Group \{1, -1, i, -i\} Cordial Labeling of Certain Splitting Graphs

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Abstract. Let G be a (p, q) graph and A be a group. Let \(f : V(G) \rightarrow A\) be a function. The order of \(a \in A\) is the least positive integer \(n\) such that \(a^n = e\). We denote the order of \(a\) by \(o(a)\). For each edge uv assign the label 1 if \(o(f(u)), o(f(v)) = 1\) or 0 otherwise. \(f\) is called a group A Cordial labeling if \(|v\_{f}(a) - v\_{f}(b)| \leq 1\) and \(|e\_{f}(0) - e\_{f}(1)| \leq 1\), where \(v\_{f}(x)\) and \(e\_{f}(n)\) respectively denote the number of vertices labeled with an element \(x\) and number of edges labeled with \(n\) \((n = 0, 1)\). A graph which admits a group A Cordial labeling is called a group A Cordial graph. The Splitting graph of G, \(S'(G)\) is obtained from G by adding for each vertex \(v\) of G, a new vertex \(v'\) so that \(v'\) is adjacent to every vertex that is adjacent to \(v\). Note that if G is a (p, q) graph then \(S'(G)\) is a (2p, 3q) graph. In this paper we prove that Splitting graphs of Star \(S'(K_{1,n})\), Fan \(S'(F_n)\), Comb \(S'(P_n \Theta K_1)\), Ladder \(S'(L_n)\), Friendship graph \(S'(C_{\infty}^{(3)})\), Umbrella graph \(S'(U_{a,n})\) and Book \(S'(B_3)\) are group \{1, -1, i, -i\} Cordial for every \(n\).

Keywords: Cordial labeling, group A Cordial labeling, group \{1, -1, i, -i\} Cordial labeling, splitting graph.

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Graphs considered here are finite, undirected and simple. Let A be a group. The order of \(a \in A\) is the least positive integer \(n\) such that \(a^n = e\). We denote the order of \(a\) by \(o(a)\). Cahit [3] introduced the concept of Cordial labeling. He has done an extensive study on cordial labeling of graphs[4, 5]. Cordial labeling behaviour of several graphs are also studied by Diab [7, 8], Salehi [12], Cichacz [6] and many others. Several authors have defined several types of cordial labeling [11]. Motivated by this, we defined group A
cordial labeling and investigated some of its properties. We also defined group \{1,−1, i,−i\} cordial labeling and discussed the behaviour of that labeling for some standard graphs \[1,2\].

The Splitting graph of G, \(S'(G)\) is obtained from G by adding for each vertex v of G, a new vertex \(v'\) so that \(v'\) is adjacent to every vertex that is adjacent to v. Note that if G is a \((p, q)\) graph then \(S'(G)\) is a \((2p, 3q)\) graph. In this paper we discuss the labeling for Splitting graphs of some graphs. Terms not defined here are used in the sense of Harary [10] and Gallian [9].

2. Preliminaries

The greatest common divisor of two integers m and n is denoted by \((m, n)\) and m and n are said to be relatively prime if \((m, n) = 1\). For any real number \(x\), we denote by \(\lceil x \rceil\), the greatest integer smaller than or equal to \(x\) and by \(\lfloor x \rfloor\), we mean the smallest integer greater than or equal to \(x\). A path is an alternating sequence of vertices and edges, \(v_1, e_1, v_2, e_2, \ldots, e_{n-1}, v_n\), which are distinct, such that \(e_i\) is an edge joining \(v_i\) and \(v_{i+1}\) for \(1 \leq i \leq n-1\). A path on \(n\) vertices is denoted by \(P_n\). A path \(v_1, e_1, v_2, e_2, \ldots, e_{n-1}, v_n\) is called a cycle and a cycle on \(n\) vertices is denoted by \(C_n\). A bipartite graph is a graph whose vertex set \(V(G)\) can be partitioned into two subsets \(V_1\) and \(V_2\) such that every edge of \(G\) joins a vertex of \(V_1\) with a vertex of \(V_2\). If \(G\) contains every edge joining \(V_1\) and \(V_2\), then \(G\) is a complete bipartite graph. If \(|V_1| = m\) and \(|V_2| = n\), then the complete bipartite graph is denoted by \(K_{m,n}\). \(K_{1,n}\) is called a star graph. Given two graphs \(G\) and \(H\), \(G+H\) is the graph with vertex set \(V(G) \cup V(H)\) and edge set \(E(G) \cup E(H) \cup \{uv/u \in V(G), v \in V(H)\}\). A Wheel \(W_n\) is defined as \(K_{1,n} \times P_1\). The book \(B_m\) is the graph \(K_1 \times P_m\). The ladder \(L_n = P_n \times P_2\) is called a ladder.

3. Main results

Definition 3.1. Let G be a \((p,q)\) graph and consider the group \(A = \{1,−1, i,−i\}\) with multiplication. Let \(f : V(G) \rightarrow A\) be a function. For each edge uv assign the label 1 if \((f(u)) \times (f(v)) = 1\) or 0 otherwise. f is called a group \(\{1,−1, i,−i\}\) Cordial labeling if \(|v_f(a) − v_f(b)| \leq 1\) and \(|e_f(0) − e_f(1)| \leq 1\), where \(v_f(x)\) and \(e_f(n)\) respectively denote the number of vertices labeled with an element \(x\) and number of edges labeled with \(n\) (\(n = 0, 1\)). A graph which admits a group \(\{1,−1, i,−i\}\) Cordial labeling is called a group \(\{1,−1, i,−i\}\) Cordial graph.

Example 3.2. A simple example of a group \{1,−1, i,−i\} Cordial graph is given in Fig. 3.1.

![Figure 3.1](image-url)
Definition 3.3. The Splitting graph of $G$, $S'(G)$ is obtained from $G$ by adding for each vertex $v$ of $G$, a new vertex $v'$ so that $v'$ is adjacent to every vertex that is adjacent to $v$. Note that if $G$ is a $(p, q)$ graph then $S'(G)$ is a $(2p, 3q)$ graph.

We now investigate the group $\{1, -1, i, -i\}$ Cordial labeling of Splitting Graph of some graphs.

Theorem 3.4. The splitting graph of the Star $S'(K_{1,n})$, is group $\{1, -1, i, -i\}$ cordial for every $n$.

Proof. Let $u$ be the vertex of $K_{1,n}$ of degree $n$ and let $u_1, u_2, \ldots, u_n$ be the vertices of degree 1 adjacent to $u$. Let $v$ be the vertex corresponding to $u$ in $S'(K_{1,n})$ and $v_1, v_2, \ldots, v_n$ be the newly added vertices so that for $1 \leq i \leq n$, $v_i$ is adjacent to the neighbours of $u_i$. Also $v$ is adjacent to the neighbours of $u$. Number of vertices in $S'(K_{1,n})$ is $2n + 2$ and number of edges is $3n$.

Case 1. $n$ is even.
Let $n = 2k$, $(k \geq 1, k \in Z)$. If $k = 1$, a group $\{1, -1, i, -i\}$ cordial labeling of $S'(K_{1,2})$ is given in Fig 3.2. Suppose $k \geq 2$. Two vertex labels should appear $k + 1$ times and two should appear $k$ times in a group $\{1, -1, i, -i\}$ cordial labeling. Each edge label should appear $3k$ times. Define a labeling $f$ of $S'(K_{1,n})$ as follows.

Label the vertices $v, v_1, v_2, \ldots, v_k$ with 1. Label the remaining vertices arbitrarily so that $k + 1$ of them get label $-1$, $k$ of them get label $i$ and $k$ of them get label $-i$. Number of edges with label 1 = $n + k = 3k$.

![Figure 3.2:](image)

Case 2. $n$ is odd.
Let $n = 2k + 1$, $(k \geq 0, k \in Z)$. If $k = 0$, then $S'(K_{1,2})$ is $P_4$ which is known to be group $\{1, -1, i, -i\}$ cordial. Suppose $k \geq 1$. Each vertex label should appear $k + 1$ times. One edge label should appear $3k + 1$ times and another should appear $3k + 2$ times. Define a labeling $f$ of $S'(K_{1,n})$ as follows. Label the vertices $v, v_1, v_2, \ldots, v_k$ with 1. Label the remaining vertices arbitrarily so that $k + 1$ of them get label $-1$, $k + 1$ of them get label $i$ and $k + 1$ of them get label $-i$. Number of edges with label 1 = $n + k = 3k + 1$. Table 3.1 shows that $f$ is a group $\{1, -1, i, -i\}$ cordial labeling.
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Table 3.1:

<table>
<thead>
<tr>
<th>N</th>
<th>$v_f(1)$</th>
<th>$v_f(-1)$</th>
<th>$v_f(i)$</th>
<th>$v_f(-i)$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2k$, $k \geq 2$, $k \in \mathbb{Z}$</td>
<td>$k+1$</td>
<td>$k+1$</td>
<td>$k$</td>
<td>$k$</td>
<td>$3k$</td>
<td>$3k$</td>
</tr>
<tr>
<td>$2k+1$, $k \geq 1$, $k \in \mathbb{Z}$</td>
<td>$k+1$</td>
<td>$k+1$</td>
<td>$k+1$</td>
<td>$k+1$</td>
<td>$3k+2$</td>
<td>$3k+1$</td>
</tr>
</tbody>
</table>

Table 3.2:

**Definition 3.5.** The graph $F_n = P_n + K_1$ is called a fan graph where $P_n$ is a path and $V(K_1) = u$.

**Theorem 3.6.** The splitting graph of the fan $S'(F_n)$ is group $\{1, -1, i, -i\}$ cordial for every $n$.

**Proof.** Let $u_1, u_2, \ldots, u_n$ be the vertices of the path $P_n$ in $F_n$ and $u$ be the vertex of degree $n$ in $F_n$. Let $v$ be the vertex corresponding to $u$ in $S'(F_n)$ and $v_1, v_2, \ldots, v_n$ be the newly added vertices so that for $1 \leq i \leq n$, $v_i$ is adjacent to the neighbours of $u_i$ Also $v$ is adjacent to the neighbours of $u$. Number of vertices in $S'(F_n)$ is $2n + 2$ and number of edges is $6n - 3$.

**Case 1.** $n$ is even.

Let $n = 2k$, ($k \geq 1$, $k \in \mathbb{Z}$). Two vertex labels should appear $k + 1$ times and two should appear $k$ times in a group $\{1, -1, i, -i\}$ cordial labeling. One edge label should appear $6k - 1$ times and another edge label appears $6k - 2$ times. Define a labeling $f$ of $S'(F_n)$ as follows.

Label the vertices $u_1, u_3, u_5, \ldots, u_{2k-1}$ with 1. Label the remaining vertices arbitrarily so that $k + 1$ of them get label $-1$, $k + 1$ of them get label $i$ and $k$ of them get label $-i$. Number of edges with label 1 = $4 + (k - 1)6 = 6k - 2$.

**Case 2.** $n$ is odd.

Let $n = 2k + 1$, ($k \geq 0$, $k \in \mathbb{Z}$). If $k = 0$, then $S'(F_0)$ is $P_3$ which is known to be group $\{1, -1, i, -i\}$ cordial. Suppose $k \geq 1$. Each vertex label should appear $k + 1$ times. One edge label should appear $6k + 1$ times and another should appear $6k + 2$ times. Define a labeling $f$ of $S'(F_n)$ as follows.

Label the vertices $v_2, u_2, u_4, \ldots, u_{2k+1}$ with 1. Label the remaining vertices arbitrarily so that $k + 1$ of them get label $-1$, $k + 1$ of them get label $i$ and $k + 1$ of them get label $-i$. Number of edges with label 1 = $6k + 2$. Table 3.2 shows that $f$ is a group $\{1, -1, i, -i\}$ cordial labeling.

<table>
<thead>
<tr>
<th>n</th>
<th>$v_f(1)$</th>
<th>$v_f(-1)$</th>
<th>$v_f(i)$</th>
<th>$v_f(-i)$</th>
<th>$e_f(0)$</th>
<th>$e_f(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2k$, $k \geq 1$, $k \in \mathbb{Z}$</td>
<td>$k$</td>
<td>$k+1$</td>
<td>$k+1$</td>
<td>$k$</td>
<td>$6k-1$</td>
<td>$6k-2$</td>
</tr>
<tr>
<td>$2k+1$, $k \geq 1$, $k \in \mathbb{Z}$</td>
<td>$k+1$</td>
<td>$k+1$</td>
<td>$k+1$</td>
<td>$k+1$</td>
<td>$6k+1$</td>
<td>$6k+2$</td>
</tr>
</tbody>
</table>
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**Definition 3.7.** Let \(G_1, G_2\) respectively be two \((p_1, q_1), (p_2, q_2)\) graphs. The corona of \(G_1\) with \(G_2\), \(G_1 \Theta G_2\) is the graph obtained by taking one copy of \(G_1\), \(p_1\) copies of \(G_2\) and joining the \(i^{th}\) vertex of \(G_1\) with an edge to every vertex in the \(i^{th}\) copy of \(G_2\).

**Definition 3.8.** The graph \(P_n \Theta K_1\) is called a comb.

**Theorem 3.9.** The splitting graph of the Comb \(S'(P_n \Theta K_1)\) is group \{1, -1, i, -i\} cordial for every \(n\).

**Proof.** Let \(u_i (1 \leq i \leq n)\) be the vertices of the path \(P_n\) and \(v_i (1 \leq i \leq n)\) be the pendent vertices of \(P_n \Theta K_1\) adjacent to the vertices of \(P_n\). Let \(u_i', v_i' (1 \leq i \leq n)\), be the vertices corresponding to \(u_i, v_i (1 \leq i \leq n)\) in \(S'(P_n \Theta K_1)\). Number of vertices in \(S'(P_n \Theta K_1)\) is 4\(n\) and number of edges is 6\(n\) – 3. Each vertex label should appear \(n\) times. One edge label should appear 3\(n\) – 1 times and another should appear 3\(n\) – 2 times. Let \(f\) be a labeling of \(S'(P_n \Theta K_1)\) defined as follows:

\[
\begin{align*}
  f(u_i) &= -1 (1 \leq i \leq n); \\
  f(v_i) &= 1 (1 \leq i \leq n); \\
  f(u_i') &= 1 (1 \leq i \leq n); \\
  f(v_i') &= -1 (1 \leq i \leq n).
\end{align*}
\]

Number of edges with label 1 = 3\(n\) – 1 and so \(f\) is a group \{1, -1, i, -i\} cordial labeling of \(S'(P_n \Theta K_1)\).

**Theorem 3.10.** The splitting graph of the Ladder \(S'(L_n)\) is group \{1, -1, i, -i\} cordial for every \(n\).

**Proof.** Let \(u_i, v_i (1 \leq i \leq n)\) be the vertices of the two paths in \(L_n\) so that \(u_i\) and \(v_i\) are adjacent. Let \(u_i'\) and \(v_i' (1 \leq i \leq n)\) be the vertices corresponding to \(u_i\) and \(v_i (1 \leq i \leq n)\). Number of vertices in \(S'(L_n)\) is 4\(n\) and number of edges is 9\(n\) – 6.

**Case 1.** \(n\) is even.

Let \(n = 2k\), \((k \geq 1, k \in \mathbb{Z})\). If \(k = 1\), a group \{1, -1, i, -i\} cordial labeling of \(S'(L_2)\) is given in Fig 3.3.
Figure 3.3:
Suppose \( k \geq 2 \). Each vertex label should appear 2\( k \) times and each edge label should appear 9\( k - 3 \) times. Define a labeling \( f \) of \( S'(L_n) \) as follows.

\[
\begin{align*}
&f(u_2) = f(u_4) = \ldots = f(u_{2k}) = 1; \\
&f(u'_1) = f(v'_1) = 1; \\
&f(u_1) = f(u_3) = \ldots = f(u_{2k-1}) = -1; \\
&f(v_1) = f(u'_3) = \ldots = f(u'_{2k-1}) = -1; \\
&\text{For } 2 \leq i \leq 2k, f(v_i) = i \text{ and } f(u'_{2k-2}) = i. \\
&\text{For } 2 \leq i \leq 2k, f(v'_i) = -i \text{ and } f(u'_{2k}) = 1.
\end{align*}
\]

Number of edges with label 1 = 9\( k - 3 \).

**Case 2.** \( n \) is odd.

Let \( n = 2k + 1, (k \geq 0, k \in \mathbb{Z}) \). If \( k = 0 \), then \( S'(L_1) \) is \( P_4 \) which is known to be group \( \{1, -1, i, -i\} \) cordial. Suppose \( k \geq 1 \). Each vertex label should appear 2\( k + 1 \) times and one edge label should appear 9\( k + 1 \) times and another should appear 9\( k + 2 \) times. Define a labeling \( f \) of \( S'(L_n) \) as follows.

\[
\begin{align*}
&f(u_2) = f(u_4) = \ldots = f(u_{2k}) = 1; \\
&f(u'_1) = f(u'_2) = f(u'_4) = \ldots = f(u'_{2k}) = 1 \text{ and } f(u'_1) = 1 \\
&f(u_1) = f(u_3) = \ldots = f(u_{2k+1}) = -1; \\
&f(v_1) = f(u'_3) = \ldots = f(u'_{2k+1}) = -1; \\
&\text{For } 1 \leq i \leq 2k+1, f(v_i) = i \text{ and } f(v'_i) = -i.
\end{align*}
\]

Number of edges with label 1 = 9\( k + 1 \). Thus \( f \) is a group \( \{1, -1, i, -i\} \) cordial labeling of \( S'(L_n) \).

**Definition 3.11.** Let \( C_3^{(t)} \) denote the one-point union of \( t \) cycles of length \( n \). The graph \( C_3^{(t)} \) is called a friendship graph.

**Theorem 3.12.** The splitting graph of the Friendship graph \( S'(C_3^{(t)}) \) is group \( \{1, -1, i, -i\} \) cordial for every \( n \).

**Proof.** Let \( u \) be the vertex of degree 2\( n \) in \( S'(C_3^{(t)}) \) and \( u_i, v_i (1 \leq i \leq n) \) be the vertices of degree 2 in each \( C_3^{(t)} \). Let \( u' \) be the vertex corresponding to \( u \) in \( S'(C_3^{(t)}) \) and \( u'_i, v'_i (1 \leq i \leq n) \) be the vertices corresponding to \( u_i \) and \( v_i (1 \leq i \leq n) \). Number of vertices in \( S'(C_3^{(t)}) \) is \( 4n + 2 \) and number of edges is \( 9n \).

**Case 1.** \( n \) is even.

Let \( n = 2k, (k \geq 1, k \in \mathbb{Z}) \). Two vertex labels should appear 2\( k + 1 \) times and two other vertex labels should appear 2\( k \) times. Each edge label should appear 9\( k \) times. Define a labeling \( f \) of \( S'(C_3^{(t)}) \) as follows.

\[
\begin{align*}
&f(u') = 1; \text{ For } 1 \leq i \leq k, f(u'_i) = f(u'_i) = 1; \\
&f(u) = -1; \text{ For } k + 1 \leq i \leq 2k, f(u'_i) = f(u'_i) = -1; \\
&\text{For } 1 \leq i \leq 2k, f(v_i) = i \text{ and } f(v'_i) = -i.
\end{align*}
\]

Number of edges with label 1 = 4\( k + 3k + 2k = 9k \).

**Case 2.** \( n \) is odd.

---

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Let \( n = 2k + 1 \), \( (k \geq 0, k \in \mathbb{Z}) \). If \( k = 0 \), then \( S'(C_3^{(0)}) \) is \( S'(C_3) \) which is known to be group \( \{1,-1,i,-i\} \) cordial. Suppose \( k \geq 1 \). Two vertex labels should appear \( 2k + 2 \) times and two other vertex labels should appear \( 2k + 1 \) times. One edge label should appear \( 9k + 4 \) times and another should appear \( 9k + 5 \) times. Define a labeling \( f \) of \( S'(C_3^{(0)}) \) as follows.

\[
\begin{align*}
f(u'_1) &= 1; \quad & \text{for } 1 \leq i \leq k, \ f(u_i) = 1 \text{ and for } 1 \leq i \leq k + 1, \ f(u'_i) = 1; \\
f(u'_i) &= 1; \quad & \text{for } k + 1 \leq i \leq 2k + 1, \ f(u_i') = -1 \text{ and for } k + 2 \leq i \leq 2k + 1, \ f(u'_i) = -1; \\
f(u) &= -1; \quad & \text{for } 1 \leq i \leq 2k + 1, \ f(v_i) = i \text{ and for } k + 2 \leq i \leq 2k + 1, \ f(v'_i) = -i; \\
\end{align*}
\]

Number of edges with label 1 = 9k + 4. Table 3.3 shows that \( f \) is a group \( \{1,-1,i,-i\} \) cordial labeling of \( S'(C_3^{(0)}) \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( v_i (1) )</th>
<th>( v_i (-1) )</th>
<th>( v'_i (i) )</th>
<th>( v'_i (-i) )</th>
<th>( e_i (0) )</th>
<th>( e_i (1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2k, k \geq 1, k \in \mathbb{Z} )</td>
<td>2k + 1</td>
<td>2k + 1</td>
<td>2k</td>
<td>2k</td>
<td>9k</td>
<td>9k</td>
</tr>
<tr>
<td>( 2k + 1, k \geq 0, k \in \mathbb{Z} )</td>
<td>2k + 2</td>
<td>2k + 2</td>
<td>2k + 1</td>
<td>2k + 1</td>
<td>9k + 5</td>
<td>9k + 4</td>
</tr>
</tbody>
</table>

\[\text{Table 3.3:}\]

**Definition 3.13.** The Umbrella graph \( U_{n,m}, m > 1 \) is obtained from a fan \( F_n \) by pasting the end vertex of the path \( P_m : v_1v_2 \ldots v_m \) to the vertex of \( K_1 \) of the fan \( F_n \).

**Theorem 3.14.** The splitting graph of the Umbrella graph \( S'(U_{n,n}) \) is group \( \{1,-1,i,-i\} \) cordial for every \( n \).

**Proof.** Let \( u_i, v_i (1 \leq i \leq n) \) be the vertices of \( U_{n,n} \) where \( u_i (1 \leq i \leq n) \) are the vertices of the path in \( F_n \) and \( v_i (1 \leq i \leq n) \) are the vertices of the path in \( U_{n,n} \) where \( v_1 \) is identified with \( K_1 \). Let \( u'_i, v'_i (1 \leq i \leq n) \) be the corresponding vertices of \( S'(U_{n,n}) \). Number of vertices in \( S'(U_{n,n}) \) is \( 4n \) and number of edges is \( 9n - 6 \).

**Case 1.** \( n \) is even.

Let \( n = 2k, (k \geq 1, k \in \mathbb{Z}) \). Each vertex label should appear \( 2k \) times. Each edge label should appear \( 9k - 3 \) times. Define a labeling \( f \) of \( S'(U_{n,n}) \) as follows:

\[
\begin{align*}
f(u_1) &= f(u_3) = \ldots = f(u_{2k-3}) = 1; \ f(un) = 1; \\
f(v_2) &= f(v_3) = \ldots = f(v_{2k-1}) = 1; \\
f(u_2) &= f(u_4) = \ldots = f(u_{2k-2}) = 1 = f(u_{2k-1}) = -1; \\
f(v_1) &= f(v_{2k-2}) = \ldots = f(v_{2k}) = -1; \\
\end{align*}
\]

For \( 1 \leq i \leq 2k, f(u'_i) = i; \)

For \( 1 \leq i \leq 2k, f(v'_i) = -i; \)

Number of edges with label 1 = \( 4 + (k - 2)6 + 4 + 4 + (k - 1)3 = 9k - 3 \).

**Case 2.** \( n \) is odd.

Let \( n = 2k + 1, (k \geq 0, k \in \mathbb{Z}) \). If \( k = 0 \), then \( S'(U_{1,1}) \) is \( P_4 \) which is trivially group \( \{1,-1,i,-i\} \) cordial. Suppose \( k \geq 1 \). Each vertex label should appear \( 2k + 1 \) times. One edge label should appear \( 9k + 2 \) times and another should appear \( 9k + 1 \) times. Define a labeling \( f \) of \( S'(U_{n,n}) \) as follows.
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\[ f(u_1) = f(u_3) = \ldots = f(u_{2k-3}) = 1; f(u_n) = 1; \]
\[ f(v_2) = f(v_3) = \ldots = f(v_{k+1}) = f(v_{k+3}) = 1; \]
\[ f(u_2) = f(u_4) = \ldots = f(u_{2k-2}) = 1 = f(u_{2k-1}) = f(u_{2k}) = -1; \]
\[ f(v_1) = f(v_{k+2}) = \ldots = f(v_{2k}) = f(v_{2k+1}) = -1; \]

For \( 1 \leq i \leq 2k+1 \), \( f(u'_i) = i; \) for \( 1 \leq i \leq 2k+1 \), \( f(v'_i) = -i; \)

Number of edges with label 1 = \( 4 + (k - 2)6 + 4 + 4 + (k - 1)3 + 4 = 9k + 1. \)

Table 3.4 shows that \( f \) is a group \( \{1, -1, i, -i\} \) cordial labeling of \( S'(Un,n) \).

<table>
<thead>
<tr>
<th>n</th>
<th>( v_i ) (1)</th>
<th>( v_i ) (-1)</th>
<th>( v_i ) (i)</th>
<th>( v_i ) (-i)</th>
<th>( e_i ) (0)</th>
<th>( e_i ) (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2k, k ( \geq 1 ), k ( \in \mathbb{Z} )</td>
<td>2k</td>
<td>2k</td>
<td>2k</td>
<td>9k - 3</td>
<td>9k - 3</td>
<td></td>
</tr>
<tr>
<td>2k+1, k ( \geq 0, k \in \mathbb{Z} )</td>
<td>2k+1</td>
<td>2k+1</td>
<td>2k+1</td>
<td>9k + 2</td>
<td>9k + 1</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4:

**Theorem 3.15.** The splitting graph of the Book \( S'(Bn) \) is group \( \{1, -1, i, -i\} \) cordial for every \( n \).

**Proof.** Let \( u \) and \( v \) be the center vertices of the two \( K_{1,n} \)'s and \( u_i, v_i(1 \leq i \leq n) \) be the pendant vertices adjacent to \( u, v \) respectively. Let \( u'_i, v'_i, v'_i(1 \leq i \leq n) \) be the corresponding vertices of \( S'(Bn) \). Number of vertices in \( S'(Bn) \) is \( 4n + 4 \) and number of edges is \( 9n + 3 \).

**Case 1.** \( n \) is even.

Let \( n = 2k, (k \geq 1, k \in \mathbb{Z}) \). Each vertex label should appear \( 2k + 1 \) times. One edge label should appear \( 9k + 1 \) times and another edge label should appear \( 9k + 2 \) times. Define a labeling \( f \) of \( S'(Bn) \) as follows:

\[ f(u) = 1; \text{ For } 1 \leq i \leq k, f(u_i) = 1; \]
\[ \text{For } k + 1 \leq i \leq 2k, f(v'_i) = 1; \]
\[ f(v) = -1; \text{ For } k + 1 \leq i \leq 2k, f(u_i) = -1; \]
\[ \text{For } 1 \leq i \leq k, f(v'_i) = -1; \]
\[ f(u'_i) = i; \text{ For } 1 \leq i \leq 2k, f(u'_i) = i; \]
\[ f(v'_i) = -i; \text{ For } 1 \leq i \leq 2k, f(v'_i) = -i; \]

Number of edges with label 1 = \( 2(2k + 1) + 3k + 2k = 9k + 2. \)

**Case 2.** \( n \) is odd.

Let \( n = 2k + 1, (k \geq 0, k \in \mathbb{Z}) \). Each vertex label should appear \( 2k + 2 \) times. Each edge label should appear \( 9k + 6 \) times. Define a labeling \( f \) of \( S'(Bn) \) as follows:

\[ f(u) = 1; \text{ For } 1 \leq i \leq k, f(u_i) = 1; \]
\[ \text{For } k + 1 \leq i \leq 2k + 1, f(v'_i) = 1; \]
\[ f(v) = -1; \text{ For } k + 1 \leq i \leq 2k + 1, f(u_i) = -1; \]
\[ \text{For } 1 \leq i \leq k, f(v'_i) = -1; \]
\[ f(u'_i) = i; \text{ For } 1 \leq i \leq 2k + 1, f(u'_i) = i; \]
\[ f(v'_i) = -i; \text{ For } 1 \leq i \leq 2k + 1, f(v'_i) = -i; \]

Number of edges with label 1 = \( 2(2k + 1 + 1) + 3k + 2(k + 1) = 9k + 6. \) Table 3.5 shows that \( f \) is a group \( \{1, -1, i, -i\} \) cordial labeling of \( S'(Bn) \).
Group \{1,−1, i, −i\} Cordial Labeling of Certain Splitting Graphs

<table>
<thead>
<tr>
<th>n</th>
<th>(v_i(1))</th>
<th>(v_i(-1))</th>
<th>(v_i(i))</th>
<th>(v_i(-i))</th>
<th>(e_i(0))</th>
<th>(e_i(1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2k, k \geq 1, k \in \mathbb{Z})</td>
<td>2k</td>
<td>2k</td>
<td>2k</td>
<td>2k</td>
<td>9k-3</td>
<td>9k-3</td>
</tr>
<tr>
<td>(2k+1, k \geq 0, k \in \mathbb{Z})</td>
<td>2k+1</td>
<td>2k+1</td>
<td>2k+1</td>
<td>2k+1</td>
<td>9k+2</td>
<td>9k+1</td>
</tr>
</tbody>
</table>

Table 3.5:

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1. S. Athisayanathan, R. Ponraj and M. K Karthik Chidambaram, Group\{1,−1, i,−i\} Cordial labeling of sum of \(P_n\) and \(K_n\), *Journal of Mathematical and Computational Science*, 7(2) (2017) 335-346.