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On Some Labelings of Subdivision of Snake Graphs

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Abstract. The graphs considered here are finite, undirected and simple. A triangular snake T_n is obtained from a path $u_1u_2...u_n$ by joining u_i and u_{i+1} to a new vertex w_i $for 1 \le i \le n-1$. A double triangular snake $D(T_n)$ is obtained from T_n by adding new vertex w_i for $1 \le i \le n-1$ and edges v_iw_i and w_iv_{i+1} for $1 \le i \le n-1$. A quadrilateral snake Q_n is obtained from a path $v_1v_2...\mathrm{v}_n$ by joining v_i and v_{i+1} to two new vertices u_i and w_i then joining u_i and w_i. Then the double quadrilateral snake $D(\mathrm{Q}_n)$ is obtained from Q_n by adding new vertices u_i' and w_i' for $1 \le i \le n-1$ and new edges $v_iu_i', \mathrm{w}_i' v_{i+1}, u_i'w_i'$ for $1 \le i \le n-1$. The subdivision of a graph is the graph obtained by subdividing each edge of a graph G is called the subdivision of G and is denoted by S(G). In this paper, the ways to construct square sum, square difference, strongly Multiplicative labeling for subdivision of triangular snake, double triangular snake, quadrilateral snake and double quadrilateral snake are reported.

Keywords: Triangular snake graph, double triangular snake graph, quadrilateral snake graph, double quadrilateral snake graph, Subdivision graph, Square sum labeling, Square difference labeling, Strongly Multiplicative labeling.

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1. Introduction

The graphs considered here are finite, undirected and simple. The concept of graph labeling was introduced by Rosa in 1967. Let G (V, E) be a (p, q) graph. The graph labeling is a very important topic of graph theory. It has many real life applications, for example see [9]. The square sum and square difference labelings were introduced by Ajitha, Arumugam and Germina [2]. Some basic concepts are taken from [1]. Some basic notations and definitions are taken from [3-8]. In this paper, we prove that the existence of square sum, square difference, strongly multiplicative labelings of subdivision of triangular snake S(T_n), subdivision of double triangular snake S(D(T_n)) and subdivision of quadrilateral snake S(Q_n), subdivision of double quadrilateral snake S(D(Q_n)) for all $n \geq 2$.

Definition 1.1. A triangular snake T_n is obtained from a path $v_1 v_2 \ldots v_n$ by joining v_i and v_{i+1} to a new vertex u_i for $1 \leq i \leq n-1$. A double triangular snake $D(T_n)$ is obtained from T_n by adding new vertex w_i for $1 \leq i \leq n-1$ and edges $v_i w_i$ and $w_i v_{i+1}$ for $1 \leq i \leq n-1$.

Definition 1.2. A quadrilateral snake Q_n is obtained from a path $v_1 v_2 \ldots v_n$ by joining v_i and v_{i+1} to two new vertices u_i and w_i then joining u_i and w_i. Then the double quadrilateral snake $D(Q_n)$ is obtained from Q_n by adding new vertices u_i' and w_i' for $1 \leq i \leq n-1$ and new edges $v_i u_i', w_i' v_{i+1}, u_i' w_i'$ for $1 \leq i \leq n-1$.

Definition 1.3. Let G be a (p, q) graph. A one-one map f : V(G) → {0, …, p–1} is said to be a square sum labeling if the induced map $f^*(uv) = (f(u))^2 + (f(v))^2$ is injective. It is said to be a square difference labeling if the induced map $f^*(uv) = (f(u))^2 - (f(v))^2$ is injective.

Definition 1.4. A (p,q) graph said to be strongly multiplicative if there exist a one-one map $f : V(G) \to \{1, 2, ..., p\}$ such that the induced map defined by $f^*(uv) = f(u) f(v)$ giving distinct edge values.

2. Main results

Theorem 2.1. The subdivision of triangular snake graph S(T_n) is a square sum graph for all $n \geq 2$.

Proof: Let P_n be the path $u_1, u_2, ..., u_n$.

Let $V(T_n) = V(P_n) \cup \{V_i : 1 \leq i \leq n-1\}$

Let $V(S(T_n)) = \{x_i, y_i, w_i : 1 \leq i \leq n-1\} \cup V(T_n)$ and

$E(S(T_n)) = \{u_i x_i, x_i v_i, y_i v_i, y_i u_{i+1}, u_i w_i, w_i u_{i+1}, 1 \leq i \leq n-1\}$

Define a map $f : V(S(T_n)) \to \{0, 1, 2, ..., (p-1)\}$ as follows

$$f(u_i) = i - 1, \ 1 \leq i \leq n-1$$
$$f(v_i) = (n-1) + i, \ 1 \leq i \leq n-1$$
$$f(x_i) = (2n-2) + i, \ 1 \leq i \leq n-1$$
$$f(y_i) = (3n-3) + i, \ 1 \leq i \leq n-1$$
$$f(w_i) = (4n-4) + i, \ 1 \leq i \leq n-1$$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.
Hence S(T_n) is a square sum labeling.

Theorem 2.2. The Subdivision of triangular snake graph S(T_n) is a square difference graph for all $n \geq 2$.

Proof: Let P_n be the path $u_1, u_2, ..., u_n$.

Let $V(T_n) = V(P_n) \cup \{V_i : 1 \leq i \leq n-1\}$

Let $V(\mathrm{S}(T_n)) = \{x_i, y_i, w_i : 1 \leq i \leq n-1\} \cup V(T_n)$ and

$\mathrm{E}(\mathrm{S}(T_n)) = \{\mathrm{u}_i\, x_i, x_i v_i, y_i v_i, y_i u_{i+1}, u_i w_i, \mathrm{w}_i\, u_{i+1}, 1 \leq i \leq n-1\}$

Define a map $f : V(S(T_n)) \rightarrow \{0, 1, 2, ..., (p-1)\}$ as follows

$f(u_i) = i-1,\ 1 \leq i \leq n-1$

$f(\mathrm{v}_i) = (n-1)+i,\ 1 \leq i \leq n-1$

$f(x_i) = (2n-2)+i,\ 1 \leq i \leq n-1$

$f(y_i) = (3n-3)+i,\ 1 \leq i \leq n-1$

$f(w_i) = (4n-4)+i,\ 1 \leq i \leq n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.
Hence S(T_n) is a square difference labeling.

Theorem 2.3. The Subdivision of triangular snake graph S(T_n) is a Strongly Multiplicative graph for all $n \geq 2$.

Proof: Let P_n be the path $u_1, u_2, ..., u_n$.

Let $V(T_n) = V(P_n) \cup \{V_i : 1 \leq i \leq n-1\}$

Let $V(\mathrm{S}(T_n)) = \{x_i, y_i, w_i : 1 \leq i \leq n-1\} \cup V(T_n)$ and

$\mathrm{E}(\mathrm{S}(T_n)) = \{\mathrm{u}_i\, x_i, x_i v_i, y_i v_i, y_i u_{i+1}, u_i w_i, \mathrm{w}_i\, u_{i+1}, 1 \leq i \leq n-1\}$

Define a map $f : V(S(T_n)) \rightarrow \{1, 2, ..., p\}$ as follows

$f(u_i) = i,\ 1 \leq i \leq n-1$

$f(\mathrm{v}_i) = n+i,\ 1 \leq i \leq n-1$

$f(x_i) = (2n-1)+i,\ 1 \leq i \leq n-1$

$f(y_i) = (3n-2)+i,\ 1 \leq i \leq n-1$

$f(w_i) = (4n-3)+i,\ 1 \leq i \leq n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.
Hence S(T_n) is a Strongly Multiplicative labeling.

Theorem 2.4. The Subdivision of quadrilateral snake graph S(Q_n) is a square sum graph for all $n \geq 2$.

Proof: Let P_n be the path $u_1, u_2, ..., u_n$ and

Let $V(\mathrm{Q}_n) = \{v_i, w_i : 1 \leq i \leq n-1\} \cup V(\mathrm{P}_n)$

Let $V(\mathrm{S}(\mathrm{Q}_n)) = \{x_i, \mathrm{u}_i^{'}, \mathrm{z}_i, y_i : 1 \leq i \leq n-1\} \cup V(\mathrm{Q}_n)$ and

$\mathrm{E}(\mathrm{S}(\mathrm{Q}_n)) = \{\mathrm{u}_i\, u_i^{'}, \mathrm{u}_i^{'}\, u_{i+1}, y_i u_{i+1} : 1 \leq i \leq n-1\} \cup$

$\{\mathrm{u}_i\, x_i, x_i v_i, v_i z_i, z_i w_i, w_i y_i : 1 \leq i \leq n-1\}$

Define a map $f : V(S(\mathrm{Q}_n)) \to \{0,1,2,...,(p-1)\}$ as follows

$f(u_i) = i-1,\ 1 \leq i \leq n-1$

$f(\mathrm{v}_i) = (n-1)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{w}_i) = (2n-2)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{x}_i) = (3n-3)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{u}_i^{'}) = (4n-4)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{z}_i) = (5n-5)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{y}_i) = (6n-6)+i,\ 1 \leq i \leq n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.

Hence S(Q_n)is a square sum labeling

Theorem 2.5. The Subdivision of quadrilateral snake graph S(Q_n) is a square difference graph for all $n \geq 2$.

Proof: Let P_n be the path $u_1, u_2, ..., u_n$ and

Let $V(\mathrm{Q}_n) = \{v_i, w_i : 1 \leq i \leq n-1\} \cup V(\mathrm{P}_n)$

Let $V(\mathrm{S}(\mathrm{Q}_n)) = \{x_i, \mathrm{u}_i^{'}, \mathrm{z}_i, y_i : 1 \leq i \leq n-1\} \cup V(\mathrm{Q}_n)$ and

$\mathrm{E}(\mathrm{S}(\mathrm{Q}_n)) = \{\mathrm{u}_i\, u_i^{'}, \mathrm{u}_i^{'}\, u_{i+1}, y_i u_{i+1} : 1 \leq i \leq n-1\} \cup$

$\{\mathrm{u}_i\, x_i, x_i v_i, v_i z_i, z_i w_i, w_i y_i : 1 \leq i \leq n-1\}$

Define a map $f : V(S(\mathrm{Q}_n)) \to \{0,1,2,...,(p-1)\}$ as follows

$f(u_i) = i-1,\ 1 \leq i \leq n-1$

$f(\mathrm{v}_i) = (n-1)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{w}_i) = (2n-2)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{x}_i) = (3n-3)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{u}_i^{'}) = (4n-4)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{z}_i) = (5n-5)+i,\ 1 \leq i \leq n-1$

$f(\mathrm{y}_i) = (6n-6)+i,\ 1 \le i \le n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.

Hence S(Q_n) is a square difference labeling.

Theorem 2.6. The subdivision of quadrilateral snake graph S(Q_n) is a strongly multiplicative graph for all $n \ge 2$.

Proof: Let P_n be the path $u_1, u_2, ..., u_n$ and

Let $V(\mathrm{Q}_n) = \{v_i, w_i : 1 \le i \le n-1\} \cup V(\mathrm{P}_n)$

Let $V(\mathrm{S}(\mathrm{Q}_n)) = \{x_i, \mathrm{u}_i^{'}, \mathrm{z}_i, y_i : 1 \le i \le n-1\} \cup V(\mathrm{Q}_n)$ and

$\mathrm{E}(\mathrm{S}(\mathrm{Q}_n)) = \{\mathrm{u}_i\, u_i^{'}, \mathrm{u}_i^{'}\, u_{i+1}, y_i u_{i+1} : 1 \le i \le n-1\} \cup$

$\{\mathrm{u}_i\, x_i, x_i v_i, v_i z_i, z_i w_i, w_i y_i : 1 \le i \le n-1\}$

Define a map $f : V(S(\mathrm{Q}_n)) \to \{1, 2, ..., p\}$ as follows

$f(u_i) = i,\ 1 \le i \le n-1$

$f(\mathrm{v}_i) = n+i,\ 1 \le i \le n-1$

$f(\mathrm{w}_i) = (2n-1)+i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i) = (3n-2)+i,\ 1 \le i \le n-1$

$f(\mathrm{u}_i^{'}) = (4n-3)+i,\ 1 \le i \le n-1$

$f(\mathrm{z}_i) = (5n-4)+i,\ 1 \le i \le n-1$

$f(\mathrm{y}_i) = (6n-5)+i,\ 1 \le i \le n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.

Hence S(Q_n) is a strongly multiplicative labeling.

Theorem 2.7. The Subdivision of double triangular snake graph S(D(T_n)) is a square sum graph for all $n \ge 2$.

Proof: Let $V(\mathrm{S}(\mathrm{D}(T_n))) = \{u_i : 1 \le \mathrm{i} \le \mathrm{n}\} \cup \{x_i, y_i, \mathrm{v}_i, z_i, x_i^{'}, y_i^{'}, w_i : 1 \le i \le n-1\}$ and

$\mathrm{E}(\mathrm{S}(\mathrm{D}(T_n))) = \{\mathrm{u}_i\, z_i, \mathrm{z}_i\, u_{i+1}, \mathrm{u}_i\, x_i, y_i u_{i+1}, \mathrm{x}_i\, v_i, \mathrm{y}_i\, v_i, 1 \le i \le n-1\} \bigcup$

$\{x_i^{'} w_i, w_i y_i^{'}, y_i^{'} u_{i+1}, u_i x_i^{'} : 1 \le i \le n-1\}$

Define a map $f : V(S(\mathrm{D}(T_n))) \to \{0, 1, 2, ..., (p-1)\}$ as follows

$f(y_i) = (3n-3)+i,\ 1 \le i \le n-1$

$f(\mathrm{z}_i) = (4n-4)+i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i^{'}) = (5n-5)+i,\ 1 \le i \le n-1$

$f(u_i) = i-1,\ 1 \le i \le n-1$

$f(\mathrm{v}_i) = (n-1)+i,\ 1 \le i \le n-1$

$f(x_i) = (2n-2)+i,\ 1 \le i \le n-1$

$f(\mathrm{y}_i^{'}) = (6n-6)+i,\ 1 \le i \le n-1$

$f(\mathrm{w}_i) = (7n-7)+i,\ 1 \le i \le n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.
Hence S(D(T_n)) is a square sum labeling.

Theorem 2.8. The Subdivision of double triangular snake graph S(D(T_n)) is a square difference graph for all $n \ge 2$.

Proof: Let $V(\mathrm{S}(\mathrm{D}(T_n))) = \{u_i : 1 \le \mathrm{i} \le \mathrm{n}\} \cup \{x_i, y_i, \mathrm{v}_i, z_i, x_i^{'}, y_i^{'}, w_i : 1 \le i \le n-1\}$ and

$$\mathrm{E}(\mathrm{S}(\mathrm{D}(T_n))) = \{\mathrm{u}_i\, z_i, \mathrm{z}_i\, u_{i+1}, \mathrm{u}_i\, x_i, y_i u_{i+1}, \mathrm{x}_i\, v_i, \mathrm{y}_i\, v_i, 1 \le i \le n-1\} \bigcup$$
$$\{x_i^{'} w_i, w_i y_i^{'}, y_i^{'} u_{i+1}, u_i x_i^{'} : 1 \le i \le n-1\}$$

Define a map $f : V(S(\mathrm{D}(T_n))) \rightarrow \{0,1,2,...,(p-1)\}$ as follows

$f(u_i) = i-1,\ 1 \le i \le n-1$

$f(\mathrm{v}_i) = (n-1)+i,\ 1 \le i \le n-1$

$f(x_i) = (2n-2)+i,\ 1 \le i \le n-1$

$f(y_i) = (3n-3)+i,\ 1 \le i \le n-1$

$f(\mathrm{z}_i) = (4n-4)+i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i^{'}) = (5n-5)+i,\ 1 \le i \le n-1$

$f(\mathrm{y}_i^{'}) = (6n-6)+i,\ 1 \le i \le n-1$

$f(\mathrm{w}_i) = (7n-7)+i,\ 1 \le i \le n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.
Hence S(D(T_n)) is a square difference labeling.

Theorem 2.9. The Subdivision of double triangular snake graph S(D(T_n)) is a Strongly Multiplicative graph for all $n \ge 2$.

Proof: Let $V(\mathrm{S}(\mathrm{D}(T_n))) = \{u_i : 1 \le \mathrm{i} \le \mathrm{n}\} \cup \{x_i, y_i, \mathrm{v}_i, z_i, x_i^{'}, y_i^{'}, w_i : 1 \le i \le n-1\}$ and

$$\mathrm{E}(\mathrm{S}(\mathrm{D}(T_n))) = \{\mathrm{u}_i\, z_i, \mathrm{z}_i\, u_{i+1}, \mathrm{u}_i\, x_i, y_i u_{i+1}, \mathrm{x}_i\, v_i, \mathrm{y}_i\, v_i, 1 \le i \le n-1\} \bigcup$$
$$\{x_i^{'} w_i, w_i y_i^{'}, y_i^{'} u_{i+1}, u_i x_i^{'} : 1 \le i \le n-1\}$$

Define a map $f : V(S(\mathrm{D}(T_n))) \rightarrow \{1,2,..., p\}$ as follows

$f(u_i) = i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i) = (n+i),\ 1 \le i \le n-1$

$f(\mathrm{y}_i) = (2n-1)+i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i^{'}) = (3n-2)+i,\ 1 \le i \le n-1$

$f(\mathrm{y}_i^{'}) = (4n-3)+i,\ 1 \le i \le n-1$

$f(\mathrm{v}_i) = (5n-4)+i,\ 1 \le i \le n-1$

$f(\mathrm{w}_i) = (6n-5)+i,\ 1 \le i \le n-1$

$f(\mathrm{z}_i) = (7n-6)+i,\ 1 \le i \le n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.
Hence S(D(T_n)) is a Strongly Multiplicative labeling.

Theorem 2.10. The Subdivision of double quadrilateral snake graph S(D(Q_n))is a square sum graph for all $n \ge 2$.
Proof: Let

$V(\mathrm{S(D(Q}_n))) = \{u_i : 1 \le \mathrm{i} \le \mathrm{n}\} \cup \{\mathrm{v}_i, \mathrm{w}_i, \mathrm{v}_i^{'}, \mathrm{w}_i^{'}, x_i, y_i, z_i, x_i^{'}, y_i^{'}, z_i^{'}, u_i^{'} : 1 \le i \le n-1\}$

and

$\mathrm{E(S(D(Q}_n))) = \{\mathrm{u}_i u_i^{'}, \mathrm{u}_i^{'} u_{i+1}, y_i u_{i+1}, \mathrm{u}_i \mathrm{x}_i, \mathrm{x}_i \mathrm{v}_i, \mathrm{v}_i \mathrm{z}_i, \mathrm{z}_i \mathrm{w}_i, \mathrm{w}_i \mathrm{y}_i : 1 \le i \le n-1\} \cup$

$\{\mathrm{u}_i x_i^{'}, x_i^{'} v_i^{'}, v_i^{'} z_i^{'}, z_i^{'} w_i^{'}, w_i^{'} y_i^{'}, y_i^{'} u_{i+1} : 1 \le i \le n-1\}$

Define a map $f : V(S(\mathrm{D(Q}_n))) \to \{0,1,2,...,(p-1)\}$ as follows

$f(u_i) = i-1,\ 1 \le i \le n-1$

$f(\mathrm{v}_i) = (n-1)+i,\ 1 \le i \le n-1$

$f(\mathrm{w}_i) = (2n-2)+i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i) = (3n-3)+i,\ 1 \le i \le n-1$

$f(\mathrm{u}_i^{'}) = (4n-4)+i,\ 1 \le i \le n-1$

$f(\mathrm{z}_i) = (5n-5)+i,\ 1 \le i \le n-1$

$f(\mathrm{y}_i) = (6n-6)+i,\ 1 \le i \le n-1$

$f(\mathrm{v}_i^{'}) = (7n-7)+i,\ 1 \le i \le n-1$

$f(\mathrm{w}_i^{'}) = (8n-8)+i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i^{'}) = (9n-9)+i,\ 1 \le i \le n-1$

$f(\mathrm{z}_i^{'}) = (10n-10)+i,\ 1 \le i \le n-1$

$f(\mathrm{y}_i^{'}) = (11n-11)+i,\ 1 \le i \le n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.

Hence S(D(Q_n)) is a square sum labeling.

Theorem 2.11. The Subdivision of double quadrilateral snake graph S(D(Q_n))is a square difference graph for all $n \geq 2$.

Proof: Let

$V(\mathrm{S(D(Q}_n))) = \{u_i : 1 \leq \mathrm{i} \leq \mathrm{n}\} \bigcup \{\mathrm{v}_i, \mathrm{w}_i, \mathrm{v}_i^{'}, \mathrm{w}_i^{'}, x_i, y_i, z_i, x_i^{'}, y_i^{'}, z_i^{'}, u_i^{'} : 1 \leq i \leq n-1\}$

and

$$\mathrm{E(S(D(Q}_n))) = \{\mathrm{u}_i u_i^{'}, \mathrm{u}_i^{'} u_{i+1}, y_i u_{i+1}, \mathrm{u}_i \mathrm{x}_i, \mathrm{x}_i \mathrm{v}_i, \mathrm{v}_i \mathrm{z}_i, \mathrm{z}_i \mathrm{w}_i, \mathrm{w}_i \mathrm{y}_i : 1 \leq i \leq n-1\} \cup \{\mathrm{u}_i x_i^{'}, x_i^{'} v_i^{'}, v_i^{'} z_i^{'}, z_i^{'} w_i^{'}, w_i^{'} y_i^{'}, y_i^{'} u_{i+1} : 1 \leq i \leq n-1\}$$

Define a map $f : V(S(\mathrm{D(Q}_n))) \to \{0,1,2,...,(p-1)\}$ as follows

$$f(u_i) = i-1,\ 1 \leq i \leq n-1$$
$$f(\mathrm{v}_i) = (n-1)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{w}_i) = (2n-2)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{x}_i) = (3n-3)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{u}_i^{'}) = (4n-4)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{z}_i) = (5n-5)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{y}_i) = (6n-6)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{v}_i^{'}) = (7n-7)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{w}_i^{'}) = (8n-8)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{x}_i^{'}) = (9n-9)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{z}_i^{'}) = (10n-10)+i,\ 1 \leq i \leq n-1$$
$$f(\mathrm{y}_i^{'}) = (11n-11)+i,\ 1 \leq i \leq n-1$$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.

Hence S(D(Q_n)) is a square difference labeling.

Theorem 2.12. The Subdivision of double quadrilateral snake graph S(D(Q_n)) is a Strongly Multiplicative graph for all $n \geq 2$.

Proof: Let

$V(\mathrm{S(D(Q}_n))) = \{u_i : 1 \leq \mathrm{i} \leq \mathrm{n}\} \bigcup \{\mathrm{v}_i, \mathrm{w}_i, \mathrm{v}_i^{'}, \mathrm{w}_i^{'}, x_i, y_i, z_i, x_i^{'}, y_i^{'}, z_i^{'}, u_i^{'} : 1 \leq i \leq n-1\}$

and

$$\mathrm{E(S(D(Q}_n))) = \{\mathrm{u}_i u_i^{'}, \mathrm{u}_i^{'} u_{i+1}, y_i u_{i+1}, \mathrm{u}_i \mathrm{x}_i, \mathrm{x}_i \mathrm{v}_i, \mathrm{v}_i \mathrm{z}_i, \mathrm{z}_i \mathrm{w}_i, \mathrm{w}_i \mathrm{y}_i : 1 \leq i \leq n-1\} \cup \{\mathrm{u}_i x_i^{'}, x_i^{'} v_i^{'}, v_i^{'} z_i^{'}, z_i^{'} w_i^{'}, w_i^{'} y_i^{'}, y_i^{'} u_{i+1} : 1 \leq i \leq n-1\}$$

Define a map $f : V(S(\mathrm{D(Q}_n))) \to \{1,2,..., p\}$ as follows

$f(u_i) = i,\ 1 \le i \le n$

$f(\mathrm{v}_i) = n + i,\ 1 \le i \le n-1$

$f(\mathrm{w}_i) = (2n-1) + i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i^{'}) = (3n-2) + i,\ 1 \le i \le n-1$

$f(\mathrm{y}_i^{'}) = (4n-3) + i,\ 1 \le i \le n-1$

$f(\mathrm{z}_i) = (5n-4) + i,\ 1 \le i \le n-1$

$f(\mathrm{u}_i^{'}) = (6n-5) + i,\ 1 \le i \le n-1$

$f(\mathrm{v}_i^{'}) = (7n-6) + i,\ 1 \le i \le n-1$

$f(\mathrm{w}_i^{'}) = (8n-7) + i,\ 1 \le i \le n-1$

$f(\mathrm{z}_i^{'}) = (9n-8) + i,\ 1 \le i \le n-1$

$f(\mathrm{x}_i^{'}) = (10n-9) + i,\ 1 \le i \le n-1$

$f(\mathrm{y}_i^{'}) = (11n-10) + i,\ 1 \le i \le n-1$

The above defined labeling pattern easy to verify that all the vertex labels are different and we get edge labels are non-zero integers and no edge label is repeated.
Hence S(D(Q_n)) is a Strongly Multiplicative labeling.

3. Conclusion

In this paper, we have examined the existence of square sum, square difference, strongly multiplicative labeling for subdivision of triangular snake graph, double triangular snake graph and subdivision of quadrilateral snake graph and double quadrilateral snake graph for all $n \ge 2$. Further investigation can be done to obtain the above labeling for some class of graphs.

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