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A Weaker form of Contra Continuous Function in Nano Topological Space

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Abstract. The purpose of this study is to introduce a weaker form of contra continuous function called Contra Nwg-continuous function in Nano topological spaces. Some of its properties are analyzed. The equivalent condition for a function to be contra Nwg-continuous function is established. Further Contra Nwg-irresolute function is defined and few of its properties are discussed.

Keywords: Nano topological space, Nano continuous function, Nano contra continuous function, Nwg-closed set, Nwg continuous function, Nwg-irresolute function.

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1. Introduction

The concept of continuity plays a major role in general topology. Many authors have studied different weaker and stronger form of continuity [11, 12, 17]. Hakawati [7] presented some strong topological aspects in Uryson spaces using closure continuity. In 1994 Dontchev et al [5] introduced the concept of contra-continuity which is a stronger form of LC-continuity in general topological space.

In 2002 [19] Zdzislaw Pawlak discussed the applications of rough set theory with an example. Based on this theory Lellis Thivagar et al [8] defined a new topology called Nano topology in terms of approximations and boundary region of a universal set using equivalence relation on it and studied some weak form of Nano open set. The author [9] established Nano continuity using Nano interior. He [10] also analyzed contra continuous function and bi contra continuous function in nano topological space. Bhuvaneswari et al [1, 2] studied various weak form of continuity in Nano topological space. Few other related works in this area can be found in [6, 13, 16, 18].

Nagaveni and Bhuvaneswari [3,14,15] applied the concept of weakly generalization in Nano topology and investigated Nwg-continuous function and Nwg-closed map. The authors [4] have also studied few properties of Nwg-continuous function in terms of Nwg - interior and Nwg - closure. In this paper we introduced weaker forms of different contra continuous functions called Contra Nwg-continuous function and Contra Nwg-irresolute function and discussed few of its characteristics.

Throughout this paper $(U, \tau_R(X))$ is a Nano Topological space with respect to X Where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R. $(V, \tau_{R'}(Y))$ is a Nano Topological space with respect to Y Where $Y \subseteq V$, R' is an equivalence relation on V, V/R' denotes the family of equivalence classes of V by R'. $(W, \tau_{R''}(Z))$ is a Nano Topological space with respect to Z where $Z \subseteq W$, R'' is an equivalence relation on W, W/R'' denotes the family of equivalence classes of W by R''.

2. Preliminaries
This section is to recall some definitions and properties which are useful in this study.

Definition 2.1. [8] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$,

1. The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is defined by $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$. Where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is defined by $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and is defined by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. [8] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms.

1. U and $\phi \in \tau_R(X)$.

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ forms a topology on U called as the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. Elements of $[\tau_R(X)]^c$ are called Nano closed sets.

Definition 2.3. [8] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), U_R(X)\}$ is the basis for $\tau_R(X)$.

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Definition 2.4. [8] If $(U, \tau_R(X))$ is a Nano Topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$ then the Nano interior of A is defined as the union of all Nano open subsets of A and it is denoted by NInt(A). Nano interior is the largest Nano open subset of A.

Definition 2.5. [8] The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A). It is the smallest Nano closed set containing A.

Definition 2.6. [8] Let $(U, \tau_R(X))$ be a Nano Topological space with respect to X and $A \subseteq U$. Then A is said to be

   (i) Nano semi- open if $A \subseteq NCl\ (NInt\ (A))$

   (ii) Nano pre- open if $A \subseteq NInt\ (NCl\ (A))$

   (iii) Nano α - open if $A \subseteq NInt\ (NCl\ (NInt\ (A)))$

   (iv) Nano Regular open if $A = NInt\ (NCl\ (A))$

Definition 2.7. [14] Let $(U, \tau_R(X))$ be a Nano Topological space. A subset A of $(U, \tau_R(X))$ is called Nano weakly generalized closed (briefly Nwg-closed) set if $Ncl\ (N \text{int}(A)) \subseteq V$ where $A \subseteq V$ and V is Nano open. The complement of Nano weakly generalized closed set is Nano weakly generalized open set.

Theorem 2.8. [14] Every Nano open (closed) set is Nano weakly generalized open (closed) set.

Defininition 2.9. The map $f : (U, \tau_R(X)) \rightarrow (V, \tau_R^{'}(Y))$ is called

   (i)   Nano continuous function [9] if the inverse image of every Nano open set in V is Nano open in U.

   (ii) Nano contra continuous function [10] if the inverse image of every Nano open set in V is Nano closed in U.

   (iii) Nwg-continuous function [15] if the inverse image of every Nano open set in V is Nano weakly generalized open in U.

   (iv) Nwg-irresolute function [15] if the inverse image of every Nano weakly generalized open set in V is Nano weakly generalized open in U.

Definition 2.10. [4] Nano weakly generalized interior of a subset A is the union of all the Nwg-open sets contained in A**.**
$\text{NInt}_{wg}(A) = \cup \{B : B \text{ is Nwg - open sets such that } B \subseteq A\}$

Definition 2.11. [4] The intersection of all the Nwg-closed set containing A is called Nwg-closure of A.
$\text{NCl}_{wg}(A) = \cap \{B : B \text{ is Nwg - closed sets such that } A \subseteq B\}$

3. Contra Nwg- continuous function

In this section we define the function called Contra Nwg- continuous function and study some of its properties.

Definition 3.1. The map $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ is Contra Nwg-continuous on U if the inverse image of every Nano open set in V is Nano weakly generalized closed set in U.

Example 3.2. Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{a\},\{b\},\{c\}\{d\},\{e\}\}$ and $X = \{a,b\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{a,b\}\}$.

Let $V = \{a,b,c,d,e\}$ with $V/R^{'} = \{\{d,e\},\{a,c\},\{b\}\}$ and $Y = \{a,b,c\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V,\phi,\{a,b,c\}\}$. Define $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ as $f(a) = d, f(b) = e, f(c) = a, f(d) = b, f(e) = c$. Then f is contra Nwg-continuous function.

Theorem 3.3. Let $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ be a function from $(U,\tau_R(X))$ to $(V,\tau_R^{'}(Y))$, f is contra Nwg - continuous function if and only if inverse image of every Nano closed set in V is Nano weakly generalized open in U.

Proof: Let $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ and B be a Nano closed set in V. Since f is contra Nwg continuous function $f^{-1}(V-B) = U - f^{-1}(B)$ is Nwg-closed in U. Hence $f^{-1}(B)$ is Nwg-open set in V.

Conversely, let B be a Nano open set in V. By assumption is $f^{-1}(V-B)$ Nwg-open set. $f^{-1}(V-B) = U - f^{-1}(B)$, $f^{-1}(B)$ is Nwg-closed set in U. Hence f is Contra Nwg- continuous function.

Theorem 3.4. Let $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ be a function from $(U,\tau_R(X))$ to $(V,\tau_R^{'}(Y))$ then the following conditions are equivalent.

Inverse image of every Nano closed set in V is Nano weakly generalized open in U.

For $x \in U$ and each nano closed set B in V with $f(x) \in B$ there exists an Nwg-open set in U such that $f(A) \subseteq B$.

Proof: (i) $\to$ (ii)

Let B be Nano closed set in V such that $f(x) \in B$, $x \in U$. Let $A = f^{-1}(B)$, $x \in A$, and $f(A) \subseteq B$.

$(ii) \to (i)$

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Let B be any Nano closed in V, $x \in U$, $f(x) \in B$. There exists an Nwg open set U_x such that $f(U_x) \subseteq B$. $f^{-1}(B) = \bigcup \{U_x, x \in f^{-1}(B) \in NO(x)\}$, $f^{-1}(B)$ is Nwg-open in U.

Remark 3.5. Composition of two contra Nwg continuous function need not be contra Nwg continuous function as shown in the following example.

Example 3.6. Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X = \{a,b\}$. Then the Nano topology is $\tau_R(X) = \{U, \phi, \{b\}, \{a,c\}, \{a,b,c\}\}$.
　　　Let $V = \{a,b,c,d,e\}$ with $V/R^{'} = \{\{a,b\},\{c,e\},\{d\}\}$ and $Y = \{a,b\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V, \phi, \{a,b\}\}$.
Let $W = \{a,b,c,d,e\}$ with $W/R^{''} = \{\{a\},\{b\},\{c\},\{d\},\{e\}\}$ and $Z = \{c,e\}$. Then the Nano topology is $\tau_R^{'}(Z) = \{W, \phi, \{c,e\}\}$.
Define $f : (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ as $f(a) = c, f(b) = d, f(c) = e, f(d) = a, f(e) = b$ and $g : (V, \tau_R^{'}(Y)) \to (W, \tau_R^{''}(Z))$ as $g(a) = b, g(b) = d, g(c) = c, g(d) = e, g(e) = a$.
Here f and g are contra Nwg continous function but their composition is not contra Nwg-continuous function since $f^{-1}(g^{-1}(\{c,e\})) = f^{-1}(\{c,d\}) = \{a,b\}$ is not Nwg-closed.

Remark 3.7. Contra Nwg-continuous function and Nwg-continuous function are independent.

Example 3.8. Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X = \{a,b\}$. Then the Nano topology is $\tau_R(X) = \{U, \phi, \{b\}, \{a,c\}, \{a,b,c\}\}$.
　　　Let $V = \{a,b,c,d,e\}$ with $V/R^{'} = \{\{a\},\{b,d\},\{c,e\}\}$ and $Y = \{c,e\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V, \phi, \{c,e\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ as $f(a) = c, f(b) = e, f(c) = a, f(d) = d, f(e) = b$. Then f is Nwg- continuous function. Since $f^{-1}(\{c,e\}) = \{a,b\}$ is not Nwg-closed set in U f is not Contra Nwg-continuous function.

Example 3.9. Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{a,c\},\{b\},\{d\},\{e\}\}$ and $X = \{a,b\}$. Then the Nano topology is $\tau_R(X) = \{U, \phi, \{b\}, \{a,c\}, \{a,b,c\}\}$.
　　　Let $V = \{a,b,c,d,e\}$ with $V/R^{'} = \{\{a\},\{b\},\{c,d\},\{e\}\}$ and $Y = \{a,b\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V, \phi, \{a,b\}\}$.
Let $f : (U, \tau_R(X)) \to (V, \tau_R^{'}(Y))$ be a function defined as
$f(a) = c, f(b) = d, f(c) = e, f(d) = a, f(e) = b$,

f then f is Contra Nwg-continuous function. since $f^{-1}(\{a,b\}) = \{d,e\}$ is not Nwg-open f is not Nwg-continuous function.

Theorem 3.10. Every Nano contra continuous function is contra Nwg-continuous function.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_R'(Y))$ be a Nano contra function from $(U,\tau_R(X))$ to $(V,\tau_R'(Y))$ and A be Nano open set in V. Since f is Nano contra function $f^{-1}(A)$ is Nano-closed set in U. Therefore $f^{-1}(A)$ is Nwg-closed in U. Hence f is contra Nwg-continous function.

Remark 3.11. If $f:(U,\tau_R(X))\to(V,\tau_R'(Y))$ is a Contra Nwg-continuous function then f need not be Nano contra continuous function as shown in the following example.

Example 3.12 . $Let \quad U \;=\; V \;=\; \{1,2,3,4,5,6,7,8,9\}$

$R \;=\; \{\{x,y\}/\ x\ and\ \ y\ are\ both\ \ even\ \ or\ odd\ \ x,y\in U\}$

$U\ /\ R \;=\; \{\{2,4,6,8\},\{1,3,5,7,9\}\}, \quad X \;=\; \{2,4,6,8,7,9\}$

The Nano topology is $\tau_R(X) \;=\; \{U,\phi,\{2,4,6,8\},\{1,3,5,7,9\}\}$.

Let $R' = \{(x,y)/\ x-y\ is\ divisible\ \ by\ 6\ and\ \ x,y\in V\}, \quad V/R' = \{\{1,7\},\{2,8\},\{3,9\}\}$, $Y = \{1,8\}$ then the Nano topology induced by R' is $\tau_R'(Y) = \{V,\phi,\{1,2,7,8\}\}$.

Let $f:(U,\tau_R(X))\to(V,\tau_R'(Y))$ be an identity map from U to V. Then f is Contra Nwg-continuous function but not Contra continuous function since $f^{-1}(\{1,2,7,8\})$ is Nwg-closed but not Nano closed.

Theorem 3.13. A function f from Nano topological space U to Nano topological space V is Contra Nwg-continuous function if the only Nano open set containing the inverse image of every Nano open A of V is U.

Proof: Let A be a Nano open set of V and U is the only Nano open set such that $f^{-1}(A)\subseteq U$. Then $Ncl(N\operatorname{int}(f^{-1}(A)))\subseteq U$. (i.e) $f^{-1}(A)$ is Nwg-closed in U. f is Contra Nwg-continuous function.

Corollary 3.14. Let f be a function defined from Nano topological space $(U,\tau_R(X))$ to Nano topological space $(V,\tau_R'(Y))$ and $N\operatorname{int}(f^{-1}(A))=\phi$ for every Nano open set A of V then f is Contra Nwg-continuous function.

Theorem 3.15. Let f be a contra Nwg-continuous function then $f(N\operatorname{Int}_{wg}(A))\subseteq NCl(f(A))$ for every subset $A\subseteq U$.

Proof: Let $A\subseteq U$ then $NCl(f(A))$ is a Nano closed set in V. Since f is contra Nwg-continous $f^{-1}(NCl(f(A)))$ is Nwg-open set in U and $N\operatorname{Int}_{wg}(f^{-1}(NCl(f(A))))=f^{-1}(NCl(f(A)))$. $f(A)\subseteq NCl(f(A))$,

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$\text{NInt}_{wg}(A) \subseteq \text{NInt}_{wg}\left(f^{-1}(NCl(f(A)))\right)$, $\text{NInt}_{wg}(A) \subseteq f^{-1}(NCl(f(A)))$, $f\left(\text{NInt}_{wg}(A)\right) \subseteq NCl(f(A))$.

Remark 3.16. The converse of the above theorem need not be true as shown in the following example.

Example 3.17. In example 3.8 $f(\text{NInt}_{wg}(A)) \subseteq NCl(f(A))$ but f is not contra Nwg-continuous function

Remark 3.18. In theorem 3.15, if $f(A)$ is Nano closed set then $f(\text{NCl}_{wg}(A)) = NCl(f(A))$.

Corollary 3.19. Let $f : (U, \tau_R(X)) \to (V, \tau_R{'}(Y))$ be a Nwg-continuous function then $f(\text{NInt}_{wg}(f^{-1}(A))) \subseteq NCl(A))$ for every subset $A \subseteq V$.

Theorem 3.20. Let $f : (U, \tau_R(X)) \to (V, \tau_R{'}(Y))$ be a Nwg-continuous function then $f^{-1}(NIntA) \subseteq NCl_{wg}\left(f^{-1}(A)\right)$ for every subset $A \subseteq V$.

Proof: Let $A \subseteq V$ then $NIntA$ is a Nano open set in V. Since f is contra Nwg-continous $f^{-1}(NIntA)$ is Nwg-closed set in U and $NCl_{wg}\left(f^{-1}(NIntA)\right) = f^{-1}(NIntA)$. $NIntA \subseteq A$, $f^{-1}(NIntA) \subseteq f^{-1}(A)$, $NCl_{wg}\left(f^{-1}(NIntA)\right) \subseteq NCl_{wg}\left(f^{-1}(A)\right)$, $\left(f^{-1}(NIntA)\right) \subseteq NCl_{wg}\left(f^{-1}(A)\right)$.

Remark 3.21. The converse of the above theorem need not be true as shown in the following example.

Example 3.22. In example 3.9 , f is not Contra Nwg- continuous function, but $f^{-1}(NIntA) \subseteq NCl_{wg}\left(f^{-1}(A)\right)$ for every subset $A \subseteq V$.

Remark 3.33. In theorem 3.20, $f^{-1}(NIntA) = NInt_{wg}\left(f^{-1}(A)\right)$ if A is Nano open.

4. Contra Nwg irresolute function
In this section we introduce Contra Nwg- irresolute function and discuss some of its properties.

Definition 4.1. The map $f : (U, \tau_R(X)) \to (V, \tau_R{'}(Y))$ is Contra Nwg irresolute function on U if the inverse image of every Nano weakly generalized open (closed) set in V is Nano weakly generalized closed (open) in U.

Example 4.2. Let $U = \{a,b,c,d,e\}$ with $U / R = \{\{a\},\{b\},\{c,d\},\{e\}\}$ and $X = \{c,d,e\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{c,d,e\}\}$.

Let $V = \{a,b,c,d,e\}$ with $V / R^{'} = \{\{a\},\{b,c\},\{d,e\}\}$ and $Y = \{d,e\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V,\phi,\{d,e\}\}$. Define $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ as $f(a)=d, f(b)=e, f(c)=b, f(d)=c, f(e)=a$. Then f is contra Nwg irresolute function.

Remark 4.3. Composition of two contra Nwg- irresolute function on U need not be contra Nwg- irresolute function as given in the following example.

Example 4.4. Let $U = \{a,b,c,d,e\}$ with $U / R = \{\{a,b\},\{c,d\},\{e\}\}$ and $X = \{c,d,e\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{c,d,e\}\}$.

Let $V = \{a,b,c,d,e\}$ with $V / R^{'} = \{\{a\},\{b,c\},\{d\},\{e\}\}$ and $Y = \{d,e\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V,\phi,\{d,e\}\}$.

Let $W = \{a,b,c,d,e\}$ with $W / R^{''} = \{\{a\},\{b\},\{c\}\{d\},\{e\}\}$ and $Z = \{a,b,d\}$. Then the Nano topology is $\tau_R^{'}(Z) = \{W,\phi,\{a,b,d\}\}$. Define $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ as $f(a)=d, f(b)=e, f(c)=b, f(d)=c, f(e)=a$

let $g:(V,\tau_R^{'}(Y)) \to (W,\tau_R^{''}(Z))$ as $g(a)=a, g(b)=b, g(c)=d, g(d)=c, g(e)=e$. Here f and g are contra Nwg –irresolute functions but their composition is not contra Nwg-irresolute function since $f^{-1}\left(g^{-1}(\{c,e\})\right) = f^{-1}(\{d,e\}) = \{ab\}$ is not Nwg-open set in U.

Theorem 4.5 . Let $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$, $g:(V,\tau_{R^{'}}(Y)) \to (W,\tau_{R^{''}}(Z))$ be two contra Nwg-irresoulute functions, then their composition is Nwg - irresolute function.

Proof: Let A be a Nwg-open set in W. $g^{-1}(A)$ is Nwg-closed set in V, since g is contra Nwg-irresolute function. $f^{-1}\left(g^{-1}(A)\right)$ is Nwg-open in U because f is contra Nwg-irresolute. Hence the composition of f and g is Nwg-continous function.

Theorem 4.6. Every Contra Nwg-irresolute function is contra Nwg-continuous function.

Proof: Let $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ be a contra Nwg-irresolute function, and A be a Nano closed set in $(V,\tau_R^{'}(Y))$. A is Nwg-closed set since every Nano closed set is Nwg-closed set. Then $f^{-1}(A)$ is Nwg-open set. Hence f is contra Nwg-continuous function.

Remark 4.7. Every Contra Nwg- continuous function need not be contra Nwg-irresolute function as shown in the following example.

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Example 4.8. Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{a\},\{b\},\{c,d\},\{e\}\}$ and $X = \{c,d,e\}$. Then the Nano topology is $\tau_R(X) = \{U,\phi,\{c,d,e\}\}$.

Let $V = \{a,b,c,d,e\}$ with $V/R^{'} = \{\{a\},\{b\},\{c\}\{d,e\}\}$ and $Y = \{b,c\}$. Then the Nano topology is $\tau_R^{'}(Y) = \{V,\phi,\{b,c\}\}$. Define $f:(U,\tau_R(X)) \to (V,\tau_R^{'}(Y))$ as $f(a) = d, f(b) = c, f(c) = e, f(d) = a, f(e) = b$. Then f is contra Nwg continuous function but not Contra Nwg-irresolute function since inverse image of Nwg-open set $\{a,b,e\}$ is not Nwg closed in V.

5. Conclusion

In this paper, we introduced a weaker form of Contra continuous function called Contra Nwg-continuous function in Nano topological space. Some of its characterization are analyzed in terms of Nwg-closure and Nwg-interior .Various conditions for a function to be Contra Nwg-continuous function is also established. In Section 4 Contra Nwg-irresolute function has been introduced and studied few of its properties.

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