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Special TSPs Considering Conveyances and Routes Through a Hybrid Algorithm

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Abstract. In this paper, Travelling Salesman Problems (TSPs) are formulated as profit maximization problems with a time constraint and solved using a hybrid algorithm. Here, in TSPs in addition to usual costs and times, between two cities, there are several paths/routes and conveyances for travel. These parameters are given and known. A Travelling Salesman (TS) visits all the cities, spends some times at each city and earns some revenues depending upon his/her spent time. Problem for a TS is to design a total tour program selecting the travel paths and vehicles between two cities and spend time at each city. Till now, this type of profit maximization of 4D TSP is not formulated so far. The model is solved using a hybrid algorithm consisting of Ant Colony Optimization (ACO) and Particle swarm optimization (PSO). These algorithms are used for the objective function successively and iteratively. The problem is broken into two parts - the tour program and spend time in cities which are determined through ACO and PSO respectively so that total profit out of the system is maximum under a time constraint. The models are illustrated numerically. Some behavioural studies of the model and convergence of the proposed hybrid algorithm with respect to iteration numbers and cost matrix sizes are presented. As particular cases, 3D and 2D TSPs are also formulated and their results are presented.

Keywords: Ant colony optimization, Particle swarm optimization, TSP, Hybrid algorithm

AMS Mathematics Subject Classification (2010): 90B06

1. Introduction

TSP [2] is a NP-hard and also one of the most complex combinatorial optimization problem which cannot be solved exactly in polynomial time. The aim of this problem is to find a shortest path passing through each city exactly once, where set of cities are given. Different types of TSPs have been extensively studied in the literature. In general, the salesman, in two dimensional (2D) TSP, travels from a city to another incurring a cost or time. In this case, it is assumed that there is only one route and one conveyance for travel [11] between any two cities. If several conveyances at each city are considered, the corresponding TSP is called three dimensional (3D) TSP or solid TSP [4]. Generally, in 2D and 3D TSPs, distance between two cities are considered as fixed. However there may be several routes between two cities. If between two cities different types of conveyances and different types of routes are considered, then it becomes four dimensional (4D) TSP.

The ACO is a technique of problem solving inspired by the behaviour of ants in finding paths from nest to food. Dorigo and Gambardella [9] described an artificial ant colony capable of solving the TSP. Stutzle and Hoos introduced Max-Min ant system (MMAS)[20], a modification of an ant system applied to TSP. In 2007, Cheng and Mao [5] presented a modified ant colony system for solving the TSP with time windows. Gaifang Dong et al.[8] presented cooperative genetic ant system for solving TSP in 2012. Bai et al. [3] proposed a model for Asymmetric TSP which include max-min ant colony optimization in 2013.

PSO is a swarm intelligent technique that is inspired by the flocking behaviour of birds. Here all the particles depend on the experiences of their neighbours to search for the optimal solution. Kennedy and Eberhart [10] first proposed PSO algorithm and has received significant attention. Recently many PSO algorithms have been developed to solve TSP [18, 15]. Moreover, most existing PSO methods are modified to improve the performance of PSO by variable parameters [19, 17] or change the updating equation [21].

The new features of the present profit maximization 4D TSP investigation are:

- In this TSP, a salesman travels from one city to another through different routes using different vehicles available at different cities. Though graphical formulation of this type of TSPs is possible, to the best of our knowledge, none has investigated mathematically 4D TSP.
- Though the visit of a salesman is organised in order to get a return for the company, most of the TSPs are concerned with the minimization of tour cost or travel time. Moreover, for selling or canvassing a product, a TS has to spend some time at each city and incurs some expenditure for this. This has also been overlooked by the TSP researchers.
- Normally, a TS is asked by his/her company to finish the entire activities including the tour (travel and stay times) within a specified time limit. This constraint has been taken into account in the present investigation.
- Though ultimate goal of a company is to make a profit through the sales representative, till now, no TSP has been formulated as a profit maximization problem considering returns and expenditures.
- In order to bridge the above mentioned gaps, one new 4D TSP is formulated and solved by a developed hybrid algorithm consisting of ACO and PSO which are operated successively and iteratively.

These are real assumptions and may be applied in different fields such as in book publishing firm or medicine firms for sealing or publishing their materials by representatives or salesmen. For a publishing firm, generally a representative or salesman is said to make a tour within a limited time for schools in town. To maximize the benefit of a tour, any one fixes the order of cities and adjust the time spending in city for sealing products. He/she incurs some costs for travel, pays for stay at stations and earns indirectly through Govt./public schools by canvassing and presentation. All these are done within the limited time fixed by the publishing firm. The proposed TSP with the above assumptions are most appropriate for the real life publishing house problems and considered in the proposed model.

In this research paper a 4D TSP is proposed, where the problem involves a set of cities which has to be travelled by the salesman. Between any two cities there are several roots (paths) and through each path different conveyances (vehicles) are available. Cost (c_{ijrv}) and time (t_{ijrv}) to travel from city-i to city-j through route-k using vehicle-v is known. At each city the salesman spends some time for canvassing and depending that time he/she incurs some profit. Also there are some stay expenditure at each city which depends upon the stay time (t). Return and expenditure at each city are assumed in a functional form of t. For a total tour salesman has a maximum time limit. Here tour schedule is to determine including the routes and vehicles and also the stay time at different cities so that total profit from the system will be a maximum in time constraint. Only travel cost and only travel time are determined in particular case to compare the result with the original problem. Numerical examples for the Models are illustrated.

Here the proposed TSP has two additional dimensions - conveyances and routes over the dimensions of a conventional TSP. In this case, choice of routes and vehicles are important as travel times through different routes and conveyances are different and total operational time is previously fixed. This model has two optimization part one is to minimize of total travel costs between the cities and second one is the allocation of stay times at different cities. These two part of the model makes a trade off so that the total outcome of the proposed problem is maximum. For these two sub-optimization TSP problem, a hybrid algorithm combining the algorithms of PSO and ACO is designed and applied successfully. Here PSO and ACO are used successively and iteratively in a generation using one's result for the other. This choice of hybrid algorithm (ACO-PSO) is done as ACO is most suitable for discrete formulation and PSO for continuous ones. The proposed TSP is illustrated with numerical examples. Some behaviours of the proposed model are presented with cost matrices of different sizes. It does not make more profit if any one spend more time at same station. Some parametric studies of the proposed hybrid algorithm with respect to size of cost matrix and iteration numbers are also represented.

Rest of the paper is organized as follows. In section 2, Models are formulated. In section 3, Hybridization of ACO-PSO system is described. Experimental results are represented in section 4. Section 5 contains formulation and illustration of some particular models. Finally a brief discussion of models' behavioural studies and conclusions are respectively presented in sections 6, 7 and 8.

2. Model formulation

2.1. Proposed 4D TSP for minimum total travel cost (Model 1A)

In a classical 2D TSP, TSP can be represented as graph $G = (Vt, E)$, where $Vt = 1,2,..., N$ is the set of nodes and E is the set of edges. A salesman has to travel N cities at minimum cost. It is an extension of classical TSP, where only one path and one vehicle are available between the cities for travel and the objective is to minimize the total cost. In the proposed model, between any two nodes there are different routes and through each route there are different vehicles. To travel from a city to another, the salesman choices a particular route and a particular vehicle so that total cost to complete his tour is minimum. In this tour, salesman starts from a city, visits all the cities exactly once and comes to the starting city incurring minimum expenditure. Let c_{ijrv} be the cost for travelling from $i-th$ city to $j-th$ city using $r-th$ route and $v-th$ vehicle. Then the model is mathematically formulated as :

$$\left.\begin{array}{l} Determine\ x_{ijrv}, i=1,2,...,N.,\ j=1,2,...,N.,\ r=1,2,...R.,\ v=1,2,...V. \\ to\ minimize\ Z_O = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv}c_{ijrv}, \\ subject\ to\ \sum_{i=1}^{N}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} = 1, \quad j=1,2,...,N; \\ \sum_{j=1}^{N}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} = 1, \quad i=1,2,...,N; \\ \sum_{i\in S}\sum_{j\in S}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} \le |S|-1, \forall S \subset Vt, \\ x_{ijrv} \in \{0,1\}. \end{array}\right\} \quad (1)$$

where x_{ijrv} = 1 if the salesman travels from city-i to city-j following r-th route using v-th vehicle, otherwise x_{ijrv} = 0

Let $(x_1, x_2, ..., x_N, x_1)$ be a complete tour of a salesman, where $x_i \in \{1,2,..,N\}$ for i = 1, 2,...,N and all x_i's are distinct. Then the above model reduces to

$$\left.\begin{array}{l} Determine\ a\ complete\ tour\ (x_1, x_2, ..., x_N, x_1)\ along\ with\ routes\ (r_1, r_2, ..., r_N) \\ and\ corresponding\ conveyance\ types\ (v_1, v_2, ..., v_N) \\ to\ minimize\ Z_C = \sum_{i=1}^{N-1} c_{x_i, x_{i+1}, r_i, v_i} + c_{x_N, x_1, r_N, v_N} \\ \quad where\ r_i \in (1,2,...,R)\ for\ i=1,2,...,N \\ \quad and\ v_i \in (1,2,...,V)\ for\ i=1,2,...,N. \end{array}\right\} \quad (2)$$

Here the salesman travels from node x_i to x_{i+1} through route r_i using vehicle v_i for i=1,2,...,N-1. and x_N to x_1 following route r_N using vehicle v_N.

Proposed 4D-TSP with time for minimum total travel time (Model 1B)

Let t_{ijrv} be the time for travelling from $i-th$ city to $j-th$ city using $r-th$ route and $v-th$ vehicle. In this case, the model formulation is the same as (3) except that the objective function is replaced by (5) given below

$$\left. To\ minimize\ Z_T = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} t_{ijrv}. \right\} \quad (3)$$

All other constraints remain unaltered.

Let $(x_1, x_2, ..., x_N, x_1)$ be a complete tour of a salesman, where $x_i \in \{1,2,..,N\}$ for i = 1, 2,...,N and all x_i 's are distinct. Then the above model reduces to

$$\left. \begin{array}{l} Determine\ a\ complete\ tour\ (x_1, x_2, ..., x_N, x_1)\ using\ routes\ (r_1, r_2, ..., r_N) \\ and\ corresponding\ conveyance\ types\ (v_1, v_2, ..., v_N) \\ To\ minimize\ Z_T = \sum_{i=1}^{N-1} t_{x_i, x_{i+1}, r_i, v_i} + t_{x_N, x_1, r_N, v_N}. \end{array} \right\} \quad (4)$$

2.2. Proposed 4D-TSP for maximum profit (Model 2)

Here a model is considered where the salesman spends some time at each city to convince the customers and hence due to this, makes some profit and incurs some expenditure also. Amounts of profit and expenditure depend on the duration of time he spends in that city. He bears some expenditure and makes profit per unit time at each city due to his/her stay. Let c_{ijrv} and t_{ijrv} be the cost and time respectively for travelling from $i-th$ city to $j-th$ city using $r-th$ route and $v-th$ vehicle, t_i the spent time in $i-th$ city, $o_i(t_i)$ the output/return, $e_i(t_i)$ the expenditure, where $o_i(t_i) = a_i + b_i t_i - c_i t_i^2$ and $e_i(t_i) = e_{0i} t_i$ for spending t_i time by TS in the $i-th$ city. Here a constraint on the total time used by the salesman is imposed. Let he/she can at most use H units of time for his/her total tour. So the model is mathematically formulated as:

$$\left. \begin{array}{l} Determine\ x_{ijrv},\ and\ t_i, i = 1,2,...N,\ j = 1,2,...N,\ r = 1,2,...R,\ v = 1,2,...V. \\ to\ maximize\ Z = \sum_{i=1}^{N}(o_i(t_i) - e_i(t_i)) - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} c_{ijrv}, \\ subject\ to\ \sum_{i=1}^{N}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} = 1, \quad j = 1,2,..., N; \\ \sum_{j=1}^{N}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} = 1, \quad i = 1,2,..., N; \\ \sum_{i \in S}\sum_{j \in S}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} \le |S| - 1, \forall S \subset Vt \\ x_{ijrv} \in \{0,1\} \\ such\ that\ \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{r=1}^{R}\sum_{v=1}^{V} x_{ijrv} t_{ijrv} + \sum_{i=1}^{N} t_i \le H, \end{array} \right\} \quad (5)$$

where x_{ijrv} = 1 if the salesman travels from city- i to city- j through r-th route using j-th conveyance, otherwise x_{ijrv} = 0, H=total allowable time for the entire tour.

Let $(x_1, x_2, ..., x_N, x_1)$ be a complete tour for the salesman, where $x_i \in \{1,2,..,N\}$ for i = 1, 2,...,N and all x_i 's are distinct. Then the above model reduces to (8) as follows

$$\left.\begin{aligned}
&Determine\ a\ complete\ tour\ (x_1, x_2, ..., x_N, x_1)\ and\ (t_1, t_2, ..., t_N)\\
&\ along\ with\ routes\ (r_1, r_2, ..., r_N)\ and\ corresponding\ conveyance\ types\ (v_1, v_2, ..., v_N)\\
&To\ maximize\ Z = (o_1(t1) + o_2(t2) + ... + o_N(tN)) - [\sum_{i=1}^{N-1} c_{x_i, x_{i+1}, r_i, v_i} + c_{x_N, x_1, r_N, v_N} +\\
&\quad (e_1(t1) + e_2(t2) + ... + e_N(tN))]\\
&subject\ to\ \sum_{i=1}^{N-1} t_{x_i, x_{i+1}, r_i, v_i} + t_{x_N, x_1, r_N, v_N} + (t_1 + t_2 + ... + t_N) \le H\\
&\quad where\ r_i \in (1,2,...,R)\ for\ i = 1,2,...,N\\
&\quad and\ v_i \in (1,2,...,V)\ for\ i = 1,2,...,N.
\end{aligned}\right\} \quad (6)$$

The defined single objective optimization problem for the 4D TSP is solved by a hybrid algorithm developed for this purpose in section 3.

3. Hybridization of ACO-PSO system

Recently, ACO and PSO algorithms are inspired by natural habits are used to solve optimization problem. ACO approach, inspired by the foraging behaviour of real ants, has become more popular to solve different type of TSP problems [9, 5, 20, 8, 12, 3]. During the last decade, many PSO algorithms [14, 18, 6] have been developed inspired by the behaviour laws of bird flocks, fish schools and human communities, for solving TSP problems.

There are some merits and demerits of ACO and PSO for solving TSP. Here a hybrid algorithm is proposed to solve TSP in some modified form by combining the features of ACO and PSO algorithm. In the proposed algorithm, at fast ant colony system is used to produce a set of paths (tours) along with route between nodes and vehicles used for the salesman. Then PSO operation is done for every path of the salesman (ant) to determine optimal stay times at different nodes for the respective path. The hybrid ACO-PSO system is presented below. In the algorithm, τ_{ijrv} represents amount of pheromone lies on the path between node i and node j following r-th route using v-th vehicle, $itn1$ and $itn2$ represent iteration counters, $maxiter1$ and $maxiter2$ respectively represent maximum iteration number of the ACO algorithm and maximum iteration number in PSO part n represent number of ants or population size and N represents number of nodes/cities in the problem. Here a set of particles of size M is used. The algorithm of the hybrid ACO-PSO is as follows

Proposed Hybrid Algorithm (HA) :

- Initialize $maxiter1$, $maxiter2$ and Set $itn1=0$.
- pheromone τ_{ijrv} initialization for $i=1,2,...,N$, $j=1,2,...,N$, $r=1,2,...,R$ and $v=1,2,...,V$.
- Do
- Generate path of n ants, i.e., construct n tour schedules (X_i, R_i, V_i),

here $X_i=(x_{i1}, x_{i2},...,x_{iN}, x_{i1})$, $R_i=(r_{i1}, r_{i2},...,r_{iN})$, $V_i=(v_{i1}, v_{i2},...,v_{iN})$, $i=1,2,...,n$ using τ_{ijrv}. where X_i represent sequence of cities to be visited using R_i and V_i respectively represent corresponding routes and vehicle.

/*For the k-th path $X_k=(x_{k1}, x_{k2},...,x_{kN}, x_{k1})$ we consider the spend times at different cities i.e., particle $(OST_{k1}, OST_{k2},..., OSTkN)$ for profit maximization of the tour using PSO.*/

- Do for each path k
- Generate randomly M size particles for the k-th path

$ST_{ki}=(ST_{ki1}, ST_{ki2},...,ST_{kiN})$,

- $i=1,2,..M$, value of $ST_{kij} \in (0.0001,1)$. ST_{ki} is i-th particle for the

swarm of path X_k.

- each ST_{ki} i.e., stay time of j-th node is calculated as

$t_{kij}=T\times ST_{kij}/(ST_{ki1}+ST_{ki2}+...+ST_{kiN})$, where j=1,2,...N and i=1,2,...,M. Here $T=H-T_k$ which is sum of all stay time at different cities, H is the total tour time and T_k is travel time for path X_k. So stay time at different cities for k-th path is represented by the vector $t_{ki}=(t_{ki1}, t_{ki2},...,t_{kiN})$.

- For each particle ST_{ki} do
- Initialize V_{ki}, i.e., velocity .
- Profit calculation (X_k, R_k, V_k, ST_{ki}), the objective value of ST_{ki}
- End for
- $itn2=0$
- Do
- For each particle ST_{ki} do
- Find $PBST_{ki}$, the personal best position .
- End for
- Find $GBST_k$, the global best position of the swarm.
- For each particle ST_{ki} do
- Update velocity V_{ki} .
- Update position ST_{ki}.

- Profit calculation (X_k, R_k, V_k, ST_{ki}), the objective value of ST_{ki}
- End for
- $itn2 = itn2 + 1$
- while($itn2 < maxiter2$).
- From $GBST_k$, Determine stay times of X_k
- End Do.
- Pheromone are evaporated.
- Pheromone Updating for all the paths.
- Find best solution.
- $itn1 = itn1 + 1$
- While ($itn1 < maxiter1$).
- Output: The best solution.
- End of Algorithm.

3.1. Procedures for the proposed hybrid algorithm

(a) Representation of a Solution: Here N cities are consider for a complete tour along with route schedule and vehicle schedule represents a path of an ant, i.e., a solution for the ACO part of the proposed algorithm. Three ' n dimensional integer vectors', $X_k = (x_{k1}, x_{k2}, ..., x_{kN})$, $R_k = (r_{k1}, r_{k2}, ..., r_{kN})$, $V_k = (v_{k1}, v_{k2}, ..., v_{kN})$, are used to represent a solution, where x_{k1}, x_{k2}, ... , x_{kN} represent N consecutive cities of a tour, $r_{k1}, r_{k2}, ..., r_{kN}$ represent routes of travel from x_{k1} , x_{k2} , ... , x_{kN} . $v_{k1}, v_{k2}, ..., v_{kN}$ represent vehicle type used to travel from x_{k1} , x_{k2} , ... , x_{kN} respectively and $k = 1,2,...,n$ where n is number of ants.

(b) Initialization of Pheromone: Here the TSP problem is designed for maximize the profit of a tour, so cost of the tour is minimized, for that the pheromone initial value is set as $\tau_{ijrv} = 1/c_{ijrv}^{1.5}$.

(c) Construction of Path: A path (X_k, R_k, V_k) for k-th ant is generated as using the following steps :

- Let $ND = \{1,2,...,N\}$, $RE = \{1,2,...,R\}$, $VE = \{1,2,...,V\}$, and $l = 1$
- A random element x_{kl} is selected from the set ND.
- Now, let $ND = ND - \{x_{kl}\}$
- Let current position of the ant is node i i.e., $x_{kl} = i$. Then the ant select the next node j, where $j \in ND$, using probability p_{ijrv} which is given in the following expression,

$$p_{ijrv} = \frac{\tau_{ijrv}^{\alpha}}{\sum_{j \in NS} \sum_{r \in RS} \sum_{v \in VS} \tau_{ijrv}^{\alpha}},$$

where r is the selected route and v is the selected vehicle i.e, $r_{kl} = r$, $v_{kl} = v$ and a positive constant α is used for control the influence of pheromone concentrations. Here Roulette-Wheel selection procedure [16] is used for the proposed algorithm.

- $l = l+1$, $r_{kl} = r$, $v_{kl} = v$, $x_{kl} = j$.
- if $l < N$ goto step (iii).
- From x_{kN} to x_{k1} is travelled using vehicle v, through route r, which are selected using

$$p_{x_{kN}x_{k1}rv} = \frac{\tau^{\alpha}_{x_{kN}x_{k1}rv}}{\sum_{j\in NS}\sum_{r\in RS}\sum_{v\in VS}\tau^{\alpha}_{x_{kN}x_{k1}rv}}$$

after selection of r and v, set $r_{kN} = r$ and $v_{kN} = v$,

n -such paths are constructed for n ants.

(d) For each path (X_k, R_k, V_k), following operations are done:

- **Initialization of Swarm :** For represent a swarm of the path, a set of M particles are generated randomly. Here a particle is considered as a set of stay times proportion at different nodes. To improve the profit of the tour, the PSO operations are done on this swarm. If T_K is the time for travel of k -th path (X_k, R_k, V_k), then the total spend time at different nodes/cities is calculated as $T = H - T_K$, where H is the total tour time which is given. N component vector is used to construct $ST_{ki} = (ST_{ki1}, ST_{ki2}, ..., ST_{kiN})$, where each $ST_{ki} \in (0.0001,1)$. The velocity for each particle is initialized between the value of V_{max} and V_{min}. For each ST_{ki}, stay time at different nodes are calculated as $t_{kij} = T \times (ST_{kij})/(ST_{ki1} + ST_{ki2} + ... + ST_{kiN})$ for j=1,2,..N and i=1,2,...,M. so $t_{ki} = (t_{ki1}, t_{ki2}, ..., t_{kiN})$.
- **Search for global best position:** Firstly profit for the tour schedule TR_k is calculated due to different particles ST_{ki}. The initial value of $GBST_k$ is considered as the maximum profit given by the particles. $GBST_k$ is updated if after each iteration particles get new better positions.
- **Search for personal best position:** Personal best position $PBST_{ki}$ is initialized with the initial position of a particle. If new position after each iteration gives better profit, then $PBST_{ki}$ is replaced by the new position.
- **Velocity Updating :** stay times at different cities i.e., for every particle of a path, velocity updation is done by following expression.

$$V_{kij}(t+1) = w \times V_{kij}(t) + C_1 r_{1j(t)}[PBST_{kij} - PPB_{kij}] + C_2 r_{2j(t)}[GBST_{kj} - PPB_{kij}]$$

Where PPB_{ki} represent the position of i -th particle of swarm k, at time step t the velocity of particle i in j -th dimension is represented as $V_{kij}(t)$. At time step $t+1$ the velocity of particle i in j-th dimension is represented as $V_{kij}(t+1)$. Here two positive acceleration constants are taken, where $C_1 = 1.49618$, $C_2 = 1.79618$ and weight $w = 0.7298$. Two random values between $[0,1]$ are consider $r_{1j}(t)$ and $r_{2j}(t)$ $: U(0,1)$

- **Updating position:** Updation of Each particle of k-th swarm is done by using the following expression

$$PPB_{ki}(t+1) = PPB_{ki}(t) + V_{ki}(t+1)$$

(e) Pheromone Evaporation: To evaporate pheromone, the following equation is used

$$\tau_{ijrv} = (1-\rho)\tau_{ijrv}$$

Here the value of constant ρ is in $[0,1]$. The ρ specifies the rate of pheromone evaporation, which helps ants to forget the previous decisions.

(f) Pheromone Updating : After construction of all paths, pheromone is increased on paths through which the ants move. The tour pheromone is increased by the following rules.

- If P_k be the profit of tour schedule TR_k, then for this path $\tau_{x_{ki}x_{ki+1}r_{ki}v_{ki}}$ is increased by $1/P_k^{\beta}$, where the parameter β is used to best fit the updating function.

4. Numerical experiments

For ten cities (N=10) using 3 routes and 3 types of vehicles, the models are illustrated. The assumed values of travel costs and times between different cities through different routes and vehicles are randomly generated within some specified values, (c_l, c_u) and (t_l, t_u) (using a relation with travel cost for the present problem). The return and expenditure of the i-th city are time dependent, i.e.,

$$\left. o_i(t) = a_i + b_i t - c_i t^2 \ \ and \ \ e_i(t) = e_{0i} t \right\} \tag{7}$$

4.1. Experiment-1: Data for different models (1A, 1B and 2)

As the input data for costs and times against different routes and conveyances are required, it becomes a huge data. For this reason, to illustrate the proposed model clearly, we consider a small 4D TSP consisting of 3 cities, 3 routes and 3 vehicles. The configuration and required data are given in Figure 1.

With these data, we formulate the Models 1A, 1B and 2 following Eq. 2, 4 an 6 respectively and solve using ACO-PSO algorithm presented in section 3. The optimum results are given in Table 1.

PN	**H**	**path**	**TT**	**ST**	**TC**	**R**	**E**	**Profit**
1A	-	$3\frac{2}{2}1\frac{2}{3}2\frac{3}{1}3$	27	-	50	-	-	-
1B	-	$3\frac{1}{2}2\frac{3}{3}1\frac{1}{1}3$	10	-	235	-	-	-
2	30	$2\frac{1}{1}1\frac{2}{2}3\frac{2}{3}1$	22	8.89,3.78,7.33	70	549.50	32.84	446.66

2	43	$1\frac{2}{3}2\frac{2}{3}3\frac{2}{2}1$	27	8.00,6.00,2.00	50	638.00	68.00	520.00
2	50	$1\frac{2}{2}3\frac{2}{3}2\frac{3}{2}1$	30	8.89,3.78,7.33	50	646.90	87.56	509.33

Table 1: Total tour times (H hours), Optimum paths, Stay Time (ST hours), Travel costs (TC in \$), Travel Time (TT hours), Returns(R), Expenditure(E) and Profit (in \$)for different models of 3 size problem.

4.2. Experiment-2: Data and result for models (1A, 1B, 2) of different cities

Now, we demonstrate our model formulation and algorithm for large 4D- TSP problems- say $((10\times10),(20\times20),(40\times40),(60\times60)\ and\ (100\times100)\times3\times3)$ with some virtual data.

The values of returns $o_i(t)'s$ and expenditures $e_i(t)'s$ for first 10 cities are given in Table 2. For other cities, returns and expenditure values are obtained by repeating the these values in the same order.

i/j	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$\mathbf{a_i}$	150	60	140	70	80	110	90	100	60	140
$\mathbf{b_i}$	24	33	60	47	65	64	31	18	110	116
$\mathbf{c_i}$	2	1	3	4	2	3	2	1	4	5
$\mathbf{(e_{0i})}$	8	9	12	7	13	10	7	8	14	6

Table 2: Returns (o_i)'s with constants a_i, b_i, c_i and Expenditure (e_{0i}) in different cities.

TSP with virtual data set

Here we generate large TSP problems with virtual data set, travel costs (c_{ijrv}) and travel times (t_{ijrv}) for 20, 40, 60 and 100 cities. These values are randomly generated within some specified values, (c_l, c_u) and (t_l, t_u) (Normally, when travel time is less (high), then travel cost is high (less). The feature is followed in some cases during the creation of these data). The above ranges differ for different problems. Random data sets are generated using rand() function in C++ language. For the present problem, t_{ijrv} s are obtained as

$$t_{ijrv} = random\ number\ in\ [t_l, t_u] + (c_n/c_{ijrv}),$$

where $t_l = 0, t_u = 5\ and\ c_n = 200$. The returns and expenditures at different cities for large TSPs are assumed respectively as the returns $o_i(t)s$ and expenditures $e_i(t)s$ at the i-th city following the earlier Eq. 7 .

With these data, TSPs of different orders are solved by the proposed hybrid algorithm. The detailed results including paths and vehicles are given only for $((10\times10)\times3\times3)$ TSP in Table 3.

For other large TSPs, only the main results are given in Table 4.

PN	H	Path	TT	ST	TC	R	E	Profit
1A	-	$0\frac{3}{1}7\frac{1}{2}1\frac{1}{2}2\frac{3}{1}4\frac{1}{1}5\frac{3}{2}8\frac{1}{1}3\frac{3}{3}6\frac{3}{3}9\frac{1}{1}0$	98	-	70	-	-	-
1B	-	$5\frac{1}{2}6\frac{3}{1}1\frac{2}{3}8\frac{3}{1}0\frac{1}{2}4\frac{1}{2}3\frac{3}{3}7\frac{1}{1}9\frac{1}{1}2\frac{3}{3}5$	17	-	725	-	-	-
2	140	$4\frac{3}{2}0\frac{1}{1}1\frac{1}{1}8\frac{3}{2}3\frac{1}{1}7\frac{1}{2}5\frac{2}{1}9\frac{3}{3}6\frac{3}{3}2\frac{1}{2}4$	73	9.67,1.97,8.86,10.52,4.12, 3.04,7.95,10.28,4.46,6.13	145	4277.5	667.6	3464.9
2	160	$6\frac{3}{3}2\frac{1}{2}4\frac{3}{1}5\frac{1}{1}8\frac{3}{2}3\frac{1}{1}7\frac{1}{2}1\frac{2}{3}0\frac{1}{1}9\frac{3}{3}6$	80	5.45,7.63,12.11,8.88,11.42, 4.47,4.22,11.22,3.98,10.63	80	4477.3	797.3	3599.9
2	180	$4\frac{3}{1}5\frac{1}{1}8\frac{3}{2}3\frac{1}{1}6\frac{3}{3}9\frac{1}{1}0\frac{3}{1}7\frac{1}{2}1\frac{3}{1}2\frac{1}{2}4$	98	12.69,8.80,11.84,4.83,5.67, 10.88,3.70,4.38,11.39,7.80	70	4503.5	818.4	3615.2
2	183	$7\frac{1}{2}1\frac{3}{1}2\frac{1}{2}4\frac{3}{1}5\frac{1}{1}8\frac{3}{2}3\frac{1}{1}6\frac{3}{3}9\frac{1}{1}0\frac{3}{1}7$	98	5.03,12.00,8.00,13.01,9.00, 11.99,5.00,6.00,11.00,3.97	70	4533	846.1	3617
2	186	$0\frac{3}{1}7\frac{1}{1}3\frac{3}{2}8\frac{1}{1}5\frac{3}{1}4\frac{1}{2}2\frac{3}{1}1\frac{1}{2}6\frac{3}{3}9\frac{1}{1}0$	103	3.83,4.80,4.86,11.89,8.84, 12.64,7.83,11.61,5.78,10.92	70	4512.7	826.67	3616.1
2	200	$1\frac{3}{1}2\frac{1}{2}0\frac{3}{1}7\frac{1}{2}3\frac{3}{3}6\frac{1}{1}9\frac{3}{1}4\frac{1}{1}5\frac{3}{2}8\frac{1}{1}1$	103	9.82,14.24,8.81,14.47,7.22, 11.49,5.24,7.49,5.63,12.61	70	4613.7	956.3	3587.4
2	220	$1\frac{3}{1}2\frac{1}{2}0\frac{3}{1}7\frac{1}{2}3\frac{3}{3}6\frac{1}{1}9\frac{3}{1}4\frac{1}{1}5\frac{3}{2}8\frac{1}{1}1$	108	17.56,9.84,6.8,10.5,6.37, 8.8,12.12,15.78,10.83,13.3	95	4631.5	1094.3	3442.2

Table 3: H, ST, TC, TT, R, E and Profit for different models of 10 size problem using 3 root and 3 vehicle.

Sl no	problem size	H	TT	ST	TC	Returns(R)	Expenditure(E)	Profit
1	20	300 350 400	198 236 282	102 114 118	205 164 148	7142.2 8390.2 8512.5	1593.7 1723.6 1995.91	5343.4 6502.6 6368.6
2	40	750 800 850	422 458 494	328 342 356	629 498 481	15102.4 15050.7 15218.7	3196.6 3335.2 3452.6	11276.7 11216.7 11284.3
3	60	1000 1100 1200	470 504 954	530 596 546	1173 999 1003	24248.4 24335.1 24829.6	4711.3 4827.35 5321.18	18364.1 18508.8 18435.4
4	100	1800 1900 2000	833 1218 1065	976 682 935	1241 1298 1312	36918.6 38075.9 39038.6	8321.3 8491.2 8645.2	21879.32 27723.64 26111.3

Table 4: H, ST, TC, TT, R, E and Profit for different Models of different city problem.

5. Particular models: formulation and illustration

Formulation

From these 4D TSPs (Models - 1A, 1B and 2), in particular, two 3D TSPs and one general (2D) TSP for each model are formulated and solved.

3D TSP with single route multiple conveyance (Models - 1A-3a, 1B-3a, 2-3a)

In the above expressions of Models - 1A, 1B and 2, putting $r = r_1$ (i,e. choosing only one route), we get the 3D (solid) TSP with cities and different conveyances.

3D TSP with multiple route single conveyance (Models - 1A-3b, 1B-3b, 2-3b)

As before, putting $v = v_1$ (i,e. choosing only one vehicle), in Eq. 2, 4 and 6, we get another type of solid TSPs with cities and different routes.

2D general TSP (Models - 1A-2, 1B-2, 2-2)

Putting $r = r_1$ and $v = v_1$ (i,e. choosing only one route and one vehicle) in Eq. 2, 4 and 6 in mathematical representations of general/conventional TSPs are obtained.

5.1. Experiment-3: Pictorial representations of 3D and 2D TSPs of $((3\times3)\times3\times3)$

The Models-1A-3a, 1B-3a, 2-3a, 1A-3b, 1B-3b, 2-3b and Models- 1A-2, 1B-2, 2-2 for the $((3\times3)\times3\times3)$ TSP are depicted in Figures 2, 3 and 4 respectively. The optimum results of these models are obtained by the proposed hybrid algorithm and placed in Table 5.

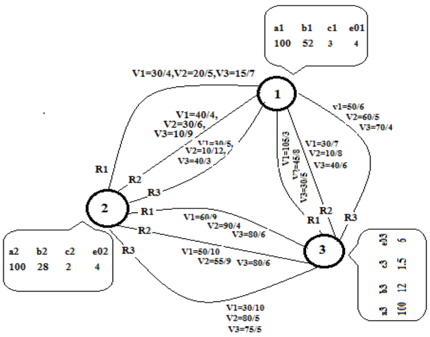

Figure 1: $((3\times3)\times3\times3)$ TSP for Models 1A, 1B, 2

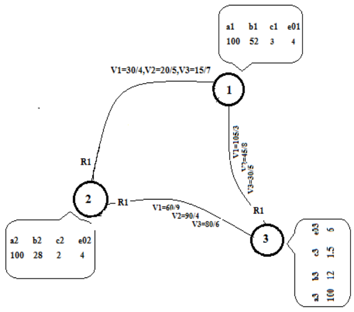

Figure 2: $((3\times3)\times1\times3)$ TSP for Models 1A-3a, 1B-3a, 2-3a

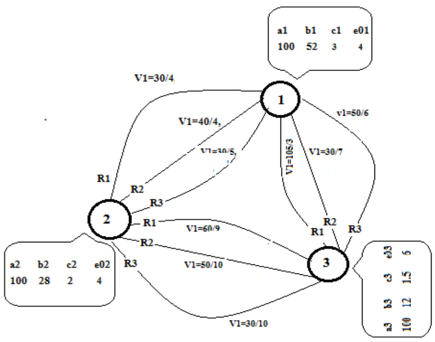

Figure 3: $((3\times3)\times3\times1)$ TSP for Models 1A-3b, 1B-3b, 2-3b

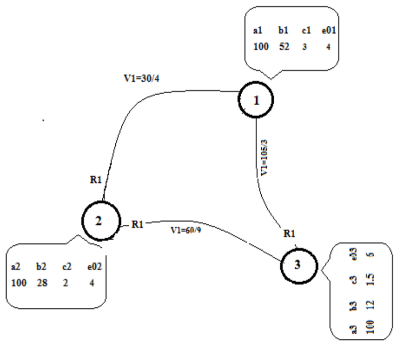

Figure 4: $((3\times3)\times1\times1)$ TSP for Models 1A-2, 1B-2, 2-2

PN	H	Path	TT	ST	TC	R	E	Profit
1A-3a	-	$1\frac{1}{3}3\frac{1}{1}2\frac{1}{3}1$	21	-	105	-	-	-
1B-3a	-	$1\frac{1}{1}3\frac{1}{2}2\frac{1}{1}1$	11	-	225	-	-	-
2-3a	37	$3\frac{1}{2}2\frac{1}{3}1\frac{1}{1}3$	21	2.0,6.0,8.0	105	630	89	436
1A-3b	-	$1\frac{2}{1}3\frac{3}{1}2\frac{1}{1}1$	21	-	90	-	-	-
1B-3b	-	$1\frac{1}{1}3\frac{1}{1}2\frac{1}{1}1$	16	-	195	-	-	-
2-3b	37	$3\frac{1}{1}2\frac{3}{1}1\frac{1}{1}3$	21	2.0,6.0,8.0	165	620	92	363
1A-2	-	$1\frac{1}{1}3\frac{1}{1}2\frac{1}{1}1$	16	-	195	-	-	-
1B-2	-	$1\frac{1}{1}3\frac{1}{1}2\frac{1}{1}1$	16	-	195	-	-	-
2-2	32	$3\frac{1}{1}2\frac{1}{1}1\frac{1}{1}3$	16	2.0,6.0,8.01	195	570	92	283

Table 5: H, ST, TC, TT, R, E and Profit for different models of 3 size problem.

5.2. Experiment-4: Ten cities 3D and 2D TSPs

As before, the input data of $((10\times10\times)3\times3)$ size 3D and 2D TSPs are randomly generated within two bounds (30, 85) and the optimum results of different TSP are obtained and given in Table 6.

Model no.	H	path	TT	ST	TC	R	E	Profit
1A-3a	-	$2\frac{1}{1}4\frac{1}{2}8\frac{1}{3}5\frac{1}{1}7\frac{1}{2}6\frac{1}{1}3\frac{1}{3}9\frac{1}{1}0\frac{1}{2}1\frac{1}{1}2$	96	-	124	-	-	-
1B-3a	-	$1\frac{1}{1}3\frac{2}{1}4\frac{2}{1}0\frac{2}{1}8\frac{3}{1}7\frac{2}{1}9\frac{3}{1}2\frac{2}{1}5\frac{1}{1}6\frac{3}{1}1$	25	-	745	-	-	-
2-3a	160	$3\frac{1}{3}0\frac{1}{1}7\frac{1}{2}2\frac{1}{1}6\frac{1}{1}5\frac{1}{1}9\frac{1}{2}1\frac{1}{3}4\frac{1}{1}8\frac{1}{2}3$	88	11.66,8.71,4.45,11.02,4.22,10.89,4.90,6.16,8.04,11.95	129	4442.35	813.98	3482.37
	170	$3\frac{1}{1}7\frac{1}{2}1\frac{1}{1}2\frac{2}{2}4\frac{1}{2}5\frac{1}{3}8\frac{1}{1}9\frac{1}{1}0\frac{1}{1}6\frac{1}{1}3$	86	5.01,4.84,11.84,7.97,12.92,8.79,11.97,10.92,3.86,5.87	134	4523.64	837.22	3552.41
	180	$9\frac{1}{2}6\frac{1}{1}3\frac{1}{1}7\frac{1}{2}5\frac{1}{3}8\frac{1}{1}4\frac{1}{2}2\frac{1}{1}1\frac{1}{2}0\frac{1}{1}9$	94	11.05,5.98,5.06,5.73,8.95,11.95,12.75,8.12,12.36,4.07	124	4539.10	853.97	3562.13
	190	$4\frac{1}{2}2\frac{1}{1}1\frac{1}{2}0\frac{1}{1}9\frac{1}{2}3\frac{1}{1}6\frac{1}{3}7\frac{1}{2}5\frac{1}{1}8\frac{1}{1}4$	99	13.57,8.38,13.25,4.60,11.26,5.31,6.64,6.28,9.39,12.31	127	4580.1	900.67	3552.44
	200	$7\frac{1}{1}3\frac{1}{1}6\frac{1}{2}2\frac{1}{1}1\frac{1}{3}0\frac{1}{1}9\frac{1}{3}4\frac{1}{1}8\frac{1}{1}5\frac{1}{2}7$	101	8.05,5.95,7.92,9.38,13.67,5.55,12.01,13.68,12.75,10.04	141	4612.8	970.6	3500.2
1A-3b	-	$9\frac{3}{1}8\frac{1}{1}5\frac{3}{1}4\frac{1}{1}2\frac{3}{1}1\frac{3}{1}3\frac{1}{1}7\frac{1}{1}6\frac{3}{1}0\frac{1}{1}9$	94	-	100	-	-	-
1B-3b	-	$1\frac{1}{1}3\frac{1}{1}4\frac{1}{1}0\frac{1}{1}8\frac{1}{1}7\frac{1}{1}9\frac{3}{1}2\frac{1}{1}5\frac{1}{1}6\frac{3}{1}1$	25	-	690	-	-	-
2-3b	160	$7\frac{1}{1}6\frac{3}{1}1\frac{3}{1}2\frac{1}{1}4\frac{3}{1}5\frac{1}{1}8\frac{2}{1}9\frac{1}{1}0\frac{1}{1}3\frac{1}{1}7$	88	3.00,5.06,9.12,7.10,10.71,8.27,10.70,10.67,3.14,4.22	123	4364.35	717.98	3524.37
	175	$7\frac{1}{1}6\frac{3}{1}0\frac{1}{1}9\frac{3}{1}8\frac{2}{1}5\frac{3}{1}4\frac{1}{1}2\frac{3}{1}1\frac{3}{1}3\frac{1}{1}7$	94	4.21,5.66,3.86,10.78,11.83,8.90,11.77,7.81,11.29,4.89	100	4488.64	806.22	3581.41
	185	$6\frac{1}{1}3\frac{3}{1}1\frac{2}{1}2\frac{1}{1}4\frac{2}{1}5\frac{1}{1}8\frac{3}{1}9\frac{1}{1}0\frac{2}{1}7\frac{1}{1}6$	101	5.90,4.95,11.80,7.94,12.91,8.93,11.94,10.96,3.89,4.79	100	4523.10	836.97	3586.13
	190	$3\frac{3}{1}1\frac{3}{0}2\frac{1}{1}4\frac{2}{0}5\frac{1}{1}8\frac{2}{1}9\frac{1}{1}0\frac{3}{1}7\frac{1}{1}6\frac{1}{1}3$	101	5.21,12.82,8.26,13.38,9.19,12.21,11.13,4.40,6.02,6.39	100	4566.1	882.67	3583.44
	200	$8\frac{1}{1}5\frac{3}{1}4\frac{3}{1}9\frac{1}{1}0\frac{3}{1}7\frac{1}{1}3\frac{1}{1}6\frac{1}{1}2\frac{3}{1}1\frac{1}{1}8$	106	12.48,10.04,12.99,11.24,5.70,7.50,5.50,7.07,8.54,12.94	100	4589.8	923.6	3565.2
1A-2	-	$0\frac{1}{}3\frac{1}{1}6\frac{1}{1}7\frac{1}{1}5\frac{1}{1}8\frac{1}{1}4\frac{1}{1}2\frac{1}{1}1\frac{1}{1}9\frac{1}{1}0$	93	-	155	-	-	-
1B-2	-	$1\frac{1}{1}3\frac{1}{1}4\frac{1}{1}0\frac{1}{1}8\frac{1}{1}7\frac{1}{1}9\frac{1}{1}2\frac{1}{1}5\frac{1}{1}6\frac{1}{1}1$	31	-	689	-	-	-
2-2	160	$2\frac{1}{1}4\frac{1}{1}5\frac{1}{1}8\frac{1}{1}9\frac{1}{1}0\frac{1}{1}6\frac{1}{1}3\frac{1}{1}7\frac{1}{1}1\frac{1}{1}2$	86	7.31,11.65,7.79,11.82,9.99,3.56,4.87,4.4,3.52,9.09	159	4403.43	745.08	3498.35
	168	$5\frac{1}{1}8\frac{1}{1}9\frac{1}{1}0\frac{1}{1}1\frac{1}{1}2\frac{1}{1}4\frac{1}{1}6\frac{1}{1}3\frac{1}{1}7\frac{1}{1}5$	88	8.6,11.78,10.78,3.55,10.93,7.69,12.52,5.5,4.75,3.82	156	4482.29	800.69	3526.59
	180	$2\frac{1}{1}4\frac{1}{1}8\frac{1}{1}5\frac{1}{1}7\frac{1}{1}6\frac{1}{1}3\frac{1}{1}0\frac{1}{1}9\frac{1}{1}1\frac{1}{1}2$	95	8.6,12.73,11.91,8.91,5.27,5.9,5.04,4.06,11.10,11.98	155	4530.54	843.37	3531.17
	190	$7\frac{1}{1}6\frac{1}{1}3\frac{1}{1}0\frac{1}{1}9\frac{1}{1}1\frac{1}{1}2\frac{1}{1}4\frac{1}{1}8\frac{1}{1}5\frac{1}{1}7$	95	7.09,7.1,5.54,5.03,11.38,14.00,8.7,14.00,12.54,9.59	155	4604.24	937.71	3511.61
	200	$4\frac{1}{1}2\frac{1}{1}1\frac{1}{1}0\frac{1}{1}9\frac{1}{1}3\frac{1}{1}6\frac{1}{1}7\frac{1}{1}5\frac{1}{1}8\frac{1}{1}4$	97	14.8,9.26,15.35,6.13,11.7,5.88,7.89,8.73,10.25,12.90	168	4631.01	1011.48	3452.53

Table 6: H, ST, TC, TT, R, E and Profit for different models of 10 size problem.

6. Discussion

Expected results are obtained from Models 1A and 1B, which is given in Table 3. In Model

1A, travel cost is minimized. Here the obtained minimum travel cost is 70$ but the travel time is high i.e., 98 hrs. reverse condition occurs in Model 1B. Here the total travel time is minimum i.e.,17 hrs but the travel cost is high (725$). In Model 2 the total time of a tour (H) is fixed. As the problem is of profit maximization, so, salesman tries to earn much by adjusting the time spend in different cities. At first if time for total tour is less i.e., upto H=80, TS adopts the minimum travel time. After this, if H is increased, TS makes a tread off between the travel cost and savings due to spend in cities. Now for reducing the travel cost, TS increases the travel time. So taking more travel time rather than minimum travel time TS manages the rest time of the total tour time by spending at different cities due to availability of more tour time. Here profit increases upto H=183 hours. If H is more, profit decreases because stay at different cities is not profitable any more, this is more clearly explained in Figure 5.

7. Model's behavioural studies

In this section, behaviour of the models by changing some model's parameters, are determined and these are graphically represented .

Profit vs tour time: Maximum profits for 10 cities problem are calculated by Model 2, against different total tour times and is plotted in figure 5. Here it is observed that maximum profit is at the tour time $TT = 183$. The interesting behaviour of the model is that with the increase of total tour time profit also increases upto certain TT and after that certain time even the TT increases the profit decreases. In this evolutionary process, TS makes maximum total return by allocating the total spend time at different cities and then suitable tour path is selected. If more time is available for travel, the TS adopts the comparatively minimum travel cost path. But if the TT is very high, then TS selects the minimum travel cost path and rest of the time is spend for stay time. Here we know that, if the stay time at each city exceeds a certain time, then return is less because of more expenditure. So in this model after certain tour time profit goes down. From this study, manager can fix the total tour time for a salesman which is very helpful in real life.

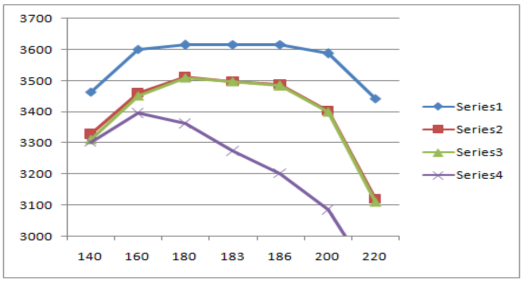

Figure 5: Tour time Vs. Profit of 10 cities

8. Conclusion

Till now, in the literature, profit maximization 4D TSP with cities, routes and vehicles are not formulated and solved. Here, a new real world TSP is proposed and solved. The conventional TSPs such general TSP(2D) and solid TSP(3D) can be derived from the presented model as particular cases, moreover, profit maximization of TSP which is realistic and practicable in the field of salesmen is formulated and solved. Here, a hybrid algorithm a combination of ACO and PSO are iteratively applied for the minimization tour cost/tour time and maximization of profit at the cities. Here, TSP Models are formulated as maximization problems. The model is illustrated with numerical examples and some interesting expected results are derived. For the first time, a hybrid algorithm- a combination of ACO and PSO has been successfully applied for the above mentioned TSP. Here, the model formulation and algorithm are quite general. The proposed algorithm can be modified by An Alternative Method to Find Initial Basic Feasible Solution [13]. The algorithm can also be designed for the transportation problems [1] and assessing problem [22]. The present TSP can also be extended to Fuzzy, Rough, Fuzzy-rough, Random etc., environments. TSPs with time windows also can be formulated and solved by the present algorithm.

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