

## On Decompositions of $(r^*g^*)^*$ Closed Set in Topological Spaces

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*Received 20 November 17; accepted 10 December 2017*

**Abstract.** The aim of this paper is to obtain decompositions of  $(r^*g^*)^*$  closed set. The concept of  $(r^*g^*)^*$  locally closed sets and  $(r^*g^*)^*$  locally continuous functions are introduced and some of their properties are investigated. Furthermore the notions of  $P^*$  sets,  $P^{**}$  sets,  $Q^{**}$  sets,  $W^*$  sets and  $A^*$  sets are introduced and are used to obtain the decompositions of  $(r^*g^*)^*$  closed sets.

**Keywords:**  $(r^*g^*)^*$  closed set,  $(r^*g^*)^*$  closure,  $(r^*g^*)^*$  continuous functions,  $(r^*g^*)^*$  irresolute functions,  $(r^*g^*)^*$  open sets.

**AMS Mathematics Subject Classification (2010):** 54A05

### 1. Introduction

Levin [10] introduced the concept of generalized closed set in topological spaces. The concept of locally closed sets in a topological space was introduced by Bourbaki [4]. Ganster and Reilly [5] further studied the properties of locally closed sets and defined the LC–continuity and LC–irresoluteness. Balachandran et al. [3] introduced the concept of generalized locally closed sets and GLC – continuous functions and investigated some of their properties. Arockiarani, Balachandran and Ganster [2] introduced regular generalized locally closed sets and RGL- continuous functions. The Authors [12] have already introduced  $(r^*g^*)^*$  closed sets and investigated some of their properties. The aim of this paper is to introduce  $(r^*g^*)^*$  locally closed set and  $(r^*g^*)^*$  locally continuous function and investigate some of their properties. Furthermore the notions of  $P^*$  sets,  $P^{**}$  sets,  $Q^{**}$  sets,  $W^*$  sets and  $A^*$  sets are used to obtain the decompositions of  $(r^*g^*)^*$  closed sets.

### 2. Preliminaries

**Definition 2.1.** A subset  $A$  of a Topological space  $X$  is called

- 1) A generalized closed set ( $g$ -closed) [10] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open.

- 2) A regular generalized closed set (rg-closed) [10] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open.
- 3) A  $(r^*g^*)^*$  closed set [12] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $r^*g^*$ - open. The complement of  $(r^*g^*)^*$  closed set is  $(r^*g^*)^*$  open.
- 4) A locally closed set [5] if  $A = S \cap F$  where  $S$  is open and  $F$  is closed.
- 5) A generalized locally closed set [3] if  $A = S \cap F$  where  $S$  is g-open and  $F$  is g-closed.
- 6) A  $glc^*$ -set [3] if  $A = S \cap F$  where  $S$  is g-open and  $F$  is closed.
- 7) A  $glc^{**}$ -set [3] if  $A = S \cap F$  where  $S$  is open and  $F$  is g-closed.
- 8) A regular generalized locally closed set [2] is  $S = G \cap F$  where  $G$  is rg-open and  $F$  is rg-closed in  $(X, \mathfrak{S})$ .
- 9) A  $rglc^*$  [2] if there exists a rg-open set  $G$  and a closed set  $F$  of  $(X, \mathfrak{S})$  such that  $S = G \cap F$ .
- 10) A  $rglc^{**}$ [2] if there exists an open set  $G$  and a rg-closed set  $F$  such that  $B = G \cap F$ .

**Definition 2.2.** A subset  $S$  of a topological space is called a

1.  $t$  set [17] if  $int(S) = int(cl(S))$ .
2.  $t^*$  set [7] if  $cl(S) = cl(int(S))$ .
3.  $\alpha^*$  set if [15]  $int(S) = int(cl(int(S)))$ .
4.  $C$  set [16] if  $S = G \cap F$  where  $G$  is open and  $F$  is a  $t$  set.
5.  $Cr$  set [16] if  $S = L \cap M$  where  $L$  is rg open and  $M$  is a  $t$  set.
6.  $Cr^*$  set [16] if  $S = L \cap M$  where  $L$  is rg open and  $M$  is a  $\alpha^*$  set.
7.  $A$  set if [18]  $S = G \cap F$  where  $G$  is open and  $F$  is a regular closed set.

**Definition 2.3.** Let  $X$  be a Topological space. Let  $A$  be a subset of  $X$ .  $(r^*g^*)^*$  closure [14] of  $A$  is defined as the intersection of all  $(r^*g^*)^*$  closed sets containing  $A$ .

**Definition 2.3.** A function  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  is called

- (i) g- continuous [10] if  $f^{-1}(V)$  is g closed in  $(X, \mathfrak{S})$  for every closed set  $V$  of  $(Y, \sigma)$ .
- (ii)  $(r^*g^*)^*$ -continuous [13] if the inverse image of every closed set in  $(Y, \sigma)$  is  $(r^*g^*)^*$ -closed in  $(X, \mathfrak{S})$
- (iii)  $(r^*g^*)^*$ -irresolute map [13] if  $f^{-1}(V)$  is a  $(r^*g^*)^*$ -closed set in  $(X, \mathfrak{S})$  for every  $(r^*g^*)^*$  closed set  $V$  of  $(Y, \sigma)$ .
- (iv) LC-continuous [5] if  $f^{-1}(V)$  is a locally closed set in  $(X, \mathfrak{S})$  for every open set  $V$  of  $(Y, \sigma)$ .
- (v) G LC-continuous [3] if  $f^{-1}(V)$  is a gl-closed set in  $(X, \mathfrak{S})$  for every open  $V$  of  $(Y, \sigma)$ .
- (vi) Rgl continuous [2] if  $f^{-1}(V)$  is a rgl closed set in  $(X, \mathfrak{S})$  for every open  $V$  of  $(Y, \sigma)$ .

### 3. $(r^*g^*)^*$ locally closed sets

**Definition 3.1.** A Subset  $S$  of  $(X, \mathfrak{S})$  is called  $(r^*g^*)^*$  Locally closed if  $S = A \cap B$  where  $A$  is  $(r^*g^*)^*$  open and  $B$  is  $(r^*g^*)^*$  closed.

**Example 3.2.** Let  $X = \{ a, b, c \}$ . Let  $\mathfrak{S} = (\emptyset, X, \{a\}, \{b\}, \{a,b\})$ .

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Closed sets are  $\{\varnothing, X, \{c\}, \{b,c\}, \{a,c\}\}$

$(r^*g^*)^*$  closed sets are  $\{\varnothing, X, \{c\}, \{b,c\}, \{a,c\}\}$

$(r^*g^*)^*$  open sets are  $\{\varnothing, X, \{a,b\}, \{a\}, \{b\}\}$

Now  $\{a\} = \{a,b\} \cap \{a,c\}$  where  $\{a,b\}$  is  $(r^*g^*)^*$  open and  $\{a,c\}$   $(r^*g^*)^*$  closed and hence  $\{a\}$  is a  $(r^*g^*)^*$  locally closed set.

Here  $\{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}$  are  $(r^*g^*)^*$  Locally closed sets.

**Definition 3.3.** A Subset  $S$  of  $(X, \mathfrak{T})$  is called  $(r^*g^*)^*$  Locally \* closed if  $S = A \cap B$  where  $A$  is  $(r^*g^*)^*$  open and  $B$  is closed.

**Example 3.4.** In Example 3.2  $\{b\} = \{a,b\} \cap \{b,c\}$  is  $(r^*g^*)^*$  Locally \* closed.

**Definition 3.5.** A subset  $S$  of  $(X, \mathfrak{T})$  is called  $(r^*g^*)^*$  locally \*\* closed if  $S = A \cap B$  where  $A$  is open and  $B$  is  $(r^*g^*)^*$  closed.

**Example 3.6.** In Example 3.2  $\{a\} = \{a,b\} \cap \{a,c\}$  is  $(r^*g^*)^*$  locally \*\*closed.

**Remark 3.7.** Every closed set is  $(r^*g^*)^*$  locally closed set.

#### Theorem 3.8.

- (i) Every Locally closed sets is  $(r^*g^*)^*$  locally closed.
- (ii) Every  $g^*$  locally closed set is  $(r^*g^*)^*$  locally closed
- (iii) Every  $(r^*g^*)^*$  locally closed set is gpr locally closed
- (iv) Every  $(r^*g^*)^*$  locally closed set is rwg locally closed
- (v) Every  $(r^*g^*)^*$  locally closed set is rg locally closed

#### Proof:

(i) Let  $S = A \cap B$  where  $A$  is open and  $B$  is closed in  $X$ . But every open set is  $(r^*g^*)^*$  open and every closed set is  $(r^*g^*)^*$  closed and hence  $S$  is  $(r^*g^*)^*$  locally closed set.

(ii) Proof follows from the fact that every  $g^*$  closed set is  $(r^*g^*)^*$  closed set [12].

(iii) Proof follows from the fact that every  $(r^*g^*)^*$  closed set is gpr closed set [12].

(iv) Proof follows from the fact that every  $(r^*g^*)^*$  closed set is rwg closed set [12].

(v) Proof follows from the fact that every  $(r^*g^*)^*$  closed set is rg closed set [12].

The converse of the above statements need not true as seen from the following example.

#### Example 3.9.

(i) Let  $X = \{a, b, c\}$ . Let  $\mathfrak{T} = \{\varnothing, X, \{c\}, \{b,c\}\}$

Closed sets are  $\{\varnothing, X, \{a\}, \{a,b\}\}$

$(r^*g^*)^*$  closed sets are  $\{\varnothing, X, \{a\}, \{a,b\}, \{a,c\}\}$

$(r^*g^*)^*$  open sets are  $\{\varnothing, X, \{b,c\}, \{c\}, \{b\}\}$

Here  $\{a,c\}$  is  $(r^*g^*)^*$  locally closed set but not locally closed set.

(ii) Let  $X=\{a,b,c\}$  ,  $\mathfrak{S} = \{ \varnothing ,X ,\{a\} \}$

Closed sets are  $\{ \varnothing ,X,\{a\} \}$

Here  $\{b\}$  is not a  $g^*$  locally closed set but it is a  $(r^*g^*)^*$  closed set.

(iii) let  $X=\{a,b,c,d\}$ ,  $\mathfrak{S} = ( \varnothing ,X ,\{a\},\{a,c\},\{a,d\},\{a,c,d\} )$

Closed sets are  $\{ \varnothing , X,\{b,c,d\},\{b,d\},\{b,c\},\{b\} \}$

Here  $\{c,d\}$  is gpr closed set but not  $(r^*g^*)^*$  closed set.

(iv) In the above example  $\{a,c,d\}$  is rwg closed set but not  $(r^*g^*)^*$  closed set.

(v) In the above example  $\{c,d\}$  is rg closed set but not  $(r^*g^*)^*$  closed set.

**Remark 3.10.**

$(r^*g^*)^*$  locally closed sets are independent of semilocally closed sets,  $\alpha$  locally closed sets, wg locally closed sets and the following examples support our statement.

**Example 3.11.**

Let  $X = \{a,b,c,d\}$  ,  $\mathfrak{S} = \{ \varnothing , X, \{a,b\} \{c,d\} \}$ . Here  $\{a\}$  is  $(r^*g^*)^*$  locally closed but not semi locally closed set.

Let  $X=\{a,b,c,d\}$ ,  $\mathfrak{S} = \{ \varnothing , X, \{a\},\{a,c\},\{a,d\},\{a,c,d\}$ . Here  $\{c,d\}$  is semi locally closed set but not  $(r^*g^*)^*$  locally closed set.

**Example 3.12.** Let  $X = \{a,b,c,d\}$ ,  $\mathfrak{S} = \{ \varnothing , X, \{a,c\}, \{a,d\} \{a\} , \{a,c,d\} \}$

Here  $\{a,c\}$  in not  $\alpha$  locally closed set but  $\{a,c\}$  is  $(r^*g^*)^*$  locally closed set

Here  $\{c,d\}$  is  $\alpha$  locally closed set but  $\{c,d\}$  is not  $(r^*g^*)^*$  locally closed.

**Example 3.13.** From the above example,  $\{c,d\}$  is wg locally closed set but  $\{c,d\}$  is not  $(r^*g^*)^*$  locally closed set.

Let  $X=\{a,b,c,d\}$ ,  $\mathfrak{S} = \{ \varnothing , X,\{a,b,c\},\{a,c,d\},\{a,c\},\{a\},\{c\} \}$ . Here  $\{a,d\}$  is  $(r^*g^*)^*$  closed set but not wg locally closed set.

**Theorem 3.14.** Every locally closed set is  $(r^*g^*)^*$  locally\* closed.

The converse need not be true as seen from the following example.

**Example 3.15.** In example 3.8  $\{a,c\}$  is  $(r^*g^*)^*$  locally\*closed but not locally closed.

**Theorem 3.16.** Every locally closed set is  $(r^*g^*)^*$  locally\*\* closed.

The converse need not be true as seen from the following example.

In example 3.9  $\{a,c\}$  is  $(r^*g^*)^*$  locally\*\*closed but not locally closed.

**Theorem 3.17.** If A is  $(r^*g^*)^*$  locally closed in X and B is  $(r^*g^*)^*$  open then  $A \cap B$  is  $(r^*g^*)^*$  locally closed in X.

**Proof:** Since A is  $(r^*g^*)^*$  locally closed  $A=P \cap Q$  where P is  $(r^*g^*)^*$  Closed and Q is  $(r^*g^*)^*$  open. Now  $A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B)$ .

Since  $(Q \cap B)$  is  $(r^*g^*)^*$  open and P is  $(r^*g^*)^*$  closed  $A \cap B$  is  $(r^*g^*)^*$  locally closed.

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**Theorem 3.18.** A subset  $S$  of  $(X, \mathfrak{T})$  is  $(r^*g^*)$  Locally closed  $(X - S)$  is the union of a  $(r^*g^*)$  open and a  $(r^*g^*)$  closed set.

**Proof:** If  $S$  is  $(r^*g^*)$  locally closed then  $S = P \cap Q$  where  $P$  is  $(r^*g^*)$  closed and  $Q$  is  $(r^*g^*)$  open. Now  $X - S = X - (P \cap Q) = (P \cap Q)^c = P^c \cup Q^c$ . Now  $P^c$  is  $(r^*g^*)$  open &  $Q^c$  is  $(r^*g^*)$  closed. Hence the result.

**Result 3.19.** The complement of a  $(r^*g^*)$  locally closed set need not be locally closed.

**Example 3.20.** Let  $X = \{a, b, c\}$ ,  $\mathfrak{T} = \{\emptyset, X, \{c\}, \{b, c\}\}$ .

Closed sets are  $\{\emptyset, X, \{a, b\}, \{a\}\}$

Here  $\{b\}$  is  $(r^*g^*)$  locally closed. But its complement  $\{a, c\}$  is not locally closed.

**Theorem 3.21.** Let  $A$  and  $B$  are subsets of  $(X, \mathfrak{T})$ . If  $A$  is  $(r^*g^*)$  locally closed and  $B$  is open then  $A \cap B$  is  $(r^*g^*)$  locally closed.

**Proof:** Let  $A$  be  $(r^*g^*)$  locally closed in  $(X, \mathfrak{T})$ . Then there exists an open set  $P$  and  $(r^*g^*)$  closed set  $Q$  such that  $A = P \cap Q$ . Now  $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$  which is  $(r^*g^*)$  locally closed set.

**Theorem 3.22.** If  $A$  is  $(r^*g^*)$  locally closed subset of  $(X, \mathfrak{T})$  and  $B$  is  $(r^*g^*)$  closed then  $A \cap B$  is  $(r^*g^*)$  locally closed.

**Proof:** Let  $A$  be  $(r^*g^*)$  locally closed. Then  $A = P \cap Q$  where  $P$  is  $(r^*g^*)$  open and  $Q$  is closed. Now  $A \cap B = (P \cap Q) \cap B = P \cap (B \cap Q)$ . Hence  $A \cap B$  is  $(r^*g^*)$  locally closed.

**Theorem 3.23.** If  $A$  is  $(r^*g^*)$  locally closed subset of  $(X, \mathfrak{T})$  and  $B$  is  $(r^*g^*)$  open then  $A \cap B$  is  $(r^*g^*)$  locally closed.

**Proof:** Let  $A$  be  $(r^*g^*)$  locally closed. Then  $A = P \cap Q$  where  $P$  is  $(r^*g^*)$  open and  $Q$  is  $(r^*g^*)$  closed. Now  $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$ . Hence  $A \cap B$  is  $(r^*g^*)$  locally closed.

#### 4. $(r^*g^*)$ locally continuous functions

**Definition 4.1.** A function  $f : (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  is called  $(r^*g^*)$  locally continuous, if  $f^{-1}(V)$  is  $(r^*g^*)$  locally closed in  $(X, \mathfrak{T})$  for every open set  $V$  in  $(Y, \sigma)$ .

**Example 4.2.** Let  $X = \{a, b, c\}$  and  $\mathfrak{T} = \{X, \emptyset, \{c\}, \{b, c\}\}$ .

Closed sets are  $\{X, \emptyset, \{a, b\}, \{a\}\}$ .  $(r^*g^*)$  Closed sets are  $\{X, \emptyset, \{a, b\}, \{a, c\}\}$ .  $(r^*g^*)$  open sets are  $\{X, \emptyset, \{b, c\}, \{c\}, \{b\}\}$ .

$(r^*g^*)$  locally closed set are  $\{X, \emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c\}, \{b\}\}$ .

Let  $Y = \{a, b, c\}$   $\sigma = \{Y, \emptyset, \{b\}\}$

Define  $f : (X, \mathfrak{T}) \rightarrow (Y, \sigma)$  defined by  $f(a) = a, f(b) = c, f(c) = b$ .

Now  $\{b\} \in \sigma$  and  $f^{-1}(\{b\}) = \{c\}$  which is  $(r^*g^*)$  locally closed in  $(X, \mathfrak{T})$ .

Hence  $f$  is  $(r^*g^*)$  locally continuous function.

**Definition 4.3.** A function  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  is said to be a  $(r^*g^*)^*$  locally irresolute function if  $f^{-1}(V)$  is a  $(r^*g^*)^*$  locally closed set in  $(X, \mathfrak{S})$  for every  $(r^*g^*)^*$  locally closed set  $V$  of  $(Y, \sigma)$ .

**Example 4.4.** Let  $X = \{a, b, c\}$   $\mathfrak{S} = \{\phi, X, \{a\}\}$ . Closed sets =  $\{\phi, X, \{b, c\}\}$   
 $(r^*g^*)^*$  closed sets are  $\{\phi, X, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, c\}\}$   
 $(r^*g^*)^*$  open sets are  $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c\}, \{a, c\}\}$   
 $(r^*g^*)^*$  locally closed sets are  $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, c\}\}$   
 $Y = \{a, b, c\}, \sigma = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ . Closed set of  $Y = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$   
 $(r^*g^*)^*$  closed set of  $Y$  are  $\{\phi, Y, \{c\}, \{b, c\}, \{a, c\}\}$   
 $(r^*g^*)^*$  open sets of  $Y$  are  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}\}$   
 $(r^*g^*)^*$  locally closed sets are  $\{\phi, Y, \{a\}, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, c\}\}$   
 Here Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  be defined by  $f(c)=c, f(b)=a, f(a)=b$   
 $f^{-1}(\{a\})=\{b\}, f^{-1}(\{b\})=\{a\}, f^{-1}(\{c\})=\{c\}$   $f^{-1}(\{a, b\})=\{a, b\}, f^{-1}(\{b, c\})=\{a, c\}, f^{-1}(\{a, c\})=\{b, c\}$   
 which are  $(r^*g^*)^*$  locally closed in  $(X, \mathfrak{S})$ . Hence  $f$  is a  $(r^*g^*)^*$  locally closed irresolute map.

**Theorem 4.5.** Every locally continuous function is  $(r^*g^*)^*$  locally continuous.

**Proof:** Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  be a locally continuous map. Let  $F$  be an open set in  $(Y, \sigma)$ . Then  $f^{-1}(F)$  is locally closed in  $(X, \mathfrak{S})$ . Since every locally closed set is  $(r^*g^*)^*$  locally closed,  $f^{-1}(F)$  is  $(r^*g^*)^*$  locally closed set. Therefore  $f$  is  $(r^*g^*)^*$  locally continuous. The converse need not be true as seen from the following example.

**Example 4.6.** Let  $X = \{a, b, c\}$   $\mathfrak{S} = \{\phi, X, \{c\}, \{b, c\}\}$ .  
 Closed set of  $X = \{\phi, X, \{a, b\}, \{a\}\}$   
 Locally closed sets are  $\{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$   
 $(r^*g^*)^*$  closed sets are  $\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$   
 $(r^*g^*)^*$  open sets are  $\{\phi, X, \{b, c\}, \{c\}, \{b\}\}$   
 $(r^*g^*)^*$  locally closed sets are  $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{c\}, \{b, c\}, \{a, c\}\}$   
 Let  $Y = \{a, b, c\}, \sigma = \{\phi, Y, \{b\}\}$ . Closed set of  $Y = \{\phi, Y, \{a, c\}\}$   
 Locally closed sets are  $\{\phi, Y, \{b\}, \{a, c\}\}$ . Define  $f$  by  $f(c)=a, f(b)=b, f(a)=c$ .  
 Now  $\{b\}$  is open in  $(Y, \sigma)$ .  $f^{-1}(\{b\})=\{b\}$  which is  $(r^*g^*)^*$  locally closed set. Therefore  $f$  is  $(r^*g^*)^*$  locally continuous. But  $f^{-1}(\{b\})=\{b\}$  is not locally closed in  $(X, \mathfrak{S})$ . Hence  $f$  is not locally continuous.

Similarly we can prove the following results.

**Theorem 4.7.**

- (i) Every  $g^*$  locally continuous function is  $(r^*g^*)^*$  locally continuous.
- (ii) Every  $(r^*g^*)^*$  locally continuous function set is  $gpr$  locally continuous function
- (iii) Every  $(r^*g^*)^*$  locally continuous function set is  $rwg$  locally continuous function
- (iv) Every  $(r^*g^*)^*$  locally continuous function set is  $rg$  locally continuous function.

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**Definition 4.8.** A function  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  is called  $(r^*g^*)^*$  locally  $*$ continuous, if  $f^{-1}(V)$  is  $(r^*g^*)^*$  locally  $*$ closed in  $(X, \mathfrak{S})$  for every  $V \in \sigma$ .

**Example 4.9.** In example 4.6 the function  $f$  is  $(r^*g^*)^*$  locally  $*$ continuous function.

**Definition 4.10.** A map  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  is said to be a  $(r^*g^*)^*$  locally  $*$  irresolute map if  $f^{-1}(V)$  is a  $(r^*g^*)^*$  locally  $*$ closed set in  $(X, \mathfrak{S})$  for every  $(r^*g^*)^*$  locally  $*$ closed set  $V$  of  $(Y, \sigma)$ .

**Example 4.11.** In example 4.4  $f$  is  $(r^*g^*)^*$  locally  $*$ irresolute.

**Definition 4.12.** A function  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  is  $(r^*g^*)^*$  locally  $**$  continuous, if  $f^{-1}(V)$  is  $(r^*g^*)^*$  locally  $**$  closed in  $(X, \mathfrak{S})$  for every  $V$  open in  $(Y, \sigma)$ .

In example 4.6 the function  $f$  is  $(r^*g^*)^*$  locally  $**$ closed continuous

**Definition 4.13.** A function  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  is said to be a  $(r^*g^*)^*$  locally  $**$ irresolute function if  $f^{-1}(V)$  is a  $(r^*g^*)^*$  locally  $**$ closed set in  $(X, \mathfrak{S})$  for every  $(r^*g^*)^*$  locally  $**$ closed set  $V$  of  $(Y, \sigma)$ .

**Example 4.14.** The function  $f$  defined in example 4.4 is a  $(r^*g^*)^*$  locally  $**$  irresolute.

**Theorem 4.15.** Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  be a function. If  $f$  is locally continuous, then it is  $(r^*g^*)^*$  locally  $*$ continuous and  $(r^*g^*)^*$  locally  $**$ continuous.

**Proof:** Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  be a locally continuous function. Let  $F \in \sigma$ . Then  $f^{-1}(F)$  is locally closed in  $(X, \mathfrak{S})$ . Since every locally closed set is  $(r^*g^*)^*$  locally  $*$ closed,  $f^{-1}(F)$  is  $(r^*g^*)^*$  locally  $*$  closed set. Therefore  $f$  is  $(r^*g^*)^*$  locally  $*$ continuous. Also since every locally closed set is  $(r^*g^*)^*$  locally  $**$ closed,  $f^{-1}(F)$  is  $(r^*g^*)^*$  locally  $**$ closed set. Hence  $f$  is  $(r^*g^*)^*$  locally  $**$ continuous.

The converse need not be true as seen from the following example.

**Example 4.16.** Let  $X = \{a, b, c\}$  and  $\mathfrak{S} = \{X, \phi, \{c\}, \{b, c\}\}$ .

Closed sets are  $\{X, \phi, \{a, b\}, \{a\}\}$

Locally closed sets of  $X$  are  $\{X, \phi, \{b\}, \{a, c\}\}$ .

$(r^*g^*)^*$  Closed sets of  $X$  are  $\{X, \phi, \{a\}, \{a, b\}, \{a, c\}\}$

$(r^*g^*)^*$  open sets of  $X$  are  $\{X, \phi, \{b, c\}, \{c\}, \{b\}\}$

$(r^*g^*)^*$  locally closed set of  $X$  are  $\{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c\}, \{b\}\}$

$(r^*g^*)^*$  locally  $*$  closed set of  $X$  are  $\{X, \phi, \{a\}, \{a, b\}, \{b, c\}, \{c\}, \{b\}\}$

$(r^*g^*)^*$  locally  $**$  closed set of  $X$  are  $\{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c\}\}$

Let  $Y = \{a, b, c\}$ ,  $\sigma = \{\phi, Y, \{a\}\}$  Closed set =  $\{\phi, Y, \{b, c\}\}$

Define a mapping  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  by  $f(a)=a$ ,  $f(b)=c$ ,  $f(c)=b$ .

Here  $f^{-1}\{a\}=\{a\}$  is  $(r^*g^*)^*$  locally  $*$ closed and  $(r^*g^*)^*$  locally  $**$ closed but not a locally closed set. Hence  $f$  is  $(r^*g^*)^*$  locally  $*$ closed continuous and  $(r^*g^*)^*$  locally  $**$ continuous but not locally continuous.

**Theorem 4.17.** Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  be a function. If  $f$  is  $(r^*g^*)^*$  locally continuous, then it is  $(r^*g^*)^*$  locally continuous.

**Proof:** Let  $f$  be  $(r^*g^*)^*$  locally continuous. Let  $V \in \sigma$ . Then  $f^{-1}(V)$  is  $(r^*g^*)^*$  locally closed.  $\therefore f^{-1}(V) = F \cap G$  where  $F$  is  $(r^*g^*)^*$  open and  $G$  is closed. But every closed set is  $(r^*g^*)^*$  closed.  $\therefore G$  is  $(r^*g^*)^*$  closed and.

$\therefore f^{-1}(V)$  is  $(r^*g^*)^*$  locally closed. Hence  $f$  is  $(r^*g^*)^*$  locally continuous.

The converse need not be true as seen from the following example.

**Example 4.18.** In example 4.16 Let  $Y = \{a,b,c\}$ ,  $\sigma = \{\phi, Y, \{b,c\}\}$ .

Closed set =  $\{\phi, Y, \{a\}\}$  Define  $f$  by  $f(a)=b$ ,  $f(c)=c$   $f(b)=a$ .

$f^{-1}\{b,c\}=\{a,c\}$  is  $(r^*g^*)^*$  locally closed and hence  $f$  is  $(r^*g^*)^*$  locally continuous but  $f^{-1}\{b,c\}=\{a,c\}$  is not  $(r^*g^*)^*$  locally closed. Therefore  $f$  is not  $(r^*g^*)^*$  locally continuous.

Similarly, we can prove the following theorem.

**Theorem 4.19.** Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  be a map. If  $f$  is  $(r^*g^*)^*$  locally continuous, then it is  $(r^*g^*)^*$  locally continuous.

The converse need not be true as seen from the following example.

In example 4.16 let  $Y = \{a,b,c\}$ ,  $\sigma = \{\phi, Y, \{b\}\}$ . Closed set =  $\{\phi, Y, \{a,c\}\}$  define  $f$  by  $f(b)=b$ ,  $f(c)=a$ ,  $f(a)=c$ . Now  $f^{-1}\{b\}=\{b\}$  is  $(r^*g^*)^*$  locally closed and hence  $f$  is  $(r^*g^*)^*$  locally continuous. But  $f^{-1}\{b\}=\{b\}$  is not  $(r^*g^*)^*$  locally closed in  $X$ . Therefore  $f$  is not  $(r^*g^*)^*$  locally continuous.

**Theorem 4.20.** Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  be a map. If  $f$  is  $(r^*g^*)^*$  locally irresolute, then it is  $(r^*g^*)^*$  locally continuous.

**Proof:** Let  $V \in \sigma$ . Then  $V=V \cap Y$ . Hence  $V$  is  $(r^*g^*)^*$  locally closed in  $Y$ . Since  $f$  is  $(r^*g^*)^*$  locally irresolute,  $f^{-1}(V)$  is  $(r^*g^*)^*$  locally closed. Now  $f^{-1}(V) = F \cap G$ , where  $F$  is  $(r^*g^*)^*$  open and  $G$  is closed. But every closed set is  $(r^*g^*)^*$  closed.  $\therefore f^{-1}(V)$  is  $(r^*g^*)^*$  locally closed. Hence  $f$  is  $(r^*g^*)^*$  locally continuous.

The converse need not be true as seen from the following example.

In example 4.16, let  $Y = \{a,b,c\}$ ,  $\sigma = \{\phi, Y, \{a,b\}\}$ . Closed set =  $\{\phi, Y, \{c\}\}$ . Let  $f$  be defined by  $f(a)=a$ ,  $f(c)=b$ ,  $f(b)=c$ . Now  $f^{-1}\{a,b\}=\{a,c\}$  is  $(r^*g^*)^*$  locally closed but not  $(r^*g^*)^*$  locally closed. Hence  $f$  is  $(r^*g^*)^*$  locally continuous but not  $(r^*g^*)^*$  locally irresolute.

**Remark 4.21.** Composition of two  $(r^*g^*)^*$  locally continuous function need not be  $(r^*g^*)^*$  locally continuous.

Let  $X=Y=\{a,b,c,d\}$ .

Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  where  $\mathfrak{S}=\{\phi, X, \{a,c\}, \{a,d\}, \{a\}, \{a,c,d\}\}$ .

$\sigma = \{\phi, X, \{a,b\}, \{c,d\}\}$ .

Let  $f$  be defined by  $f(a) = a$ ,  $f(d) = c$ ,  $f(c) = b$ ,  $f(b) = d$

$f^{-1}(a,b) = \{a,c\}$ ,  $f^{-1}(c,d) = \{d,b\}$

$f^{-1}(a,b)$  is  $(r^*g^*)^*$  locally closed.  $f^{-1}(\{c,d\})$  is  $(r^*g^*)^*$  locally closed. Hence

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$f : X \rightarrow Y$  is  $(r^*g^*)^*$  locally continuous

let  $g : (Y, \sigma) \rightarrow (Z, \eta)$  where  $\eta = \{ \varphi, X, \{a, b\} \}$  be defined by  $g(c) = b, g(a) = c, g(d) = d, g(b) = a$ .

$g^{-1}(\{a, b\}) = \{b, c\}$  is  $(r^*g^*)^*$  locally closed and hence  $g$  is  $(r^*g^*)^*$  locally continuous but  $(g \circ f)^{-1}(\{a, b\}) = f^{-1}(g^{-1}(\{a, b\})) = f^{-1}(\{b, c\}) = \{c, d\}$  is not  $(r^*g^*)^*$  locally closed. Hence  $g \circ f$  is not  $(r^*g^*)^*$  locally continuous.

The following theorem gives the condition under which the composition of two functions is  $(r^*g^*)^*$  locally continuous.

**Theorem 4.22.** Let  $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$  and  $g : (Y, \sigma) \rightarrow (Z, \eta)$  be two function. Then

- 1)  $g \circ f$  is  $(r^*g^*)^*$  locally continuous if  $g$  is  $(r^*g^*)^*$  locally continuous and  $f$  is  $(r^*g^*)^*$  locally irresolute
- 2)  $g \circ f$  is  $(r^*g^*)^*$  locally irresolute if both  $f$  and  $g$  are  $(r^*g^*)^*$  locally irresolute.
- 3)  $g \circ f$  is  $(r^*g^*)^*$  locally\* continuous if  $g$  is  $(r^*g^*)^*$  locally\* continuous and  $f$  is  $(r^*g^*)^*$  locally\* irresolute.

### 5. Another decomposition of $(r^*g^*)^*$ closed sets

The following definitions are introduced to obtain decompositions of  $(r^*g^*)^*$  closed set.

**Definitions 5.1.** A subset  $A$  of a topological space  $X$  is called a

- 1)  $P^*$  set if  $A = L \cap M$  where  $L$  is  $(r^*g^*)^*$  open and  $M$  is a  $t$  set.
- 2)  $P^{**}$  set if  $A = L \cap M$  where  $L$  is  $(r^*g^*)^*$  open and  $M$  is a  $t^*$  set.
- 3)  $Q^{**}$  set if  $A = L \cap M$  where  $L$  is  $(r^*g^*)^*$  open and  $M$  is a  $C$  set.
- 4)  $W^*$  set if  $A = L \cap M$  where  $L$  is  $(r^*g^*)^*$  open and  $M$  is an  $\alpha^*$  set.
- 5)  $A^*$  set if  $A = L \cap M$  where  $L$  is  $(r^*g^*)^*$  open and  $M$  is a regular closed set.

### Propositions 5.2.

1. Every  $C$  set is a  $P^*$  set
- 1) Every  $P^*$  set is  $Cr$  set.
- 2) Every  $W^*$  set is  $Cr^*$  set.
- 3) Every  $A$  set is  $A^*$  set.
- 4) Every  $A^*$  set is  $P^{**}$  set.
- 5) Every  $t$  set is  $P^*$  set.
- 6) Every  $C$  set is  $Q^{**}$  set.
- 7) Every  $\alpha^*$  set is  $W^*$  set.
- 8) Every  $(r^*g^*)^*$  open set is  $P^*$  set.
- 9) Every  $(r^*g^*)^*$  open set is  $W^*$  set.

**Remark 5.3.** The converses need not be true as seen from the following examples.

Example 1. Let  $X = \{a, b, c\}$   $\mathfrak{S} = \{\emptyset, X, \{a\}, \{b, c\}\}$ . Here  $\{b\}$  is  $P^*$  but not  $C$ .

Example 2. Let  $X = \{a, b, c\}$   $\mathfrak{S} = \{\emptyset, X, \{b\}, \{a, b\}\}$ . Here  $\{b, c\}$  is a  $Cr$  set but not a  $P^*$  set.

Example 3. In example 2  $\{b, c\}$  is  $Cr^*$  but not  $W^*$ .

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Example 4. In example 2  $\{b,c\}$  is  $P^{**}$  but not  $A^*$ .

Example 5. In example 2  $\{a\}$  is  $A^*$  but not  $A$ .

Example 6. In example 2  $\{a,b\}$  is  $P^*$  but not  $t$

Example 7. In example 1  $\{a,b\}$  is  $Q^{**}$  but not  $C$ .

Example 8. In example 2  $\{b\}$  is  $w^*$  but not  $\alpha^*$

Example 9. In example 2  $\{c\}$  is  $P^*$  but not  $(r^*g^*)^*\text{open}$ .

Example 10. In example 2  $\{a,c\}$  is  $W^*$  but not  $(r^*g^*)^*\text{open}$ .

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