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Bounded Sum and Bounded Product of Fuzzy Matrices

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***Abstract*.** Fuzzy matrices have been proposed to represent fuzzy relations in finite universes. Different algebraic operations are involved in the study of fuzzy matrices. In this paper, we defined bounded difference of fuzzy matrices and by using bounded sum and bounded product. we proved some new inequalities connected with fuzzy matrices. Also, some results on existing operators along with these are presented.

Keywords: Fuzzy matrix, bounded sum, bounded product, bounded difference

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1. Introduction

The fuzzy matrices are successfully used when fuzzy uncertainty occurs in a problem. In 1977, Thomason [27] initiated the study on convergence of powers of fuzzy matrix. Kim and Roush [4] gave a systematic development to fuzzy matrix theory. Among the basic operations which can be performed on fuzzy matrix are $\vee, \wedge$, complement, algebraic sum, algebraic product, bounded sum, bounded product, max-min composition and so on. Ragab and Emam [17] studied some properties of the min-max composition of fuzzy matrices. Shyamal and Pal [19,20] introduced two binary fuzzy operators $\oplus$ and $\odot$ for fuzzy matrices and proved several properties on $\oplus$ and $\odot$. Also they extended these binary operators for Intuitionistic fuzzy matrix. Sriram and Boobalan [25] studied the algebraic properties of these binary operators and proved some results on existing operators along with these operators. Bounded sum and bounded product of fuzzy matrices are introduced by Zhang and Zheng [28] and presented several properties on these operations.

In this paper, we defined bounded difference of fuzzy matrices and by using bounded sum, bounded product. we proved some new inequalities connected with fuzzy matrices. Also, some results on existing operators along with these are presented.

The paper is organized in three sections. In section 2, the definitions and operations on fuzzy matrices are given. In section 3, results regarding of fuzzy matrices are proved using the operations in the preceding section. In section 4, some results on bounded difference of fuzzy matrices.

2. Preliminaries

In this section, we define some operators on fuzzy matrices whose elements are in the closed interval $F=[0,1]$. For all $x,y\in F$ the following operators are defined.

$(i)\ x\vee y=\max(x,y),$

$(ii)\ x\wedge y=\min(x,y),$

$(iii)\ x^c=1-x,$

$(iv)\ x\oplus y=(x+y)\wedge 1,$

$(v)\ x\odot y=(x+y-1)\vee 0.$

The matrix I_n is the $n\times n$ Identity matrix and the matrix J is the matrix whose elements are all **1**.

Next, we define some operations on fuzzy matrices. Let $A=(a_{ij})$ and $B=(b_{ij})$ be two fuzzy matrices of order $m\times n$.

Then $(i) A\vee B=(a_{ij}\vee b_{ij}),$

$(ii) A\wedge B=(a_{ij}\wedge b_{ij}).$

The Bounded sum of A and B is defined by $A\oplus B=((a_{ij}+b_{ij})\wedge 1).$

The Bounded product of A and B is defined by $A\odot B=((a_{ij}+b_{ij}-1)\vee 0).$

The complement of the fuzzy matrix A is given by $A^c=[1-a_{ij}]$.

$A\le B$ if and only if $a_{ij}\le b_{ij}$ for all i,j.

Theorem 2.1. [12] (i) If a,b,c are real numbers with $a\ge 0$ then the following holds:

$a.\max(b,c) \quad = \quad \max(ab,ac),$

$a.\min(b,c) \quad = \quad \min(ab,ac).$

(ii) For real numbers a,b,c with $a\ge 0$ then addition distributes over the maximum operation and also over the minimum operation:

$a+\max(b,c) \quad = \quad \max(a+b,a+c),$

$a+\min(b,c) \quad = \quad \min(a+b,a+c).$

(iii) For real numbers, the maximum operation is distributive over the minimum operation and vice versa:

$\max(a,\min(b,c)) \quad = \quad \min(\max(a,b),\max(a,c)),$

$\min(a,\max(b,c)) \quad = \quad \max(\min(a,b),\min(a,c)).$

Lemma 2.2. [12] Let x,y and z be real numbers. Then the following equalities hold:

$(i) x - \min(y, z) = \max(x - y, x - z),$

$(ii) x - \max(y, z) = \min(x - y, x - z),$

$(iii) \min(x, y) - z = \min(x - z, y - z),$

$(iv) \max(x, y) - z = \max(x - z, y - z).$

3. Some results on operators of fuzzy matrix

In [28], Zhang and Zheng introduced $\oplus$ and $\odot$ of fuzzy matrices. In this section, we proved some new inequalities connected with fuzzy matrices and it's algebraic properties.

Theorem 3.1. For any fuzzy matrices A and B of same size,

$(i) A \oplus (A \odot B) \geq A,$

$(ii) A \odot (A \oplus B) \leq A.$

Proof:

(i) $A \oplus (A \odot B) = (a_{ij}) \oplus (\max(0, a_{ij} + b_{ij} - 1)),$

$= (\min(1, a_{ij} + \max(0, a_{ij} + b_{ij} - 1))),$

$= (\min(1, \max(a_{ij}, 2a_{ij} + b_{ij} - 1)))$(by Theorem2.1)

Case 1. Suppose $(\min(1, \max(a_{ij}, 2a_{ij} + b_{ij} - 1) = 1$

Then $A \oplus (A \odot B) = J \geq A$ for all i, j.

Therefore $A \oplus (A \odot B) \geq A$.

Case 2. Suppose $(\min(1, \max(a_{ij}, 2a_{ij} + b_{ij} - 1) = \max(a_{ij}, 2a_{ij} + b_{ij} - 1)$

Subcase 2.1. Suppose $\max(a_{ij}, 2a_{ij} + b_{ij} - 1) = (a_{ij}) \geq (a_{ij})$ for all i, j

Therefore $A \oplus (A \odot B) \geq A$

Subcase 2.2. Suppose $\max(a_{ij}, 2a_{ij} + b_{ij} - 1) = (2a_{ij} + b_{ij} - 1) \geq (a_{ij})$

Therefore $A \oplus (A \odot B) \geq A$.

(ii) $A \odot (A \oplus B) = (a_{ij}) \odot (\min(1, a_{ij} + b_{ij})),$

$= (\max(0, a_{ij} + \min(1, a_{ij} + b_{ij}) - 1))$

$= (\max(0, \min(a_{ij}, 2a_{ij} + b_{ij} - 1)))$(byTheorem2.1)

Case 1. Suppose $(\max(0, \min(a_{ij}, 2a_{ij} + b_{ij} - 1) = 0$

Then $A \odot (A \oplus B) = O \leq A$ for all i, j.

Case 2. Suppose $(\max(0, \min(a_{ij}, 2a_{ij} + b_{ij} - 1) = \min(a_{ij}, 2a_{ij} + b_{ij} - 1)$

Subcase 2.1. Suppose $\min(a_{ij}, 2a_{ij} + b_{ij} - 1) = (a_{ij}) \leq (a_{ij})$ for all i, j.

Therefore $A \odot (A \oplus B) \leq A$.

Subcase 2.2. Suppose $\min(a_{ij}, 2a_{ij} + b_{ij} - 1) = (2a_{ij} + b_{ij} - 1) \leq (a_{ij})$

Therefore, $A \odot (A \oplus B) \leq A$.

Theorem 3.2. For any fuzzy matrices A and B of same size,
$(i) A \oplus (A \vee B) \geq A,$
$(ii) A \oplus (A \wedge B) \geq A.$

Proof:
$(i)\ A \oplus (A \vee B) = (a_{ij}) \oplus (\max(a_{ij}, b_{ij})),$
$= (\min(1, a_{ij} + \max(a_{ij}, b_{ij})))$ (by Theorem2.1)
$= (\min(1, \max(2a_{ij}, a_{ij} + b_{ij}))),$

Case 1. Suppose $(\min(1, \max(2a_{ij}, a_{ij} + b_{ij}))) = 1$
Then $A \oplus (A \vee B) = J \geq A$

Case 2. Suppose $(\min(1, \max(2a_{ij}, a_{ij} + b_{ij}))) = \max(2a_{ij}, a_{ij} + b_{ij})$

Subcase 2.1. Suppose $\max(2a_{ij}, a_{ij} + b_{ij}) = (2a_{ij}) \geq (a_{ij})$ for all i, j.
Therefore $A \oplus (A \vee B) \geq A.$

Subcase 2.2. Suppose $(\max(2a_{ij}, a_{ij} + b_{ij})) = (a_{ij} + b_{ij}) \geq (a_{ij})$
Therefore $A \oplus (A \vee B) \geq A.$

$(ii)\ A \oplus (A \wedge B) = (a_{ij}) \oplus (\min(a_{ij}, b_{ij})),$
$= (\min(1, a_{ij} + \min(a_{ij}, b_{ij})))$ (by Theorem2.1)
$= (\min(1, \min(2a_{ij}, a_{ij} + b_{ij}))),$

Case 1. Suppose $(\min(1, \min(2a_{ij}, a_{ij} + b_{ij}))) = 1$
Then $A \oplus (A \vee B) = J \geq A$

Case 2. Suppose $(\min(1, \min(2a_{ij}, a_{ij} + b_{ij}))) = \min(2a_{ij}, a_{ij} + b_{ij})$

Subcase 2.1. Suppose $\min(2a_{ij}, a_{ij} + b_{ij}) = (2a_{ij}) \geq (a_{ij})$ for all i, j.
Therefore $A \oplus (A \vee B) \geq A.$

Subcase 2.2. Suppose $(\min(2a_{ij}, a_{ij} + b_{ij})) = (a_{ij} + b_{ij}) \geq (a_{ij})$
Therefore, $A \oplus (A \vee B) \geq A.$

Theorem 3.3. For any fuzzy matrices A and B of same size,
$(i) A \odot (A \vee B) \leq A,$
$(ii) A \odot (A \wedge B) \leq A.$

Proof:
$(i)\ A \odot (A \vee B) = (a_{ij}) \odot (\max(a_{ij}, b_{ij})),$
$= (\max(0, a_{ij} + \max(a_{ij}, b_{ij}) - 1))),$

$= (\max(0, a_{ij} + \max(a_{ij} - 1, b_{ij} - 1)))$ (by Lemma 2.2)

$= (\max(0, \max(2a_{ij} - 1, a_{ij} + b_{ij} - 1)))$ (by Theorem 2.1)

Case 1. Suppose $\max(0, \max(2a_{ij} - 1, a_{ij} + b_{ij} - 1)) = 0$,

Then $A \odot (A \vee B) = O \leq A$.

Case 2. Suppose $\max(0, \max(2a_{ij} - 1, a_{ij} + b_{ij} - 1)) = (\max(2a_{ij} - 1, a_{ij} + b_{ij} - 1))$

Subcase 2.1. Suppose $\max(2a_{ij} - 1, a_{ij} + b_{ij} - 1)) = (2a_{ij} - 1) \leq (a_{ij})$ for all i, j.

Therefore $A \odot (A \vee B) \leq A$.

Subcase 2.2. Suppose $\max(2a_{ij} - 1, a_{ij} + b_{ij} - 1)) = (a_{ij} + b_{ij} - 1) \leq (a_{ij})$,

Therefore $A \odot (A \vee B) \leq A$.

(ii) $A \odot (A \wedge B) = (a_{ij}) \odot (\min(a_{ij}, b_{ij}))$,

$= (\max(0, a_{ij} + \min(a_{ij}, b_{ij}) - 1)))$,

$= (\max(0, \min(2a_{ij} - 1, a_{ij} + b_{ij} - 1)))$ (by Theorem2.1)

Case 1. Suppose $\max(0, \min(2a_{ij} - 1, a_{ij} + b_{ij} - 1)) = 0$,

Then $A \odot (A \wedge B) = O \leq A$.

Case 2. Suppose $\max(0, \min(2a_{ij} - 1, a_{ij} + b_{ij} - 1)) = (\max(2a_{ij} - 1, a_{ij} + b_{ij} - 1))$

Subcase 2.1. Suppose $\max(2a_{ij} - 1, a_{ij} + b_{ij} - 1)) = (2a_{ij} - 1) \leq (a_{ij})$ for all i, j.

Therefore $A \odot (A \wedge B) \leq A$.

Subcase 2.2. Suppose $\min(2a_{ij} - 1, a_{ij} + b_{ij} - 1)) = (a_{ij} + b_{ij} - 1) \leq (a_{ij})$,

Therefore $A \odot (A \wedge B) \leq A$.

Theorem 3.4. For any fuzzy matrices A and B of same size,

$(i) A \vee (A \oplus B) = A \oplus B$,

$(ii) A \wedge (A \oplus B) = A$.

Proof:

(i) $A \vee (A \oplus B) = (\max(a_{ij}, \min(1, a_{ij} + b_{ij})))$,

$= (\min(\max(a_{ij}, 1), \max(a_{ij}, a_{ij} + b_{ij})))$ (by Theorem 2.1)

$= (\min(1, a_{ij} + b_{ij}))$,

$= A \oplus B$.

(ii) $A \wedge (A \oplus B) = (\min(a_{ij}, \min(1, a_{ij} + b_{ij})))$,

$= (\min(\min(a_{ij}, 1), (a_{ij} + b_{ij})))$ (by Theorem 2.1)

$= (\min(a_{ij}, a_{ij} + b_{ij}))$,

$= A$.

Theorem 3.5. For any fuzzy matrices A and B of same size,
$(i) A \vee (A \odot B) = A,$
$(ii) A \wedge (A \odot B) = A \odot B.$

Proof:

$$
\begin{aligned}
(i)\quad A \vee (A \odot B) &= (\max(a_{ij}, \max(0, a_{ij} + b_{ij} - 1))),\\
&= (\max(\max(a_{ij}, 0), a_{ij} + b_{ij} - 1))),\\
&= (\max(a_{ij}, a_{ij} + b_{ij} - 1)),\\
&= (a_{ij}),\\
&= A.
\end{aligned}
$$

$$
\begin{aligned}
(ii)\quad A \wedge (A \odot B) &= (\min(a_{ij}, \max(0, a_{ij} + b_{ij} - 1))),\\
&= (\max(\min(a_{ij}, 0), \min(a_{ij}, a_{ij} + b_{ij} - 1))),\\
&= (\max(0, a_{ij} + b_{ij} - 1)),\\
&= A \odot B.
\end{aligned}
$$

The $\vee$ and $\wedge$ operators are not distributive laws over the $\oplus$ are proved by the following theorems.

Theorem 3.6. For any fuzzy matrices A, B and C of same size,
$A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C),$

Proof: $A \vee (B \oplus C) = (\max(a_{ij}, \min(1, b_{ij} + c_{ij})))$,

$(A \vee B) \oplus (A \vee C) = (\min(1, \max(a_{ij}, b_{ij}) + \max(a_{ij}, c_{ij}))),$

$$
(A \vee (B \oplus C) = \begin{cases} (a_{ij}), & \text{if } (\min(1, b_{ij} + c_{ij})), \\ (\min(1, (b_{ij} + c_{ij}))), & \text{if } (a_{ij}) \leq (\min(1, b_{ij} + c_{ij})), \end{cases}
$$

$$
(A \vee B) \oplus (A \vee C) = \begin{cases} J, & \text{if } 1 < (\max(a_{ij}, b_{ij}) + \max(a_{ij}, c_{ij})), \\ (\max(a_{ij}, b_{ij}) + \max(a_{ij}, c_{ij})), & \text{if } 1 \geq (\max(a_{ij}, b_{ij}) + \max(a_{ij}, c_{ij})), \end{cases}
$$

Case (a) $A \vee (B \oplus C) = (a_{ij}),$

Suppose $(A \vee B) \oplus (A \vee C) = J > A \vee (B \oplus C),$

Then $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C),$

Suppose $(A \vee B) \oplus (A \vee C) = (\min(1, \max(a_{ij}, b_{ij}) + \max(a_{ij}, c_{ij}))),$

Case 1. $a_{ij} \leq b_{ij}$ and $a_{ij} \leq c_{ij}$ for all $i, j,$

Therefore $(A \vee B) \oplus (A \vee C) = (b_{ij} + c_{ij}) > (a_{ij}),$

(ie) $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C).$

Case 2. $a_{ij} > b_{ij}$ and $a_{ij} < c_{ij}$ for all i, j ,
Therefore $(A \vee B) \oplus (A \vee C) = (a_{ij} + c_{ij}) > (a_{ij})$,
(ie) $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C)$.
Case 3. $a_{ij} \leq b_{ij}$ and $a_{ij} > c_{ij}$ for all i, j,
Therefore $(A \vee B) \oplus (A \vee C) = (b_{ij} + a_{ij}) > (a_{ij})$,
(ie) $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C)$.
Case 4. $a_{ij} > b_{ij}$ and $a_{ij} > c_{ij}$,
Therefore $(A \vee B) \oplus (A \vee C) = (a_{ij} + b_{ij}) > (a_{ij})$,
(ie) $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C)$,
Hence $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C)$.

Case (b) $A \vee (B \oplus C) = (\min(1, b_{ij} + c_{ij}))$,
Suppose $(\min(1, b_{ij} + c_{ij})) = 1$,
If $(A \vee B) \oplus (A \vee C) = J \geq (\min(1, b_{ij} + c_{ij}))$ for all i, j ,
(ie) $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C)$,
If $(A \vee B) \oplus (A \vee C) = (\max(a_{ij}, b_{ij}) + \max(a_{ij}, c_{ij}))$.
Case 1. $a_{ij} \leq c_{ij}$ and $a_{ij} \leq b_{ij}$ for all i, j,
$(A \vee B) \oplus (A \vee C) = (b_{ij} + c_{ij}) \geq (\min(1, b_{ij} + c_{ij}))$ for all i, j,
(ie) $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C)$.
Case 2. $a_{ij} \leq c_{ij}$ and $a_{ij} \geq b_{ij}$ for all i, j,
$(A \vee B) \oplus (A \vee C) = (a_{ij} + c_{ij}) \geq (b_{ij} + c_{ij}) \geq (\min(1, b_{ij} + c_{ij}))$,
Hence $A \vee (B \oplus C) \leq (A \vee B) \oplus (A \vee C)$.
Similarlly, the following theorem 3.7, we can pove it.

Theorem 3.7. For any fuzzy matrices A, B and C of same size,
$A \wedge (B \oplus C) \leq (A \wedge B) \oplus (A \wedge C)$

In the following theorem we proved some identities on fuzzy matrices.

Theorem 3.8. For a fuzzy matrix A
Proof: Let $A = (a_{ij})$ be a fuzzy matrix.

$(i) A \oplus O = A.$

$(ii) A \oplus J = J.$

$(iii) A \odot O = O.$

$(iv) A \odot J = A.$

4. Results on bounded difference of fuzzy matrix

In the section, we defined bounded difference of fuzzy matrices and proved it's algebraic properties.

Definition 4.1. Let $A = (a_{ij})$ and $B = (b_{ij})$ be two fuzzy matrices of order $m \times n$, then the bounded-difference of A and B is defined by $A \ominus B = ((a_{ij} - b_{ij}) \vee 0)$.

Theorem 4.2. For any fuzzy matrices A and B of same size,

$(i) A \oplus (B \ominus A) = A \vee B,$

$(ii) A \oplus (A \ominus B) \geq A,$

$(iii) A \ominus (A \ominus B) = A \wedge B.$

Proof:

$(i)\ A \oplus (B \ominus A) = (a_{ij}) \oplus (\max(0, b_{ij} - a_{ij})),$

$= (\min(1, a_{ij} + \max(0, b_{ij} - a_{ij}))),$

$= (\min(1, (\max(a_{ij}, a_{ij} + b_{ij} - a_{ij}))))$(byTheorem2.1)

$= (\min(1, \max(a_{ij}, b_{ij}))),$

$= (\max(\min(1, a_{ij}), \min(1, b_{ij})))$(byTheorem2.1)

$= (\max(a_{ij}, b_{ij})),$

$= A \vee B.$

$(ii)\ A \oplus (A \ominus B) = (a_{ij}) \oplus (\max(0, a_{ij} - b_{ij})),$

$= (\min(1, a_{ij} + (\max(0, a_{ij} - b_{ij})))),$

$= (\min(1, \max(a_{ij} + 0, a_{ij} + a_{ij} - b_{ij})))$(byTheorem2.1)

$= (\min(1, \max(a_{ij}, 2a_{ij} - b_{ij}))),$

Case 1. Suppose $(\min(1, \max(a_{ij}, 2a_{ij} - b_{ij}))) = 1$ for all $i, j,$

Then $A \oplus (A \ominus B) = J \geq A.$

Case 2. Suppose $(\min(1, \max(a_{ij}, 2a_{ij} - b_{ij}))) = \max(a_{ij}, 2a_{ij} - b_{ij}) = (a_{ij})$ for all $i, j,$

Subcase 2.1. Suppose $\max(a_{ij}, 2a_{ij} - b_{ij}) = (a_{ij}) \geq (a_{ij})$ for all $i, j,$

Therefore $A \oplus (A \ominus B) \geq A.$

Subcase 2.2. Suppose $\max(a_{ij}, 2a_{ij} - b_{ij}) = 2a_{ij} - b_{ij} \geq (a_{ij})$ for all i, j,
Therefore $A \oplus (A \ominus B) \geq A$.

(iii) $A \ominus (A \ominus B) = (a_{ij}) \ominus (\max(0, a_{ij} - b_{ij}))$,

$$= (\max(0, a_{ij} - \max(0, a_{ij} - b_{ij}))),$$
$$= (\max(0, (\min(a_{ij}, a_{ij} - (a_{ij} - b_{ij}))))(\text{byTheorem2.1})$$
$$= (\max(0, \min(a_{ij}, b_{ij}))),$$
$$= (\min(\max(0, a_{ij}), \max(0, b_{ij})))(\text{byTheorem2.1})$$
$$= (\min(a_{ij}, b_{ij})),$$
$$= A \wedge B.$$

Theorem 4.3. For any fuzzy matrices A and B of same size,
$(i) A \vee (A \ominus B) = A,$
$(ii) A \wedge (A \ominus B) = A \ominus B$
Proof:
(i) $A \vee (A \ominus B) = (a_{ij}) \vee (\max(0, a_{ij} - b_{ij}))$,

$$= (\max(a_{ij}, \max(0, a_{ij} - b_{ij}))),$$
$$= (\max(\max(a_{ij}, 0), a_{ij} - b_{ij})),$$
$$= (\max(a_{ij}, a_{ij} - b_{ij})),$$
$$= (a_{ij}),$$
$$= A.$$

(ii) $A \wedge (A \ominus B) = (a_{ij}) \wedge (\min(0, a_{ij} - b_{ij}))$,

$$= (\min(a_{ij}, \max(0, a_{ij} - b_{ij}))),$$
$$= (\max(\min(a_{ij}, 0), \min(a_{ij}, a_{ij} - b_{ij}))),$$
$$= (\max(0, a_{ij} - b_{ij})),$$
$$= A \ominus B.$$

In [28], Zhang and Zheng proved the De Morgan's laws for the operators $\oplus$ and $\odot$. In the following theorem, we proved some of it's further results.

Theorem 4.4. For any fuzzy matrices A and B of same size,
$(i) (A \oplus B)^C = A^C \ominus B, (ii) (A \ominus B)^C = A^C \oplus B,$
$(iii) A^C \ominus B^C = B \ominus A, (iv) A \odot B^C = A \ominus B,$.
$(v) A \ominus B^C = A \odot B$

Proof: Let $A=(a_{ij})$ and $B=(b_{ij})$ be two fuzzy matrices of same order

(i) $(A\oplus B)^C = (1-\min(1,a_{ij}+b_{ij}))$,

$=(\max(0,1-a_{ij}-b_{ij}))$ (4.1) (by Lemma 2.2)

$$A^C \ominus B = (1-a_{ij}) \ominus (b_{ij}) = (\max(0,1-a_{ij}-b_{ij})) \quad (4.2)$$

From (4.1) and(4.2), (i) is true.

$$(ii)\ (A\ominus B)^C = (1-\max(0,a_{ij}-b_{ij})) = (\min(1,1-(a_{ij}-b_{ij}))) \quad (4.3)$$

$$A^C \oplus B = (1-a_{ij})\oplus(b_{ij}) = (\min(1,1-a_{ij}+b_{ij})) \quad (4.4)$$

From (4.3) and $(4.4),(ii)$ is true. The proofs of $(iii),(iv)$ and (v) are obvious.

In the following theorem we proved some identities on fuzzy matrices,

Theorem 4.5. For a fuzzy matrix A

$(i) A\ominus O = A,$

$(ii) O\ominus A = O,$

$(iii) A\ominus J = O,$

$(iv) J\ominus A = A^C.$

5. Conclusions

In this article, bounded sum, bounded product and bounded difference of fuzzy matrices are defined and some properties are proved. Thus the bounded sum, bounded product and bounded difference of fuzzy matrices are very useful to further works.

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