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Group {1,−1, *i*, −*i*} Cordial Labeling of Certain Splitting Graphs

***M.K.Karthik Chidambaram*[1], *S.Athisayanathan*[2] and *R.Ponraj*[3]**

[1]Department of Mathematics, St.Xavier's College
Palayamkottai 627 002, Tamil Nadu, India.
E-mail: karthikmat5@gmail.com
[2]Department of Mathematics, St.Xavier's College
Palayamkottai 627 002, Tamil Nadu, India.
E-mail: athisxc@gmail.com
[3]Department of Mathematics, Sri Paramakalyani College
Alwarkuruchi 627 412, Tamil Nadu, India.
E-mail: ponrajmath@gmail.com
[1]Corresponding Author

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Abstract. Let G be a (p,q) graph and A be a group. Let f : V (G) → A be a function. The order of a ∈ A is the least positive integer n such that $a^n = e$. We denote the order of a by o(a). For each edge uv assign the label 1 if (o(f(u)), o(f(v))) = 1or 0 otherwise. f is called a group A Cordial labeling if $|v_f (a)-v_f (b)| \le 1$ and $|e_f (0)-e_f (1)| \le 1$, where $v_f (x)$ and $e_f (n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with n(n = 0, 1). A graph which admits a group A Cordial labeling is called a group A Cordial graph. The Splitting graph of G, S′(G) is obtained from G by adding for each vertex v of G, a new vertex v′ so that v′ is adjacent to every vertex that is adjacent to v. Note that if G is a (p, q) graph then S′(G) is a (2p, 3q) graph. In this paper we prove that Splitting graphs of Star S ‘ $(K_{1,n})$, Fan $S'(F_n)$, Comb $S'(P_n \Theta K_1)$, Ladder $S'(L_n)$, Friendship graph $S'(C_n^{(3)})$, Umbrella graph $S'(U_{n,n})$ and Book $S'(B_n)$ are group {1,−1, i,−i} Cordial for every n.

Keywords: Cordial labeling, group A Cordial labeling, group {1,−1, i,−i} Cordial labeling, splitting graph .

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Graphs considered here are finite, undirected and simple. Let A be a group. The order of a ∈ A is the least positive integer n such that $a^n = e$. We denote the order of a by o(a). Cahit [3] introduced the concept of Cordial labeling. He has done an extensive study on cordial labeling of graphs[4, 5]. Cordial labeling behaviour of several graphs are also studied by Diab [7, 8], Salehi [12], Cichacz [6] and many others. Several authors have defined several types of cordial labeling [11]. Motivated by this, we defined group A cordial labeling and investigated some of its properties. We also defined group {1,−1, i,−i} cordial labeling and discussed the behaviour of that labeling for some standard graphs [1,2] .

The Splitting graph of G, S′(G) is obtained from G by adding for each vertex v of G, a new vertex v′ so that v′ is adjacent to every vertex that is adjacent to v. Note that if G is a (p, q) graph then S′(G) is a (2p, 3q) graph. In this paper we discuss the labeling for Splitting graphs of some graphs. Terms not defined here are used in the sense of Harary [10] and Gallian [9].

2. Preliminaries

The greatest common divisor of two integers m and n is denoted by(m, n) and m and n are said to be relatively prime if (m, n) = 1. For any real number x, we denote by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x. A path is an alternating sequence of vertices and edges, v_1, e_1, v_2, e_2, ...,e_{n-1}, v_n, which are distinct, such that e_i is an edge joining v_i and v_{i+1} for $1 \leq i \leq n-1$. A path on n vertices is denoted by P_n. A path v_1, e_1, v_2, e_2, ..., e_{n-1},v_n, e_n, v_1 is called a cycle and a cycle on n vertices is denoted by C_n. A bipartite graph is a graph whose vertex set V (G) can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. If G contains every edge joining V_1 and V_2, then G is a complete bipartite graph. If $|V_1| = m$ and $|V_2| = n$, then the complete bipartite graph is denoted by $K_{m,n}$. $K_{1,n}$ is called a star graph. Given two graphs G and H, G+H is the graph with vertex set $V(G) \cup V(H)$and edge set $E(G) \cup E(H) \cup \{uv/u \in V(G), v \in V(H)\}$. A Wheel W_n is defined as $C_n + K_1$.The Cartesian product of two graphs $G_1$and $G_2$is the graph $G_1 \times G_2$with the vertex set V1×V2 and two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ are adjacent whenever [$u_1 = v_1$and $u_2$adj v_2] or [$u_2 = v_2$and $u_1$adj v_1]. The Book B_m is the graph $K_{1,m} \times P_2$. The graph $L_n = P_n \times P_2$is called a Ladder.

3. Main results

Definition 3.1. Let G be a (p,q)graph and consider the group A = {1,−1, i,−i} with multiplication. Let $f : V(G) \to A$ be a function. For each edge uv assign the label 1 if (o(f(u)), o(f(v))) = 1or 0 otherwise. f is called a group {1,−1, i,−i} Cordial labeling if $|v_f(a) - v_f(b)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(x)$ and $e_f(n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with n(n =0, 1). A graph which admits a group {1,−1, i,−i} Cordial labeling is called a group {1,−1, i,−i} Cordial graph.

Example 3.2. A simple example of a group {1,−1, i,−i} Cordial graph is given in Fig. 3.1.

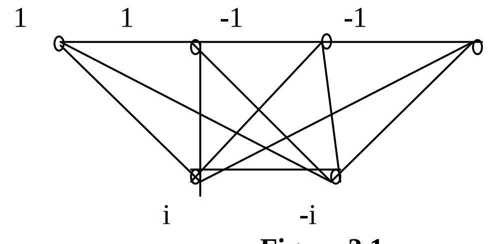

Figure 3.1:

Definition 3.3. The Splitting graph of G, S′(G) is obtained from G by adding for each vertex v of G, a new vertex v′ so that v′ is adjacent to every vertex that is adjacent to v. Note that if G is a (p, q) graph then S′(G) is a (2p, 3q) graph.

We now investigate the group {1,−1, i,−i} Cordial labeling of Splitting Graph of some graphs .

Theorem 3.4. The splitting graph of the Star S′(K1,n), is group {1,−1, i,−i} cordial for every n.

Proof. Let u be the vertex of $K_{1,n}$ of degree n and let u_1, u_2, ..., u_n be the vertices of degree 1 adjacent to u. Let v be the vertex corresponding to u in S′(K1,n) and v_1, v_2, ..., v_n be the newly added vertices so that for $1 \leq i \leq n$, v_i is adjacent to the neighbours of u_i. Also v is adjacent to the neighbours of u. Number of vertices in S′($K_{1,n}$) is 2n + 2 and number of edges is 3n.

Case 1. n is even.

Let n = 2k, ($k \geq 1$, $k \in Z$). If k = 1, a group {1,−1, i,−i} cordial labeling of S′(K1,2) is given in Fig 3.2. Suppose $k \geq 2$. Two vertex labels should appear k + 1 times and two should appear k times in a group {1,−1, i,−i}cordial labeling. Each edge label should appear 3k times. Define a labeling f of S′($K_{1,n}$) as follows.

Label the vertices v, v_1, v_2, ..., v_k with 1. Label the remaining vertices arbitrarily so that k +1 of them get label −1, k of them get label i and k of them get label −i. Number of edges with label 1 = n + k = 3k.

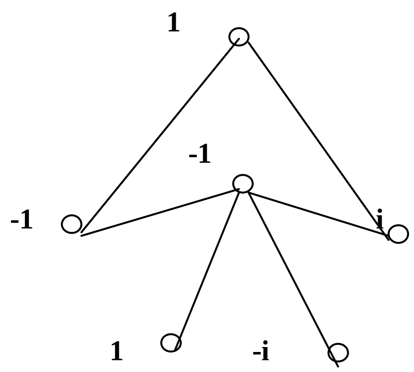

Figure 3.2:

Case 2. n is odd.

Let n = 2k + 1, ($k \geq 0$, $k \in Z$). If k = 0, then S′($K_{1,n}$) is P_4which is known to be group {1,−1, i,−i} cordial. Suppose $k \geq 1$. Each vertex label should appear k + 1 times. One edge label should appear 3k + 1 times and another should appear 3k + 2 times. Define a labeling f of S′(K1,n) as follows. Label the vertices v, v_1, v_2,..., v_k with 1. Label the remaining vertices arbitrarily so that k + 1 of them get label −1, k + 1 of them get label i and k + 1 of them get label −i. Number of edges with label 1 = n + k = 3k + 1.Table 3.1 shows that f is a group {1,−1, i,−i} cordial labeling.

N	v_f (1)	v_f (-1)	v_f (i)	v_f (-i)	e_f (0)	e_f (1)
2k, k≥2, k ∈Z	k+1	k+1	k	k	3k	3k
2k+1, k≥1,k ∈Z	k+1	k+1	k+1	k+1	3k+2	3k+1

Table 3.1:

Definition 3.5. The graph $F_n= P_n+ K_1$is called a fan graph where P_n:u_1, u_2,. . ., u_n is a path and V (K_1) = u.

Theorem 3.6. The splitting graph of the fan S′(F_n) is group {1,−1, i,−i}cordial for every n.

Proof. Let u_1, u_2, ..., u_n be the vertices of the path P_n in F_n and u be the vertex of degree n in F_n. Let v be the vertex corresponding to u in S′(F_n) andv_1, v_2, ..., v_n be the newly added vertices so that for $1 \leq i \leq n$, v_i is adjacent to the neighbours of u_i. Also v is adjacent to the neighbours of u. Number of vertices in S′(F_n) is 2n + 2 and number of edges is 6n − 3.

Case 1. n is even.

Let n = 2k, (k ≥ 1, k ∈ Z). Two vertex labels should appear k + 1 times and two should appear k times in a group {1,−1, i,−i} cordial labeling. One edge label should appear 6k−1 times and another edge label appears 6k−2 times. Define a labeling f of S′(Fn) as follows.

Label the vertices u_1, u_3, u_5, ..., $u_{2k-1}$with 1. Label the remaining vertices arbitrarily so that k + 1 of them get label −1, k + 1 of them get label i and k of them get label −i. Number of edges with label 1 = 4 + (k − 1)6 = 6k − 2.

Case 2. n is odd.

Let n = 2k + 1, (k ≥ 0, k ∈ Z). If k = 0, then S′(F_n) is P_4which is known to be group {1,−1, i,−i} cordial. Suppose k ≥ 1. Each vertex label should appear k + 1 times. One edge label should appear 6k + 1 times and another should appear 6k + 2 times. Define a labeling f of S′(F_n) as follows.

Label the vertices v_2, u_2, u_4, ..., u_{2k} with 1. Label the remaining vertices arbitrarily so that k + 1 of them get label −1, k + 1 of them get label i and k + 1of them get label −i. Number of edges with label 1 = 6k + 2.Table 3.2 shows that f is a group {1,−1, i,−i} cordial labeling.

n	v_f (1)	v_f (-1)	v_f (i)	v_f (-i)	e_f (0)	e_f (1)
2k, k ≥ 1, k ∈Z	k	k+1	k+1	k	6k-1	6k-2
2k+1, k ≥ 1,k ∈Z	k+1	k+1	k+1	k+1	6k+1	6k+2

Table 3.2:

Definition 3.7. Let G_1, G_2respectively be two (p_1, q_1), (p_2, q_2) graphs. The corona of $G_1$with G_2, $G_1 \Theta G_2$ is the graph obtained by taking one copy of $G_{1,}$ $p_1$copies of $G_2$and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.

Definition 3.8. The graph $P_n \Theta K_1$ is called a comb.

Theorem 3.9. The splitting graph of the Comb $S'(P_n \Theta K_1)$ is group {1,−1, i,−i} cordial for every n.
Proof. Let u_i ($1 \leq i \leq n$) be the vertices of the path P_n and v_i($1 \leq I \leq n$) be the pendent vertices of $P_n \Theta K_1$ adjacent to the vertices of P_n. Let u_i’, v_i’ ($1 \leq i \leq n$), be the vertices corresponding to ui, vi($1 \leq i \leq n$) in $S'(P_n \Theta K_1)$.Number of vertices in $S'(P_n \Theta K1)$ is 4n and number of edges is 6n − 3. Each vertex label should appear n times. One edge label should appear 3n−1 times and another should appear 3n − 2 times. Let f be a labeling of $S'(P_n \Theta K_1)$ defined as follows:
f (u_i) = −1($1 \leq i \leq n$);
f(vi) =I ($1 \leq I \leq n$);
f(u′i) = 1($1 \leq i \leq n$);
f(v′i) = −I ($1 \leq i \leq n$).
Number of edges with label 1 = 3n − 1 and so f is a group {1,−1, i,−i}cordial labeling of $S'(P_n \Theta K_1)$.

Theorem 3.10. The splitting graph of the Ladder S′(Ln) is group {1,−1, i,−i}cordial for every n.
Proof. Let u_i, v_i($1 \leq i \leq n$) be the vertices of the two paths in $L_n$so that uiand v_i are adjacent. Let u′i and v′I ($1 \leq i \leq n$) be the vertices corresponding to ui and vi($1 \leq i \leq n$). Number of vertices in S′(Ln) is 4n and number of edges is 9n − 6.

Case 1. n is even.
Let n = 2k, ($k \geq 1$, $k \in Z$). If k = 1, a group {1,−1, i,−i} cordial labeling of S′(L2) is given in Fig 3.3.

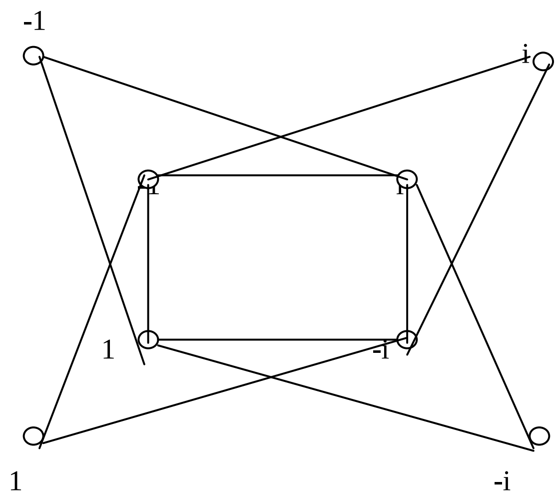

Figure 3.3:

Suppose $k \geq 2$. Each vertex label should appear 2k times and each edge label should appear $9k - 3$ times. Define a labeling f of $S'(Ln)$ as follows.
$f(u_2) = f(u_4) = \ldots = f(u_{2k}) = 1$;
$f(u_2') = f(u_4') = \ldots = f(u'2(k-2)) = 1$;
$f(u'1) = f(v'1) = 1$;
$f(u1) = f(u3) = \ldots = f(u2k-1) = -1$;
$f(v1) = f(u'3) = \ldots = f(u'2k-1) = -1$;
For $2 \leq i \leq 2k$, $f(vi) = i$ and $f(u'2k-2) = i$.
For $2 \leq i \leq 2k$, $f(v'i) = -i$ and $f(u'2k) = i$.
Number of edges with label $1 = 6k + 3(k - 2) + 1 + 2 = 9k - 3$.

Case 2. n is odd.
Let $n = 2k + 1$, $(k \geq 0, k \in Z)$. If $k = 0$, then $S'(L1)$ is P4 which is known to be group $\{1,-1, i,-i\}$ cordial. Suppose $k \geq 1$. Each vertex label should appear $2k + 1$ times. One edge label should appear $9k + 1$ times and another should appear $9k + 2$ times. Define a labeling f of $S'(Ln)$ as follows.
$f(u2) = f(u4) = \ldots = f(u2k) = 1$;
$f(u'2) = f(u'4) = \ldots = f(u'2(k)) = 1$ and $f(u'1) = 1$
$f(u1) = f(u3) = \ldots = f(u2k+1) = -1$;
$f(u'3) = \ldots = f(u'2k+1) = -1$;
For $1 \leq i \leq 2k + 1$, $f(vi) = i$ and $f(v'i)) = -i$.
Number of edges with label $1 = 9k+1$. Thus f is a group $\{1,-1, i,-i\}$ cordial labeling of $S'(Ln)$.

Definition 3.11. Let $C_n^{(t)}$ denote the one-point union of t cycles of length n. The graph $C_3^{(t)}$is called a friendship graph.

Theorem 3.12. The splitting graph of the Friendship graph $S'(C_3^{(t)})$ is group $\{1,-1, i,-i\}$ cordial for every n.
Proof. Let u be the vertex of degree 2n in $S'(C_3^{(t)})$ and ui, $vi(1 \leq i \leq n)$be the vertices of degree 2 in each $C_3^{(t)}$. Let u′ be the vertex corresponding to u in
$S'(C_3^{(t)})$ and u′i, v′I $(1 \leq i \leq n)$ be the vertices corresponding to ui and $vi(1 \leq i \leq n)$. Number of vertices in $S'(C_3^{(t)})$ is $4n + 2$ and number of edges is 9n.

Case 1. n is even.
Let $n = 2k$, $(k \geq 1, k \in Z)$. Two vertex labels should appear $2k + 1$ times and two other vertex labels should appear 2k times. Each edge label should appear 9k times. Define a labeling f of $S'(C_3^{(t)})$ as follows.
$f(u') = 1$; For $1 \leq i \leq k$, $f(ui) = f(u'i) = 1$;
$f(u) = -1$; For $k + 1 \leq i \leq 2k$, $f(ui) = f(u'i) = -1$;
For $1 \leq i \leq 2k$, $f(vi) = i$ and $f(v'i) = -i$;
Number of edges with label $1 = 4k + 3k + 2k = 9k$.

Case 2. n is odd.

Let n = 2k + 1, (k ≥ 0, k ∈ Z). If k = 0, then S′($C_3^{(t)}$) is S′(C3) which is known to be group {1,−1, i,−i} cordial . Suppose k ≥ 1. Two vertex labels should appear 2k + 2 times and two other vertex labels should appear 2k + 1 times. One edge label should appear 9k + 4 times and another should appear 9k + 5 times. Define a labeling f of S′($C_3^{(t)}$) as follows.

f(u′) = 1; For 1 ≤ i ≤ k, f(ui) = 1 and for 1 ≤ i ≤ k + 1, f(u′i) = 1;
f(u) = −1; For k + 1 ≤ i ≤ 2k + 1, f(ui) = −1 and for k + 2 ≤ i ≤2k + 1, f(u′i) = −1;
For 1 ≤ i ≤ 2k + 1, f(vi) = i and f(v′i) = −i;
Number of edges with label 1 = 9k + 4. Table 3.3 shows that f is a group {1,−1, i,−i} cordial labeling of S′($C_3^{(t)}$).

n	v_f (1)	v_f (-1)	v_f (i)	v_f (-i)	e_f (0)	e_f (1)
2k, k ≥ 1, k ∈Z	2k+1	2k+1	2k	2k	9k	9k
2k+1, k ≥0,k ∈Z	2k+2	2k+2	2k+1	2k+1	9k+5	9k+4

Table 3.3:

Definition 3.13. The Umbrella graph Un,m, m > 1 is obtained from a fan Fn by pasting the end vertex of the path Pm : v1v2 . . . vm to the vertex of K1 of the fan Fn.

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Theorem 3.14. The splitting graph of the Umbrella graph S′(Un,n) is group {1,−1, i,−i} cordial for every n.

Proof. Let ui, vi(1 ≤ i ≤ n) be the vertices of Un,n where ui(1 ≤ i ≤ n) are the vertices of the path in Fn and vi(1 ≤ i ≤ n) are the vertices of the path in Un,n where v1 is identified with K1. Let u′i, v_i’(1 ≤ i ≤ n) be the corresponding vertices of S′(Un,n). Number of vertices in S′(Un,n) is 4n and number of edges is 9n − 6.

Case 1. n is even.

Let n = 2k, (k ≥ 1, k ∈ Z). Each vertex label should appear 2k times. Each edge label should appear 9k − 3 times. Define a labeling f of S′(Un,n) as follows:

f(u1) = f(u3) = . . . = f(u2k−3) = 1; f(un) = 1;
f(v2) = f(v3) = . . . = f(vk+1) = 1;
f(u2) = f(u4) = . . . = f(u2k−2) = 1 = f(u2k−1) = −1;
f(v1) = f(vk+2) = . . . = f(v2k) = −1;
For 1 ≤ i ≤ 2k, f(u′i) = i;
For 1 ≤ i ≤ 2k, f(v′i) = −i;
Number of edges with label 1 = 4 + (k − 2)6 + 4 + 4 + (k − 1)3 = 9k − 3.

Case 2. n is odd.

Let n = 2k + 1, (k ≥ 0, k ∈ Z). If k = 0, then S′(U1,1) is P4 which is trivially group {1,−1, i,−i} cordial. Suppose k ≥ 1. Each vertex label should appear2k + 1 times. One edge label should appear 9k + 2 times and another should appear 9k + 1 times. Define a labeling f of S′(Un,n) as follows.

$f(u1) = f(u3) = . . . = f(u2k−3) = 1$; $f(un) = 1$;
$f(v2) = f(v3) = . . . = f(vk+1) = f(vk+3) = 1$;
$f(u2) = f(u4) = . . . = f(u2k−2) = 1 = f(u2k−1) = f(u2k) = −1$;
$f(v1) = f(vk+2) = . . . = f(v2k) = f(v2k+1) = −1$;
For $1 \leq i \leq 2k + 1$, $f(u'i) = i$;
For $1 \leq i \leq 2k + 1$, $f(v'i) = −i$;
Number of edges with label 1 = 4 + (k − 2)6 + 4 + 4 + (k − 1)3 + 4 = 9k + 1.
Table 3.4 shows that f is a group {1,−1, i,−i} cordial labeling of S′(Un,n) .

n	v_f (1)	v_f (-1)	v_f (i)	v_f (-i)	e_f (0)	e_f (1)
2k, $k \geq 1$, $k \in Z$	2k	2k	2k	2k	9k-3	9k-3
2k+1, $k \geq 0$, $k \in Z$	2k+1	2k+1	2k+1	2k+1	9k+2	9k+1

Table 3.4:

Theorem 3.15. The splitting graph of the Book S′(Bn) is group {1,−1, i,−i}cordial for every n.
Proof. Let u and v be the center vertices of the two K1,n’s and ui, vi($1 \leq i \leq n$) be the pendent vertices adjacent to u, v respectively. Let u′, v′, u′i, v′i($1 \leq i \leq n$) be the corresponding vertices of S′(Bn). Number of vertices in S′(Bn)is 4n + 4 and number of edges is 9n + 3.

Case 1. n is even.
Let n = 2k, ($k \geq 1$, $k \in Z$). Each vertex label should appear 2k+1 times. One edge label should appear 9k + 1 times and another edge label should appear 9k + 2 times. Define a labeling f of S′(Bn) as follows:
$f(u) = 1$; For $1 \leq i \leq k$, $f(ui) = 1$;
For $k + 1 \leq i \leq 2k$, $f(v'i) = 1$;
$f(v) = −1$; For $k + 1 \leq i \leq 2k$, $f(ui) = −1$;
For $1 \leq i \leq k$, $f(v'i) = −1$;
$f(u') = i$; For $1 \leq i \leq 2k$, $f(u'i) = i$;
$f(v') = −i$; For $1 \leq i \leq 2k$, $f(vi) = −i$;
Number of edges with label 1 = 2(2k + 1) + 3k + 2k = 9k + 2.

Case 2. n is odd.
Let n = 2k + 1, ($k \geq 0$, $k \in Z$). Each vertex label should appear 2k + 2times. Each edge label should appear 9k + 6 times. Define a labeling f of S′(Bn) as follows.
$f(u) = 1$; For $1 \leq i \leq k$, $f(ui) = 1$;For $k + 1 \leq i \leq 2k + 1$, $f(v'i) = 1$;
$f(v) = −1$; For $k + 1 \leq i \leq 2k + 1$, $f(ui) = −1$;For $1 \leq i \leq k$, $f(v'i) = −1$;
$f(u') = i$; For $1 \leq i \leq 2k + 1$, $f(u'i) = i$;$f(v') = −i$; For $1 \leq i \leq 2k + 1$, $f(vi) = −i$;

Number of edges with label 1 = 2(2k +1+1)+3k +2(k +1) = 9k +6. Table 3.5 shows that f is a group {1,−1, i,−i} cordial labeling of S′(Bn).

n	v_f (1)	v_f (-1)	v_f (i)	v_f (-i)	e_f (0)	e_f (1)
2k, k ≥ 1, k ∈Z	2k	2k	2k	2k	9k-3	9k-3
2k+1, k ≥0,k ∈Z	2k+1	2k+1	2k+1	2k+1	9k+2	9k+1

Table 3.5:

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