Annals of Pure and Applied Mathematics
Vol. 16, No. 2, 2018, 365-372
ISSN: 2279-087X (P), 2279-0888(online)
Published on 24 February 2018
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v16n2a13

Annals of **Pure and Applied Mathematics**

Further Results on Vertex Odd Divisor Cordial Labeling of Some Graphs

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Received 15 January 2018; accepted 13 February 2018

Abstract. In this paper we prove that the graphs $\prec F_n \blacktriangle F_n \succ$, Theta graph, Switching of a vertex in a Petersen graph, $\prec K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \succ$ and $\prec K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \succ$ are vertex odd divisor cordial graphs.

Keywords: Graph labeling, cordial labeling, divisor cordial labeling, vertex odd divisor cordial labeling.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Graph theory has several interesting applications in system analysis, operations research and economics. Since most of the time the aspects of graph problems are uncertain, it is nice to deal with these aspects via the methods of labeling. The concept of labeling of graphs is an active research area and it has been widely studied by several researchers. In a wide area network (WAN), several systems are connected to the main server, the labeling technique plays a vital role to label the cables. The labeling of graphs have been applied in the fields such as circuit design, communication network, coding theory, and crystallography.

A graph labeling, is a process in which each vertex is assigned a value from the given set of numbers, the labeling of edges depends on the labels of its end vertices. An excellent survey of various graph labeling problems, we refer to Gallian [2]. Two well known graph labeling methods are graceful labeling and harmonious labeling. These labelings are studied by Cahit [1].

Cordial labeling was introduced by Cahit [1]. Many labeling schemes were introduced with slight variations in cordial such as prime cordial labeling, divisor cordial labeling. Varatharajan et al. [10] have analyzed the divisor cordial labeling.The divisor cordial labeling of various types of graph is presented in [5,6,7,8,11] Muthaiyan et al. [4] introduce the concept of vertex odd divisor cordial graph. In section 2, we summarize the necessary definitions and basic results. In section 3, we proved that some standard graphs are vertex odd divisor cordial graph. We conclude in section 4.

2. Basic definitions

In this section, we provide a brief summary of the definitions and other results which are prerequisites for the present work.

All the graphs considered here are simple finite, undirected without loops and multiple edges. Let $G=(V,E)$ be a graph and as usual we denote $p=|V|$ and $q=|E|$. For terminology and notations not specifically defined here, we refer to Harary [3].
We recall the following definition from Harary [3].

Definition 2.1. Let $G=(V,E)$ be a graph. A mapping $f:V\to\{0,1\}$ is called the binary vertex labeling of G and $f(v)$ is called the label of the vertex $v\in V$ of G under f. The induced edge labeling $f^*:E\to\{0,1\}$ is given by $f^*(e)=|f(u)-f(v)|$, for all $e=uv\in E$.

We denote $v_f(i)$ is the number of vertices of G having label i under f and $e_f(i)$ is the number of edges of G having label i under f, where $i=0,1$. Now we define cordial labeling of a graph.

Definition 2.2. [1] Let $G=(V,E)$ be a graph and $f:V\to\{0,1\}$ be a binary vertex labeling of G. The map f is called a cordial labeling if $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.

A graph G is called cordial graph if it admits cordial labeling.

Definition 2.3. [10] A divisor cordial labeling of a graph $G=(V,E)$ is a bijection $f:V\to\{1,2,3,...,|V|\}$ such that if each edge uv is assigned the label 1 if $f(u)/f(v)$ or $f(v)/f(u)$ and the label 0 if $f(u) f(v)$, then $|e_f(0)-e_f(1)|\le 1$.

Definition 2.4. [4] A vertex odd divisor cordial labeling of a graph $G=(V,E)$ is a bijection $f:V\to\{1,2,3,...,2n-1\}$ such that if each edge uv is assigned the label 1 if $f(u)/f(v)$ or $f(v)/f(u)$ and the label 0 if $f(u) f(v)$, then $|e_f(0)-e_f(1)|\le 1$.

A graph which admits odd divisor cordial labeling is called a vertex odd divisor cordial graph.

Definition 2.5. [9] Consider two copies of graph G namely G_1 and G_2. Then the graph $G'=\prec G_1\blacktriangle G_2\succ$ is the graph obtained by joining the apex vertices of G_1 and G_2 by an edge as well as to a new vertex v'.

Note that $\prec G_1\blacktriangle G_2\blacktriangle G_3\succ=\prec (G_1\blacktriangle G_2)\blacktriangle G_3\succ$ and

$$\prec G_1\blacktriangle G_2\blacktriangle G_3\blacktriangle G_4\succ=\prec (G_1\blacktriangle G_2\blacktriangle G_3)\blacktriangle G_4\succ.$$

Definition 2.6. The friendship graph Fn is a planar undirected graph with 2n+1 vertices and 3n edges. The friendship graph Fn can be constructed by joining n copies of the cycle graph C_3 with a common vertex.

Definition 2.7. A vertex switching G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G.

3. Main results

Theorem 3.1. The graph $G = \prec F_n \blacktriangle F_n \succ$ is a vertex odd divisor cordial graph.

Proof: Let $G_1 = G_2 = F_n$. Let G be the graph $\prec G_1 \blacktriangle G_2 \succ$. Let u and v be the apex vertices of G_1 and G_2. Let Let $u_1, u_2, u_3, ..., u_{2n}$ be the vertices of G_1 and $v_1, v_2, v_3, ..., v_{2n}$ be the vertices of G_2 respectively. Let v' be the new vertex joining with the apex vertices u and v. Then $|V(G)| = 4n+3$ and $|E(G)| = 6n+3$. We define $f : V(G) \to \{1,3,5,...,8n+5\}$, as follows $f(u) = 1, f(v) = 3\, f(v') = p$ where p is the largest prime number such that $0 \prec p \le 8n+5$.

$$f(v_i) = \begin{cases} 9+6(i-1) & \text{if} \quad 1 \le i \le n \\ 11+6(i-n-1) & \text{if} \quad n+1 \le i \le 2n \end{cases}$$

The remaining labels are assigned to the vertices $u_1, u_2, u_3, ..., u_2 n$ in ascending order. From the above labeling pattern, we have $e_f(0) = 3n+1$ and $e_f(1) = 3n+2$. Therefore $|e_f(0) - e_f(1)| \le 1$.

Hence, G is a vertex odd divisor cordial graph.

Example 3.2. Vertex odd divisor cordial labeling of the graph $\prec F_7 \blacktriangle F_7 \succ$ is shown in Figure 1.

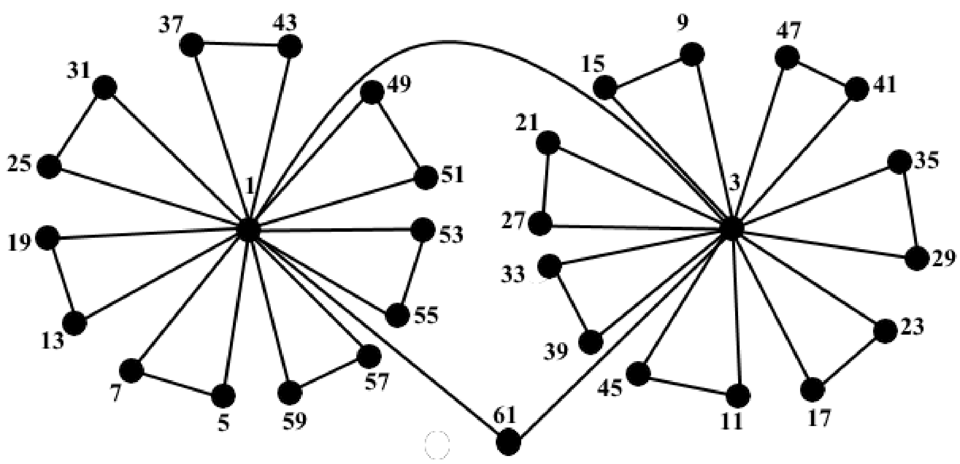

Figure 1: Vertex odd divisor cordial labeling of $\prec F_7 \blacktriangle F_7 \succ$

Theorem 3.3. Theta graph is a vertex odd divisor cordial graph.

Proof: Let $v_1, v_2, v_3, v_4, v_5, v_6$ be the vertices of Theta graph. Let G be the Theta graph. Then $|V(G)| = 6$ and $|E(G)| = 7$.

We define $f : V(G) \to \{1,3,5,7,9,11\}$ as follows

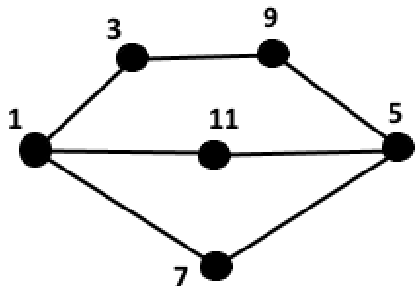

Figure 2: Vertex odd divisor cordial labeling of theta graph

$f(v_1) = 1, f(v_{i+1}) = 3 + 6(i-1), 1 \le i \le 2$.

Except the labels assigned from the set $\{1,3,5,...,11\}$ the remaining labels are assigned to v_4, v_5, v_6 in any order.We observe that, from the above labeling pattern, we have $e_f(1) = 4$ and $e_f(0) = 3$. Therefore $|e_f(0) - e_f(1)| \le 1$.

Hence G is a vertex odd divisor cordial graph.

Theorem 3.4. Switching of a vertex in a Petersen graph admits vertex odd divisor cordial graph.

Proof: Let G be the Petersen graph and let $V(G) = \{v, v_i : 1 \le i \le 9\}$ be the vertex set. Let G_v be the graph obtained from G by switching the vertex v.

Then $|V(G_v)| = 10$ and $|E(G_v)| = 18$.

We define $f : V(G) \to \{1,3,5,...19\}$, as follows

$f(v) = 1, f(v_i) = 3 + 6(i-1), 1 \le i \le 2\, f(v_3) = 7, f(v_{2i+2}) = 5 + 10(i-1), 1 \le i \le 2.$

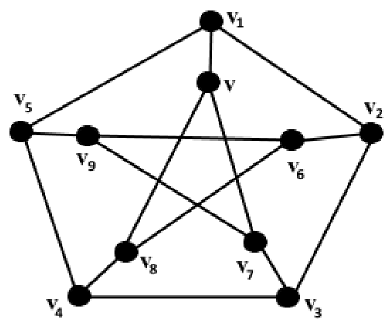

Figure 3: Petersen graph

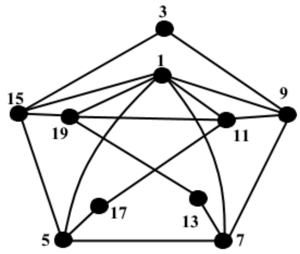

Figure 4: Switching of a vertex v in Petersen graph

Except the labels assigned from the set $\{1,3,5,...,19\}$ the remaining labels are assigned to $\{v_6, v_7, v_8, v_9, v_{10}\}$ in any order.

In view of above defined labeling pattern, we have $e_f(0) = e_f(1) = 9$.

Therefore $|e_f(0) - e_f(1)| \leq 1$.

Hence G is a vertex odd divisor cordial graph.

Theorem 3.5. The graph $G = \prec K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \succ$ is a vertex odd divisor cordial graph.

Proof: Let $G_1 = G_2 = G_3 = K_{1,n}$. Let G be the graph $G = \prec K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \succ$. Let u, v and w be the apex vertices of G_1, G_2 and G_3. Let $\{u_i, v_i, w_i : 1 \leq i \leq n\}$ be the pendant vertices, where $u_i, v_i, w_i (1 \leq i \leq n)$ are attached with u, v and w respectively. Let v' and v'' be the new vertices joining with apex vertices u and v, v and w respectively. Then $|V(G)| = 3n+5$ and $|E(G)| = 3n+6$.

We define $f : V(G) \rightarrow \{1,3,5,...,6n+9\}$, as follows

$f(u) = p, f(v) = 1, f(w) = 3, f(v') = 7, f(v'') = 5$ where p is the largest prime number such that $0 \prec p \leq 6n+9$.

case: 1 For $n = 2$.

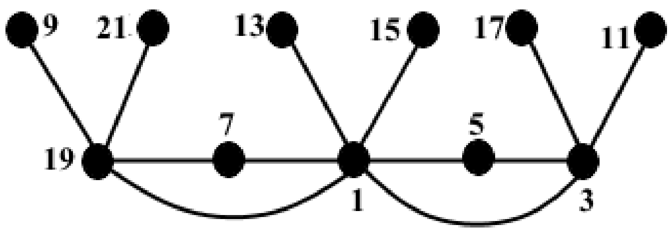

Figure 5: Vertex odd divisor cordial labeling of $\prec K_{1,2} \blacktriangle K_{1,2} \blacktriangle K_{1,2} \succ$

From the above figure we observe that $e_f(0) = 6 = e_f(1)$.

Hence $\prec K_{1,2} \blacktriangle K_{1,2} \blacktriangle K_{1,2} \succ$ is a vertex odd divisor cordial graph.

Case 2: when n is even $(n \succ 2)$

$$f(v_i)=\begin{cases}9+6(i-1) & \text{if} \quad 1\le i\le \frac{n-2}{2}\\ 11+6(i-\frac{n}{2}) & \text{if} \quad \frac{n}{2}\le i\le n\end{cases}$$

Case 3: when n is odd

$$f(v_i)=\begin{cases}9+6(i-1) & \text{if} \quad 1\le i\le \frac{n-1}{2}\\ 11+6(i-\frac{n-1}{2}) & \text{if} \quad \frac{n+1}{2}\le i\le n\end{cases}$$

Except the labels assigned from the set $\{1,3,5,...,6n+9\}$ the remaining labels are assigned to $u_i, v_i (1\le i\le n)$ in any order.
In view of above defined labeling pattern, we have

$$e_f(1)=\begin{cases}3n & \text{if} \quad \text{n is even}\\ 3n-1 & \text{if} \quad \text{n is odd}\end{cases} \text{ and}$$

$$e_f(0)=\begin{cases}3n & \text{if} \quad \text{n is even}\\ 3n-2 & \text{if} \quad \text{n is odd}\end{cases}$$

Therefore $|e_f(0)-e_f(1)|\le 1$.
Hence G is vertex odd divisor cordial labeling graph.

Example 3.6. Vertex odd divisor cordial labeling for $\prec K_{1,7} \blacktriangle K_{1,7} \blacktriangle K_{1,7} \succ$ and $\prec K_{1,8} \blacktriangle K_{1,8} \blacktriangle K_{1,8} \succ$ are shown in Figures 6 and 7.

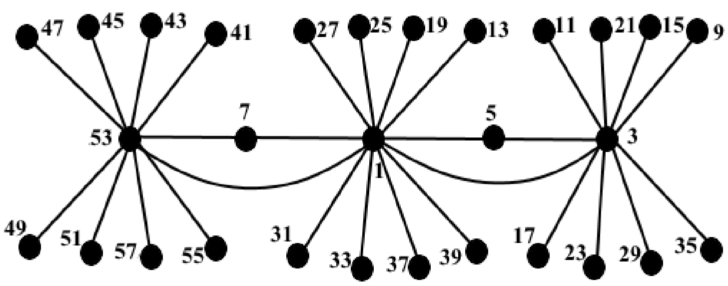

Figure 6: Vertex odd divisor cordial labeling of $\prec K_{1,7} \blacktriangle K_{1,7} \blacktriangle K_{1,7} \succ$

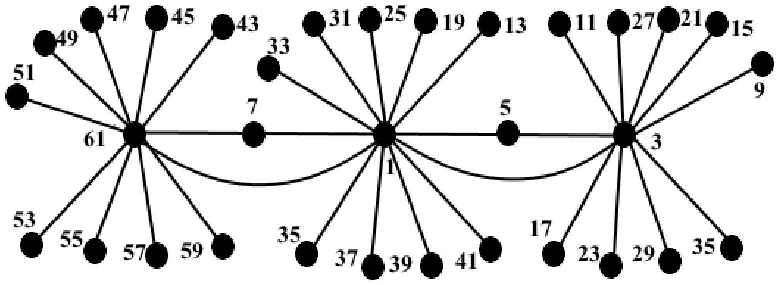

Figure 7: Vertex odd divisor cordial labeling of $\prec K_{1,8} \blacktriangle K_{1,8} \blacktriangle K_{1,8} \succ$

Theorem 3.7. The graph $G = \prec K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \succ$ is vertex odd divisor cordial graph.

Proof: Let $G_1 = G_2 = G_3 = G_4 = K_{1,n}$. Let G be the graph $G = \prec K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \succ$. Let u_1 , u_2 , u_3 and u_4 be the apex vertices of G_1 , G_2 , G_3 and G_4. Let $\{u_1^i, u_2^i, u_3^i, u_4^i : 1 \le i \le n\}$ be the pendant vertices are attached with u_1, u_2 , u_3 and u_4 respectively. Let v' , v'' and v''' be the new vertices joining with apex vertices u_1 and u_2, u_2 and u_3 , u_3 and u_4 respectively.

Then $|V(G)| = 4n+7$ and $|E(G)| = 4n+9$.

We define $f : V(G) \to \{1,3,5,...,8n+13\}$ as follows

$f(u_1) = p_1$,

$f(u_2) = 1$,

$f(u_3) = 3$,

$f(u_4) = p_2$,

$f(v') = 11$,

$f(v'') = 7$, $f(v''') = 5$

where p_1 and p_2 $(p_1 \ne p_2)$ are the largest prime number such that $0 \prec p_2 \prec p_1 \le 8n+13$. $f(u_3^i) = 9 + 6(i-1); 1 \le i \le n$ Except the labels assigned from the set $\{1,3,5,...,8n+13\}$ the remaining labels are assigned to $u_1^i, u_2^i, u_4^i (1 \le i \le n)$ in any order.

In view of above defined labeling pattern, we have $e_f(0) = 2n+5$ and $e_f(1) = 2n+4$.

Therefore $|e_f(0) - e_f(1)| \le 1$.Hence G is vertex odd divisor cordial labeling graph.

Example 3.8. Vertex odd divisor cordial labeling for $\prec K_{1,8} \blacktriangle K_{1,8} \blacktriangle K_{1,8} \blacktriangle K_{1,8} \succ$ is shown in Figure 8.

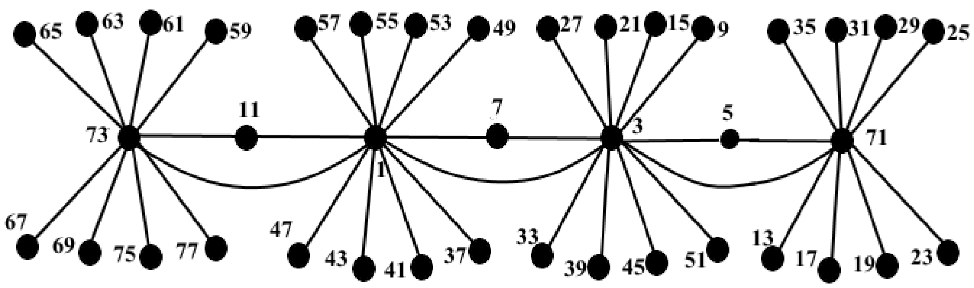

Figure 8: Vertex odd divisor cordial labeling of $\prec K_{1,8} \blacktriangle K_{1,8} \blacktriangle K_{1,8} \blacktriangle K_{1,8} \succ$

4. Conclusion

The vertex odd divisor cordial labeling is a variation of cordial labeling. It is very interesting to study graph or families of graph which are vertex odd divisor cordial as all the graphs do not admit vertex odd divisor cordial labeling. In this paper we proved that the graphs $\prec F_n \blacktriangle F_n \succ$, Theta graph, Switching of a vertex in a Petersen graph, $\prec K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \succ$ and $\prec K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \blacktriangle K_{1,n} \succ$ are vertex odd divisor cordial graphs.

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