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MHD Free Convective Flow Through Porous Medium Past an Infinite Vertical Plate

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Abstract. In this paper, the unsteady MHD free convective flow through porous medium over an infinite vertical plate under the influence of uniform transverse magnetic field of strength H_0 has been discussed. The flow is induced by a general time dependent movement of vertical plate. The exact solutions for the velocity, temperature and concentration are obtained making use of Laplace transform technique. The effect of various governing flow parameters on the velocity, temperature and concentration are analyzed, and numerical solutions for the skin friction, Nusselt number and Sherwood number are also obtained and discussed.

Keywords: Heat and mass transfer, infinite vertical flat plate, Laplace transform technique, MHD flows, porous medium

AMS Mathematics Subject Classification (2010): 80Axx, 80Mxx 76Wxx

1. Introduction
MHD free convection fluid flows frequently occur in natural world. Flow through porous medium is a great interest and attract towards the applications in the fields of science and technology namely in the area of agriculture engineering to know about the ground water resources, in fuel technology to study the moment of natural gas, oil, and water through the oil reservoirs. Studied the effects of chemical reaction, Studied radiation effects on MHD flow through a porous medium with variable temperature or variable mass diffusion. Kesavaiah et al. [1] Many authors [2] discussed MHD free convection heat and mass transfer flow embedded in porous medium with heat source and suction. Reddy et al. [3] discussed radiation absorption, chemical reaction and magnetic field effects on the free convection and mass transfer flow through porous medium. Raju et al. [4] studied unsteady MHD free convection and chemically reactive flow past an infinite vertical porous plate. Umamaheswar et al. [5] investigated heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Reddy et al. [6] had presented thermal radiation and chemical reaction effects on MHD mixed convective boundary layer slip flow in a porous medium with heat source and Ohmic heating. Rout et al. [7] discussed effect of radiation and chemical reaction on free convective MHD

flow through a porous medium with double diffusion. Effects of chemical reaction and radiation absorption on MHD flow of dusty visco-elastic fluid were considered by Prakash et al. [8] investigated combined effects of heat absorption and MHD on convective Rivlin-Ericksenflow past a semi-infinite vertical porous plate. Raghunath et al. [9] Studied MHD natural convective rotating flow of a viscous, electrically conducting and incompressible fluid over an impulsively moving vertical plate embedded in porous medium, Wahiduzzaman et al. [10] Studied Unsteady MHD-Free Convection Flow Past from a Rotating Vertical Plate with the Influence of Hall and Ion-slip Current.Md. Mizanur Rahman et al. [11] Studied Numerical Simulation on MHD Free Convection Mass and Heat Transfer Fluid Flow over a Vertical Porous Plate in a Rotating System with Induced Magnetic Field.

Keeping the above mentioned facts, in this paper, the unsteady MHD free convective flow through porous medium over an infinite vertical plate under the influence of uniform transverse magnetic field of strength H_0 has been discussed.

2. Mathematical formulation and solution of the problem

We consider the unsteady MHD free convection flow of an electrically conducting viscous incompressible fluid through porous medium over an infinite vertical plate subjected to a uniform transverse magnetic field of strength B_0 normal to plate. The physical configuration of the problem is presented in the Fig. 1.

We choose a Cartesian co-ordinate system $O(x, y, z)$ such that x, y axes respectively are in the vertical upward and perpendicular directions on the plane. i.e., the x-axis is along the plate in the upward direction, y-axis normal to it and z-axis normal to the xy-plane. The fluid is permeated by a uniform transverse magnetic field of strength B_0 applied to along the y-axis. For time $t < 0$, the stationary plate and the fluid are at the constant temperature T_∞ and species concentration C_∞
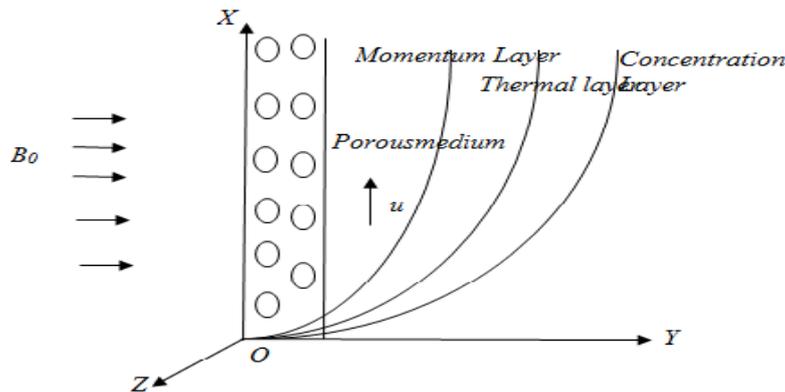

Figure 1: Physical configuration of the problem

At time $t = 0$, the plate begins to move with a time dependent velocity $u_0 f(t)$ in its own plane along the x-axis and the temperature of the plate is raised or lowered to $T_\infty + (T_w - T_\infty)\dfrac{t}{t_0}$, when $t < t_0$ and thereafter for $t > t_0$ it is maintained at a uniform

MHD Free Convective Flow Through Porous Medium Past an Infinite Vertical Plate

temperature T_w. Also for time $t > 0$ species concentration is raised to C_∞. Since the plate is of infinite extent in x and z direction, and is electrically non-conducting, all the physical quantities except pressure, are functions of y and t only. Magnetic Reynolds number is so small, and hence the induced magnetic field produced by the fluid motion is negligible in comparison to the applied magnetic field $B = (0, B_0, 0)$, the governing equations are given by,

$$\frac{\partial u}{\partial t} = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0}{\rho}u - \frac{v}{K_1}u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{1}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{Q_0}{\rho C_p}(T - T_\infty) \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial z^2} \tag{3}$$

Assuming no-slip between the plate and fluid, the initial and boundary conditions are,

$$u = 0, T = T_\infty, C = C_\infty, \text{ for } t \le 0 \text{ and } y \ge 0 \tag{4}$$

$$u = u_0 f(t), C = C_w, \text{ at } t > 0 \text{ and } y = 0 \tag{5}$$

$$T = T_\infty + (T_w - T_\infty)\frac{t}{t_0} \text{ at } y = 0 \text{ for } 0 < t \le t_0 \tag{6}$$

$$T = T_\infty \qquad \text{ at } y = 0 \text{ for } t > t_0 \tag{7}$$

$$u = 0, T \to T_\infty, C \to C_\infty, \text{ as } y \to \infty \text{ for } t > 0 \tag{8}$$

Introducing the following non-dimensional quantities:

$$y^* = \frac{y}{u_0 t_0}, u^* = \frac{u}{t_0}, t^* = \frac{t}{t_0}, T^* = \frac{T - T_\infty}{T_w - T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty}$$

Using non-dimensional quantities (dropping asterisks), the governing equations (1), (2) and (3) can be written as

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{K}\right)u + G_r T + G_m C \tag{9}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} - \varphi T \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{11}$$

where,

$M^2 = \dfrac{\sigma B_0^2 v}{\rho u_0^2}$ is the Hartmann number (Magnetic field parameter), $K = \dfrac{K_1 u_0^2}{v^2}$ is the

Porosity parameter, $G_r = \dfrac{g\beta v(T_w - T_\infty)}{u_0^3}$ is the thermal Grashof

421

number, $G_m = \dfrac{g\beta^* v(C_w - C_\infty)}{u_0^3}$ is the mass Grashof number, $\text{Pr} = \dfrac{\rho v C_p}{k}$ is Prandtl

parameter, $\varphi = \dfrac{Q_0 v}{\rho C_p u_0^2}$ is the Heat absorption parameter and $Sc = \dfrac{v}{D}$ is the Schmidt

number.

The corresponding non-dimensional boundary conditions

$u = 0, T = 0, C = 0,$ for $t \le 0$ and $y \ge 0$ (12)

$u = f(t), C = 1,$ for $t > 0$ and $y = 0$ (13)

$T = t$ for $0 < t \le 1$ at $y = 0$ (14)

$T = 1$ for $t > 1$ at $y = 0$ (15)

$u = 0, T = 0, C = 0,$ for $t > 0$ as $y \to \infty$ (16)

3. Results and discussion

The velocity, temperature and species concentration profiles are given in Figures (2-17), (18-21) and (22-23) respectively when $\omega = \pi/2$ for both ramped temperature and isothermal plates. The numerical values of the skin friction, Nusselt number and Sherwood number are presented in Tables (1-2) again for both ramped temperature and isothermal plates. The computational results carried out using Mathematica Software.

Figures (2-3 and 10-15) shown that for both ramped temperature and isothermal plates, the fluid velocity decreases with an increase in M, φ, Sc or Pr, whereas it increases with an increase in K, Gr or Gm (Figs 4-9). The application of a transverse magnetic field on an electrically conducting fluid gives rise to a resistive-type of force called the Lorentz force.

Figures (16-17) shows that for both ramped temperature and isothermal plates, the fluid velocity increases with an increase in t throughout the fluid region. The fluid velocity in the case of a flow past an isothermal plate is higher than that of a flow past a ramped temperature plate.

The computational results for the Concentration profiles for variation in t, and Sc are shown in Fig. (22-23). In the presence of porous matrix the concentration boundary layer became thinner and thinner as Schmidt number increases. It is observed that the concentration profile is asymptotic in nature. Concentration increases with time whereas it reduces with Schmidt number increases throughout the fluid region.

The numerical computation of skin friction coefficient, Nusselt number and Sherwood number is obtained and presented in Table (1-2).
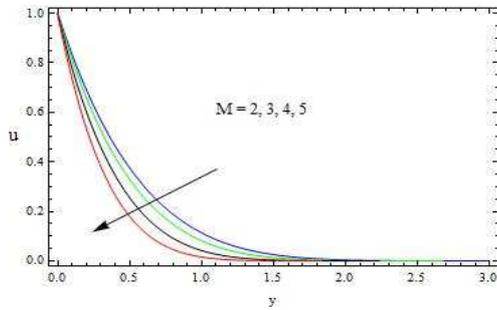
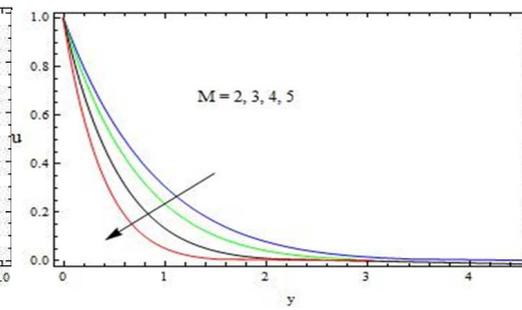**Figure 2:** The velocity profile against M for ramped temperature with K=1, Pr=0.71, Gr=2, Gm=2,

Figure 3: The velocity profile against M for isothermal plate with K=1, Pr=0.71, Gr=2, Gm=2, $\phi = 1$, Sc=0.22, t=0.2
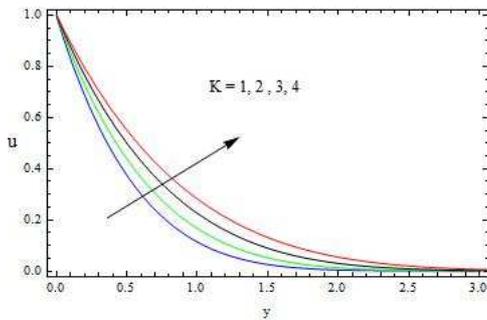
Figure 4: The velocity profile against K for ramped temperature with M=2, Pr=0.71, Gr=2, Gm=2, $\phi = 1$, Sc=0.22, t=0.2

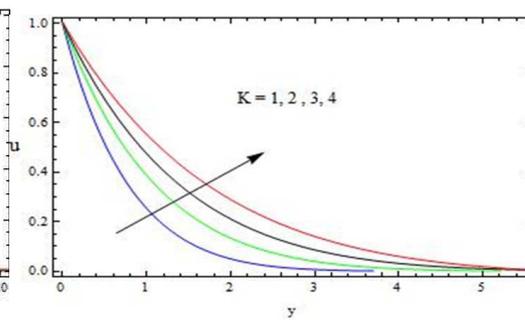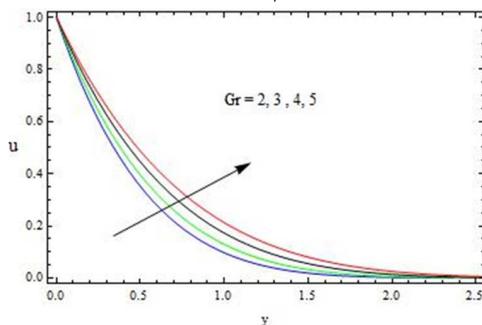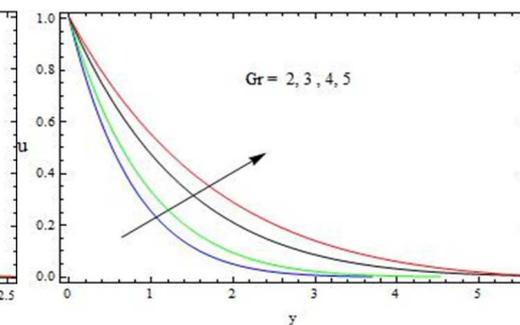**Figure 5:** Isothermal plate with M=2, Pr=0.71, Gr=2, Gm=2, $\phi = 1$, Sc=0.22, t=0.2

Figure 6: The velocity profile against Gr for ramped temperature with M=2, Pr=0.71, K=1, Gm=2, $\phi = 1$, Sc=0.22, t=0.2

Figure 7: The velocity profile against Gr for isothermal plate with M=2, Pr=0.71, K=1, Gm=2, $\phi = 1$, Sc=0.22, t=0.2

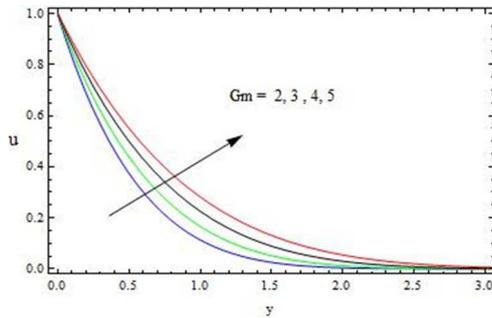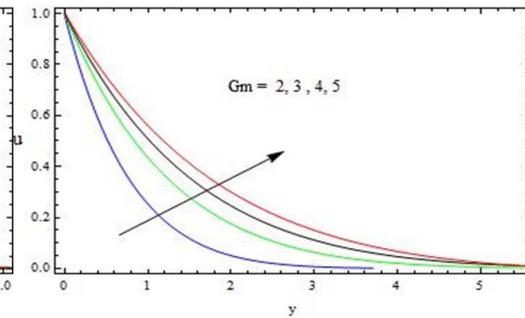**Figure 8:** The velocity profile against Gm for ramped temperature with M=2, Pr=0.71, K=1, Gr=2, $\phi = 1$, Sc=0.22,t=0.2

Figure 9: The velocity profile against Gm for isothermal plate with M=2, Pr=0.71, K=1, Gr=2, $\phi = 1$, Sc=0.22,t=0.2
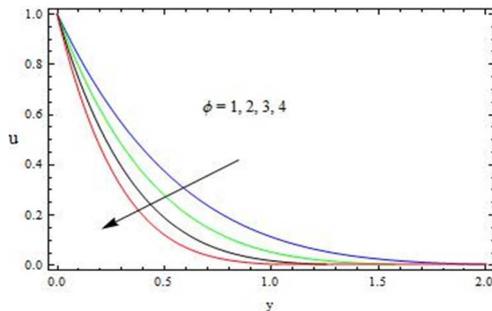

Figure 10: The velocity profile against ϕ for ramped temperature with M=2, Pr=0.71, K=1, Gr=2, Gm=2, Sc=0.22,t=0.2

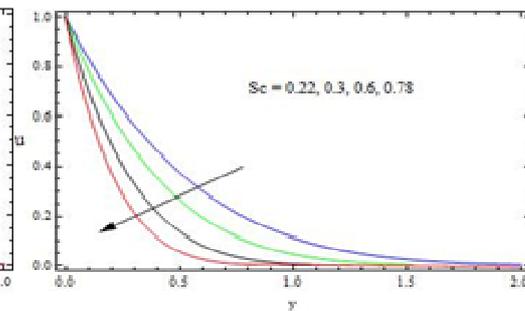**Figure 11:** The velocity profile against for isothermal plate with M=2, Pr=0.71, K=1, Gr=2, Gm=2, Sc=0.22,t=0.2

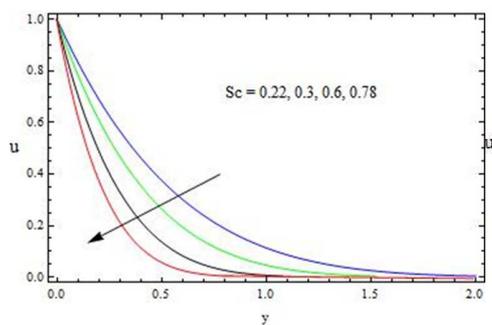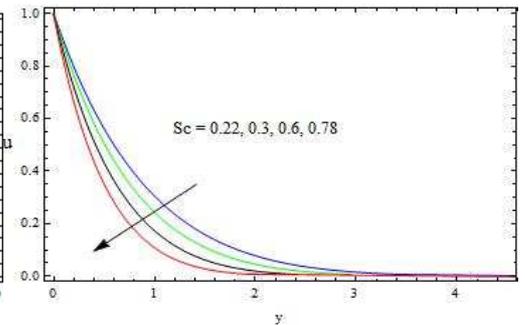**Figure 12:** The velocity profile against Sc for ramped temperature with M=2, Pr=0.71, K=1, Gr=2, Gm=2, ,t=0.2

Figure 13: The velocity profile against Sc for isothermal plate with M=2, Pr=0.71, K=1,Gr=2, Gm=2, $\phi = 1$,t=0.2
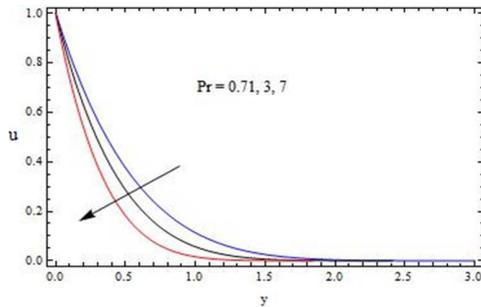
424

Figure 14: The velocity profile against Pr for ramped temperature with M=2, Sc=0.22, K=1, Gr=2, Gm=2, $\phi=1$, t=0.2
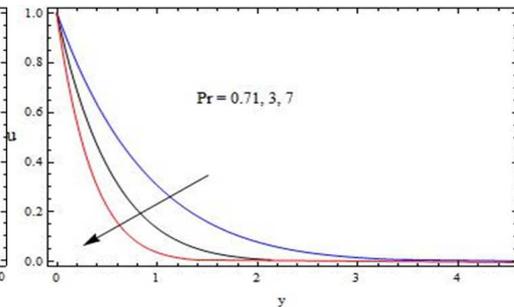
Figure 15: The velocity profile against Pr for isothermal plate with M=2, Sc=0.22, K=1, Gr=2, Gm=2, $\phi=1$, t=0.2
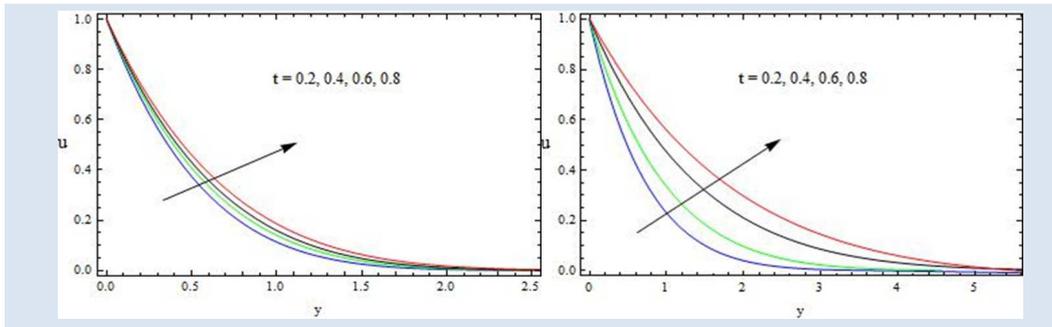

Figure 16: The velocity profile against t for ramped temperature with M=2, Sc=0.22, K=1, Gr=2, Gm=2, Pr=0.71

Figure 17: The velocity profile against t for isothermal plate with M=2, Sc=0.22, K=1, Gr=2, Gm=2, $\phi=1$, Pr=0.71
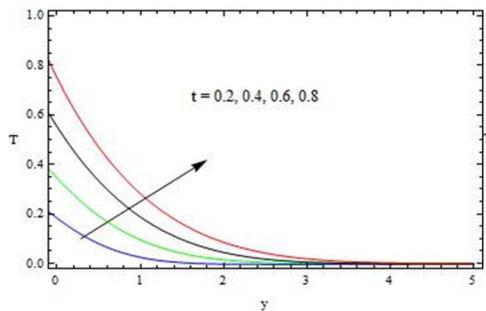

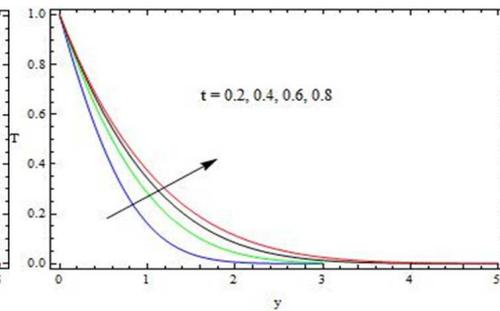**Figure 18:** The temperature profile against t for ramped temperature with $\phi=1$, Pr=0.71

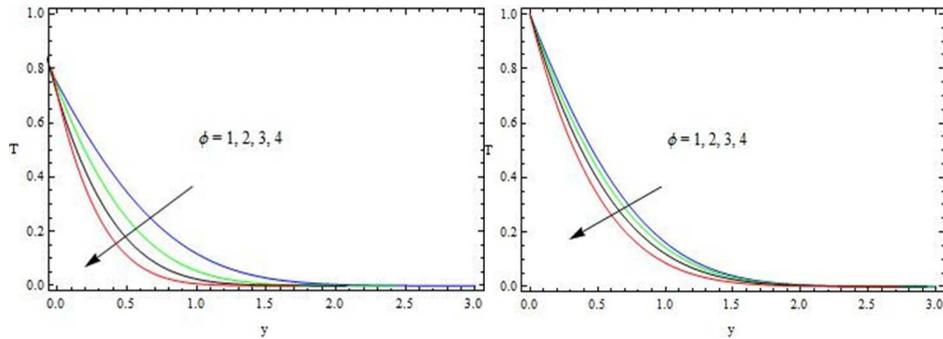**Figure 19:** The temperature profile against t for isothermal plate with $\phi=1$, Pr=0.71

Figure 20: The temperature profile against ϕ for ramped temperature with t=0.8,

Figure 21: The temperature profile against ϕ for isothermal plate with t=0.2, Pr=0.71
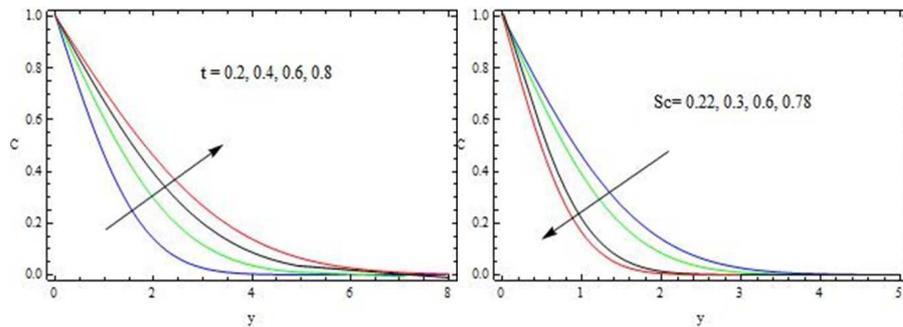

Figure 22: The concentration profile against ϕ with Sc=0.22

Figure 23: The Concentration profile against Sc with ϕ

Table 1: Skin friction

M	K	φ	Pr	Gr	Gm	Sc	t	τ_{11}	τ_{12}
2	1	1	0.71	2	2	0.22	0.2	-0.807552	-0.025256
3	1	1	0.71	2	2	0.22	0.2	-0.725452	-0.014855
4	1	1	0.71	2	2	0.22	0.2	-0.624989	-0.000455
2	2	1	0.71	2	2	0.22	0.2	-0.747748	-0.014452
2	3	1	0.71	2	2	0.22	0.2	-0.602784	-0.009996
2	1	2	0.71	2	2	0.22	0.2	-1.225632	-0.035625
2	1	3	0.71	2	2	0.22	0.2	-2.870014	-0.041475
2	1	1	3	2	2	0.22	0.2	-0.902747	-0.074452
2	1	1	7	2	2	0.22	0.2	-1.255440	-0.14445
2	1	1	0.71	3	2	0.22	0.2	-1.255522	-0.125210
2	1	1	0.71	4	2	0.22	0.2	-1.355455	-0.859963
2	1	1	0.71	2	3	0.22	0.2	-2.854748	-0.998546
2	1	1	0.71	2	4	0.22	0.2	-3.558966	-1.845510
2	1	1	0.71	2	2	0.6	0.2	-0.690748	-0.009633

Table 2: Nusselt number

t	ϕ	Pr	At Ramped temperature (Nu)	At isothermal plate (Nu)
0.2	1	0.71	0.857396	-0.658155
0.4	1	0.71	1.243430	-0.978467
0.6	1	0.71	1.589440	-1.283285
0.2	2	0.71	1.253952	-1.066595
0.2	3	0.71	1.624698	-1.447939
0.2	1	3	1.762434	-1.352882
0.2	1	7	2.692163	-2.066561

4. Conclusion

The conclusions are made as follows:
1. Heat absorption reduces the fluid temperature, whereas thermal diffusion and time have the opposite effect,
2. Heat absorption and the magnetic field tend to retard the fluid flow, whereas thermal diffusion, mass diffusion, thermal buoyancy force and mass buoyancy force and porosity have the opposite effect.
3. The mass diffusion rate and time tend to increase species concentration,
4. Heat absorption and the magnetic field tend to increase the shear stress at the plate, whereas thermal diffusion, thermal buoyancy force and mass buoyancy force have the reverse effect,
5. Heat absorption and time tend to increase the rate of heat transfer at the plate, whereas thermal diffusion has the reverse effect, and mass diffusivity and time tend to reduce the rate of mass transfer at the plate.

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