Annals of Pure and Applied Mathematics
Vol. 15, No. 2, 2017, 351-356
ISSN: 2279-087X (P), 2279-0888(online)
Published on 11 December 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v15n2a21

Annals of Pure and Applied Mathematics

On Multigranular Rough Soft Set

***M.Gilbert Rani*[1], *R.Rajeswari*[2] and *J.Jenitha Seronmani*[3]**

[1,3]Department of Mathematics, Arul Anandar College
Karumathur, Tamilnadu, India.
[2]Department of Mathematics, Fatima College, Madurai, Tamilnadu, India.
Email: gil.anto@yahoo.co.in

Received 25 November 2017; accepted 7 December 2017

Abstract. Granulate the similar things is an essential part in multivalued information system. Rough set theory plays a vital role to solve imprecise problem. In Particular, multigranular rough set is an efficient tool to work on multivalued information system. Soft set theory is also deal uncertainty. In this paper we propose multigranular rough soft set and its properties.

Keywords: Multigranular soft set, rough soft set, multigranular rough soft set

AMS Mathematics Subject Classification (2010): 54A05, 03E70

1. Introduction

With the help of granulation we can encapsulate the indiscernible thing into a single capsule. For a partition of equivalence relation or multi equivalence relations multigranulation is an emerging mathematical tool in uncertain data processing. Even though rough set is a powerful tool to solve imprecise problems occur in engineering environmental sciences, physics and so many fields, it is unigranular that is we work on single equivalence relation with rough sets. In this paper we introduce one hybrid notion of rough set.

In 1999, Molosdtov [5] recognized soft set theory to handle uncertainty. Soft set is defined as the image of the function from attribute subset to the power set of the universal set. Beside soft set theory, Pawlak [3] identified rough set as an approximations holding set. Both the sets have so many extension and applications. Vinay et al. [9] gave the definition of rough soft sets using soft relations.

2. Preliminaries

Let U be a common universe and let E be a set of parameters.

Definition 2.1. ([1]) A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U, where F is a mapping given by F: E $\rightarrow$ P (U).

In other words, the soft set is a parameterized family of subsets of the set U. Every set F(e) ($e \in$ E), from this family may be considered as the set of e-elements of the soft sets (F, E), or as the set of e-approximate elements of the soft set.

Definition 2.2. ([6]) For two soft sets (F, A) and (G, B) over U, (F, A) is called a soft subset of (G, B) if

(1) A $\subset$ B and

(2) $\forall e \in$ A, F (e) and G (e) are identical approximations.

This relationship is denoted by (F, A) $\tilde{\subset}$ (G, B).
Similarly, (F, A) is called a soft superset of (G, B) if (G, B) is a soft subset of (F, A). This relationship is denoted by (F, A) $\tilde{\supset}$ (G, B).

Definition 2.3. [6] Two soft sets (F, A) and (G, B) over U are called soft equal if (F, A) is a soft subset of(G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4. [6] The intersection of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where C = A∩B and $\forall e \in$ C, H(ε) = F (e) or G(e) (as both are same set). This is denoted by (F, A) $\tilde{\cap}$ (G, B) = (H, C).

Definition 2.5. [6] The union of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where C = A∪B and $\forall e \in$ C,

$$H(e)=\begin{cases} \mathrm{F}(e), & \text{if } e\in \mathrm{A-B} \\ \mathrm{G}(e), & \text{if } e\in \mathrm{B-A} \\ \mathrm{F}(e)\cup \mathrm{G}(e), & \text{if } e\in \mathrm{A}\cap \mathrm{B} \end{cases}$$

This is denoted by (F , A) $\tilde{\cup}$ (G, B) = (H, C).

Definition 2.6. [6] NULL SOFT SET. A soft set (F, A) over U is said to be a NULL soft set denoted by Φ, if $e \in$ A, F(e) = ∅.

Definition 2.7. [3] Let R be an equivalence relation on U. The pair (U, R) is called a Pawlak approximation space. The equivalence R is often called an indiscernibility relation R, one can define the following two rough approximations:

$$R_*(x)=\{x\in U:[x]_R \subseteq X\}$$
$$R^*(x)=\{x\in U:[x]_R \cap X = \Phi\}$$

$R_*(x)$ And $R^*(X)$ are called the pawlak lower approximation and thee pawlak upper approximation of X, repectively.

Definition 2.8. [11] Let $A, B \subseteq E$ and $(F, A),(G, B)$ be soft sets. Then a soft relation from (F, A) to (G,B) is a soft subset of $(F, A) X (G, B)$.

Definition 2.9. [12] A soft relation R on a soft set (F,A) is called

i. Soft reflexive if $F(a)\ X\ F(a) \in R, \forall a \in A$
ii. Soft symmetric if $F(a)\ X\ F(b) \in R \Rightarrow F(b)\ X\ F(a), \forall (a,b) \in A\ X\ A$
iii. Soft transitive if
$F(a)\ X\ F(b) \in R, F(b)\ X\ F(c) \Rightarrow F(a)\ X\ F(c), \forall a,b,c \in A$

Definition 3.5. [11] Let (F,A) be soft. Then
$[F(a)] = \{F(a') : F(a)\ X\ F(a') \in R, \forall a, a' \in A\}$

3. Multigranular rough soft sets

Definition 3.1. Let (F, A, R) be a soft approximation space. Let $P, Q \in R$. Then soft lower approximation and soft upper approximation of $(G, B) \subseteq (F, A)$, are defined as

$$\underline{apr}_{P+Q}(G,B) = \bigcup_{a\in A}\{F(a) \in (F,A) : [F(a)]_P \subseteq (G,B) or [F(a)]_Q \subseteq (G,B)\}$$

$$\overline{apr}^{P+Q}(G,B) = (\underline{apr}_{P+Q}(G,B)^C)^C$$

respectively. The multigranular boundary region is defined as
$BN_{P+Q}(G,B) = \overline{apr}^{P+Q}(G,B) \setminus \underline{apr}_{P+Q}(G,B)$. If $\underline{apr}_{P+Q}(G,B) \neq \overline{apr}^{P+Q}(G,B)$ then (G, B) is called multigranular rough soft set.

Example 3.2. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$,
$E = \{e_1, e_2, e_3, e_4, e_5\}, A = \{e_1, e_2, e_3, e_4, e_5\}$ and $B = \{e_1, e_3, e_5\}$. (F, A) and (G, B) defined as follows
$(F, A) = \{(e_1, \{u_1, u_2, u_3\}), (e_2, \{u_2, u_4, u_5, u_6\}), (e_3, \{u_2, u_3, u_5, u_6\}), (e_4, \{u_2, u_5\}),$
$\{e_5, \{u_1, u_3, u_4, u_6\})\}$
and $(G, B) = \{(e_1, \{u_1, u_2, u_3\}), (e_3, \{u_2, u_5\}), (e_5, \{u_1, u_3\})\}$. Also, consider a soft relation $R = \{[F(e)] : forall\ e \in A\}$.
Let $P = \{F(e_1)\ X\ F(e_2), F(e_3)\ X\ F(e_4), F(e_4)\ X\ F(e_5), F(e_3)\ X\ F(e_5)\}$ and
$Q = \{F(e_1)\ X\ F(e_4), F(e_3)\ X\ F(e_5), F(e_4)\ X\ F(e_4), F(e_2)\ X\ F(e_5), F(e_5)\ X\ F(e_5)\}$
Then
$\underline{apr}_{P+Q}(G,B) = \{F(e_1)\}$, $\overline{apr}_{P+Q}(G,B) = (F, A)$.

Proposition 3.3. For a soft approximation space (F, A, R), for all subset $(G, B), (H, C)$ of (F, A) and $P, Q \in R$

i. $\underline{apr}_{P+Q}(F_\phi) = F_\phi = \overline{apr}^{P+Q}(F_\phi)$
ii. $\underline{apr}_{P+Q}(F_A) = F_A = \overline{apr}^{P+Q}(F_A)$
iii. If $(G,B) \subseteq (H,C)$ then $\underline{apr}_{P+Q}(G,B) \subseteq \underline{apr}_{P+Q}(H,C)$ and $\overline{apr}^{P+Q}(G,B) \subseteq \overline{apr}^{P+Q}(H,C)$
iv. $\underline{apr}_{P+Q}(G,B) = (\overline{apr}^{P+Q}(G,B)^C)^C$
v. $\overline{apr}^{P+Q}(G,B) = (\underline{apr}_{P+Q}(G,B)^C)^C$
vi. $\underline{apr}_{P+Q}((G,B) \cap (H,C)) = \underline{apr}_{P+Q}(G,B) \cap \underline{apr}_{P+Q}(H,C)$

vii. $\underline{apr}_{P+Q}(G,B)\cup\underline{apr}_{P+Q}(H,C)\subseteq\underline{apr}_{P+Q}((G,B)\cup(H,C))$

viii. $\overline{apr}^{P+Q}((G,B)\cup(H,C))=\overline{apr}^{P+Q}(G,B)\cup\overline{apr}^{P+Q}(H,C)$

ix. $\overline{apr}^{P+Q}((G,B)\cap(H,C))\subseteq\overline{apr}^{P+Q}(G,B)\cap\overline{apr}^{P+Q}(H,C)$

Proof: From the definition3.1 and operations on soft set.
Each fact of the above proposition verified with the following example.

Example 3.4. Let $U=\{u_1,u_2\}$ and $E=\{e_1,e_2,e_3\}, A=\{e_1,e_2\}$. (F, A) defined as $\{(e_1,\{u_1,u_2\}),(e_2,\{u_1,u_2\})$ Then the subset of (F,A) are

$$(F,A)^1=\{(e_1,\{u_1,u_2\}),(e_2,\{u_1\})\}$$
$$(F,A)^2=\{(e_1,\{u_1,u_2\}),(e_2,\{u_2\})\}$$
$$(F,A)^3=\{(e_1,\{u_1\}),(e_2,\{u_1,u_2\})\}$$
$$(F,A)^4=\{(e_1,\{u_2\}),(e_2,\{u_1,u_2\})$$
$$(F,A)^5=\{(e_1,\{u_1\}),(e_2,\{u_1\})\}$$

$$(F,A)^6=\{(e_1,\{u_2\}),(e_2,\{u_2\})\}$$
$$(F,A)^7=\{(e_1,\{u_1\}),(e_2,\{u_2\})\}$$
$$(F,A)^8=\{(e_1,\{u_2\}),(e_2,\{u_1\})\}$$
$$(F,A)^9=\{(e_1,\{u_1,u_2\})\}$$
$$(F,A)^{10}=\{(e_2,\{u_1,u_2\})\}$$
$$(F,A)^{11}=\{(e_1,\{u_1\})\}$$
$$(F,A)^{12}=\{(e_1,\{u_2\})\}$$
$$(F,A)^{13}=\{(e_2,\{u_1\})\}$$
$$(F,A)^{14}=\{(e_2,\{u_2\})\}$$
$$(F,A)^{15}=F_\phi$$
$$(F,A)^{16}=(F,A)$$

The equivalence soft relation R,P and Q are

$R=\{F(e_1)X\ F(e_1),F(e_1)\ X\ F(e_2),F(e_2)\ X\ F(e_1),F(e_2)\ X\ F(x_2)\},$

$P=\{\ F(e_1)\ X\ F(e_1),F(e_1)\ X\ F(e_2),F(e_2)\ X\ F(e_1)\}\ and$

$Q=\{\ F(e_2)\ X\ F(e_2),F(e_2)\ X\ F(e_1),F(e_1)\ X\ F(e_2)\}$

In the above example,

$\underline{apr}_{P+Q}((F,A)^i, i=1,2,3,4,9)=\{F(e_2)\}$

$\underline{apr}_{P+Q}((F,A)^i, i=5,6,7,8,11,12,13,14,15)=F_\phi$

$\overline{apr}^{P+Q}((F,A)^i, i = 1,2,3,4,9) = (F,A)$

$\overline{apr}^{P+Q}((F,A)^i, i = 5,6,7,8,11,12,13,14,15) = F_\phi$

Proposition 3.5. Let (F, A, R) be a soft approximation space and R be soft equivalence relation. Then for all the subsets (G,B) of (F,A)

i. $(G,B) \subseteq \overline{apr}^{P+Q}(G,B)$

ii. $\underline{apr}^{P+Q}(G,B) \subseteq (G,B)$

iii. $\overline{apr}^{P+Q}(\underline{apr}_{P+Q}(G,B)) \subseteq (G,B)$

iv. $(G,B) \subseteq \underline{apr}_{P+Q}(\overline{apr}^{P+Q}(G,B))$

v. $\overline{apr}^{P+Q}(\overline{apr}^{P+Q}(G,B)) \subseteq \overline{apr}^{P+Q}(G,B)$

vi. $\underline{apr}_{P+Q}(G,B) \subseteq \underline{apr}_{P+Q}(\underline{apr}_{P+Q}(G,B))$

4. Conclusion

In this paper, we defined multigranular rough soft set and its properties alone. In future we will find the reduction of the attributes using multigranular rough soft set and define the topological structure of multigranular rough soft set.

REFERENCES

1. R.Raghavan et al., on some topological properties of multigranular rough sets, pelagia research library, *Advances in Applied Science Research*, 2(3) (2011) 536-543.
2. Y.Qian, J.Liang, Y.Yao and C.Dang, MGRS: A Multi-granulation rough set, *Inform. Sci.*, 180 (2010) 949–970.
3. Z.Pawlak, Rough sets, *International Journal of Computer & Information Sciences*, 11(5) (1982) 341-356.
4. T.A.Sunitha, Some problems in topology a study of $\breve{C}ech$ closure spaces, Thesis, School of Mathematical Sciences, Cochin University of Science and Technology, Cochin (1994).
5. D.Molodtsov, Rough soft set theory-first results, *Comput. Math. Appl.*, 37 (1999) 19-31
6. P.K.Maji, R.Biswas and A.R.Roy, Soft set theory, *Comput. Math. Appl.*, 45 (2003) 555–562.
7. M.I.Ali, A note on rough soft sets, rough rough soft sets, and fuzzy rough soft sets, *Applied Rough Soft Computing*, 11 (2011) 3329-3332.
8. F.Feng, C.Li, B.Davvaz, and M.I.Ali, Rough soft sets combined with fuzzy sets and rough sets: a tentative approach, *Rough Soft Computing*, 14(9) (2010) 899–911.
9. M.A.Ali, F.Feng, X.Liu and W.K.Min, On some new operations in rough soft set theory, *Comput. Math. Appl.*, 57(9) (2009) 1547-1553.
10. Vinay Gautam, et.al, On the topological structure of rough soft sets, Conference paper, October 2014. http://www.researchgate.net/publication/267908594.

11. S.Roy and S.Bera, Approximation of Rough soft set and its application of lattice, *Fuzzy Information and Engineering,* 7(3) (2015) 379-387.
12. K.V.Babitha and J.J.Sunil, Soft set relations and functions, *Computers and Mathematics with Applications,* 60 (2010) 1840-1849.
13. H.L.Yang and Z.L.Guo, Kernels and closures of soft set relations and soft set relation mapping, *Computers and Mathematics with Applications,* 61 (2011) 651-662.