), f^{-1}(\mu(y)) \ f^{-1}(\mu(z))\}.$$

Therefore, $f^{-1}(\mu)$ is a fuzzy ternary sub semigroup of T.

Let $x, y, z, m, n \in T_1$

Again consider $f^{-1}(\mu(xmynz)) = \mu(f(xmynz))$

$$= \mu(f(x) f(m) f(y) f(n) f(z)).$$

$$\geq \mu(f(x))\ \mu(f(y))\ \mu(f(z))).$$

$$= \min\{f^{-1}(\mu(x))\ f^{-1}(\mu(y))\ \ f^{-1}(\mu(z))\}.$$

$$f^{-1}(\mu(xmymz)) \geq \min\{f^{-1}(\mu(x)), f^{-1}(\mu(y)), f^{-1}(\mu(z))\}.$$

Therefore $f^{-1}(\mu)$ is a fuzzy bi – ideal in ternary semigroup T_1 .

4. Conclusions

We introduced the notion of fuzzy ideal, fuzzy quasi ideal, fuzzy bi-ideal in an ternary semigroup and studied their properties and relations between them. We characterize the fuzzy bi-ideals in an ternary semigroup with respect to bi ideals. In continuous of this paper we propose to study fuzzy bi-ideals over Ternary semigroups.

Acknowledgement. We are thankful to the reviewer for the reviewing our paper. Your suggestions and Comment are very helpful in making further improvement if any. Thank you for your support.

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