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Sum Divisor Cordial Labeling of Herschel Graph

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Abstract. A sum divisor cordial labeling of a graph G with vertex set $V(G)$ is a bijection f from $V(G)$ to $\{1,2,...,|V(G)|\}$ such that each edge uv assigned the label 1 if 2 divides $f(u)+f(v)$ and 0 otherwise. Further, the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that Herschel graph H_s, fusion of any two adjacent vertices of degree 3 in a Herschel graph H_s, duplication of any vertex of degree 3 in a Herschel graph H_s, switching of a central vertex in the Herschel graph H_s, path union of two copies of H_s are sum divisor cordial graphs.

Keywords: Divisor cordial labeling, sum divisor cordial labeling, fusion, duplication, switching, path union.

***AMS Mathematics Subject Classification (2010)*:** 05C78

1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory we refer to Harary [3]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Cordial labeling is extended to divisor cordial labeling, prime cordial labeling, total cordial labeling, Fibonacci cordial labeling etc.

Varatharajan et al.[10] introduced the concept of divisor cordial labeling. Vaidya and Shah [9] proved that some star and bistar related graphs are divisor cordial labeling. Rokad and Godasara [6] have discussed the Fibonacci cordial labeling of some special graphs.

For dynamic survey of various graph labeling, we refer to Gallian [1]. Lourdusamy and Patrick [5] introduced the concept of sum divisor cordial labeling. Sugumaran and Rajesh [7] proved that Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of Swastik graph Sw_n (k is odd), Jelly fish $J(n,n)$ and Petersen graph are sum divisor cordial graphs. Sugumaran and Rajesh [8] proved that the Theta graph and some graph operations in Theta graph are sum divisor cordial graphs. Ganesan and Balamurugan [2] have discussed the prime labeling of Herschel graph. Our primary objective of this paper is to prove the Herschel graph and some graph operations in Herschel graph namely fusion, duplication, switching of a central vertex, path union of two copies of Herschel graphs are sum divisor cordial graphs.

Definition 1.1. [10] Let $G=(V(G),E(G))$ be a simple graph and let $f:V(G)\to\{1,2,...,|V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u)\,|\,f(v)$ or $f(v)\,|\,f(v)$ and the label 0 otherwise. The function f is called a *divisor cordial labeling* if $|e_f(0)-e_f(1)|\le 1$. A graph which admits a divisor cordial labeling is called a *divisor cordial graph.*

Definition 1.2. [5] Let $G=(V(G),E(G))$ be a simple graph and let $f:V(G)\to\{1,2,...,|V(G)|\}$ be a bijection. For each edge $e=uv$, assign the label 1 if either $2\,|\,(f(u)+f(v))$ and assign the label 0 otherwise. The function f is called a *sum divisor cordial labeling* if $|e_f(0)-e_f(1)|\le 1$. A graph which admits a sum divisor cordial labeling is called a *sum divisor cordial graph.*

Definition 1.3. A *Herschel graph* H_s is a bipartite undirected graph with 11 vertices and 18 edges.

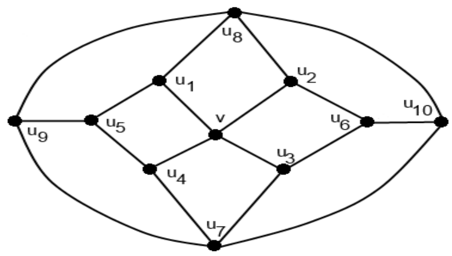

Figure 1: Herschel graph H_s

In this paper, we always fix the position of vertices $v,u_1,u_2,...,u_{10}$ of H_s as indicated in the above Figure 1, unless or otherwise specified.

Definition 1.4. Let u and v be two distinct vertices of a graph G. A new graph G_1 is constructed by *fusing* (identifying) two vertices u and v by a single vertex x in G_1

such that every edge which was incident with either u (or) v in G now incident with x in G_1.

Definition 1.5. *Duplication* of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v'_k with $N(v_k) = N(v'_k)$. In other words, a vertex v'_k is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v'_k.

Definition 1.6. A *vertex switching* G_v of a graph G is obtained by taking a vertex v of G, removing the entire edges incident with v and adding edges joining v to every vertex which are non-adjacent to v in G.

Definition 1.7. [4] Let G be a graph and let $G_1 = G_2 = \cdots = G_n = G$, where $n \geq 2$. Then the graph obtained by adding an edge from each G_i to G_{i+1} $(1 \leq i \leq n-1)$ is called the *path union* of G.

2. Main results

Theorem 2.1. The Herschel graph H_s is a sum divisor cordial graph.

Proof: Let $G = H_s$ be a Herschel graph and let v be the central vertex and $u_i (1 \leq i \leq 10)$ be the remaining vertices of the Herschel graph. Then $|V(G)| = 11$ and $|E(G)| = 18$. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v) = 11$, $f(u_2) = 1$, $f(u_8) = 4$,

$f(u_i) = i$; $i = 3, 9, 10$,

$f(u_i) = i + 1$; $i = 1, 4, 5, 6, 7$.

From the above labeling pattern, we have $e_f(0) = e_f(1) = 9$.

Hence $|e_f(0) - e_f(1)| \leq 1$.

Thus G is a sum divisor cordial graph.

Example 2.1. The sum divisor cordial labeling of Herschel graph H_s is shown in Figure 2.

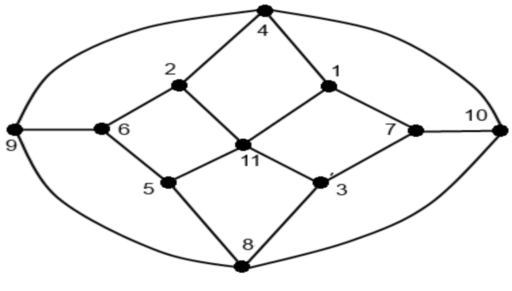

Figure 2: Sum divisor cordial labeling of Herschel graph H_s

Theorem 2.2. The fusion of any two adjacent vertices of degree 3 in the Herschel graph is a sum divisor cordial graph.

Proof: Let H_s be the Herschel graph with $|V(H_s)|$= 11 and $|E(H_s)|$= 18. Let v be the central vertex of the Herschel graph and it has 3 vertices of degree 4 and 8 vertices of degree 3. Let G be the graph obtained by fusion of two adjacent vertices of degree 3 in the Herschel graph of H_s. Then $|V(G)|$ = 10 and $|E(G)|$ = 17.

We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

Case 1. Fusion of u_6 and u_{10}.

Suppose that u_6 and u_{10} are fused together as a single vertex u.

$f(v) = 1$,

$f(u_3) = 7$,

$f(u) = 4$,

$f(u_i) = i+1$; $i = 1, 2, 4, 5, 7, 8, 9$.

Case 2. Fusion of u_6 and u_2.

Suppose that u_6 and u_2 are fused together as a single vertex u.

$f(v) = 1$, $f(u_4) = 7$, $f(u_{10}) = 4$,

$f(u) = 2$,

$f(u_i) = i+1$; $i = 5, 7, 8, 9$.

$f(u_i) = i+2$; $i = 1, 3$.

Case 3. Fusion of u_6 and u_3.

Suppose that u_6 and u_3 are fused together as a single vertex u.

$f(v) = 1$, $f(u_1) = 9$, $f(u_{10}) = 4$,

$f(u) = 2$,

$f(u_i) = i$; $i = 7, 8$.

$f(u_i) = i+1$; $i = 2, 4, 5, 9$.

Case 4. Fusion of u_5 and u_9.

Suppose that u_5 and u_9 are fused together as a single vertex u.

$f(v) = 1$, $f(u_3) = 7$,

$f(u) = 4$,

$f(u_i) = i$; $i = 6, 10$.

$f(u_i) = i+1$; $i = 1, 2, 4, 7, 8$.

Case 5. Fusion of u_5 and u_4.

Suppose that u_5 and u_4 are fused together as a single vertex u.

$f(v) = 1$, $f(u_3) = 7$, $f(u_9) = 5$,

$f(u) = 4$,

$f(u_i) = i$; $i = 6,10$.

$f(u_i) = i + 1$; $i = 1,2,7,8$.

Case 6. Fusion of u_5 and u_1.

Suppose that u_5 and u_1 are fused together as a single vertex u.

$f(v) = 1$, $f(u_3) = 7$, $f(u_9) = 4$,

$f(u) = 2$,

$f(u_i) = i$; $i = 6,10$.

$f(u_i) = i + 1$; $i = 2,4,7,8$.

From all the above cases, we have $|e_f(0) - e_f(1)| \le 1$.

Thus G is a sum divisor cordial graph.

Example 2.2. The sum divisor cordial labeling of fusion of u_6 and u_{10} in H_s is shown in Figure 3.

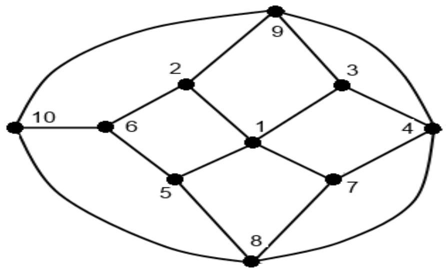

Figure 3: Sum divisor cordial labeling of fusion of u_6 and u_{10} in H_s.

Theorem 2.3. The duplication of any vertex of degree 3 in a Herschel graph is a sum divisor cordial graph.

Proof: Let H_s be the Herschel graph with $|V(H_s)|$= 11 and $|E(H_s)|$= 18. Let v be the central vertex and u_k' be the duplication of the vertex u_k in the Herschel graph H_s. Let G be the graph obtained by duplicating the vertex u_k of degree 3 in H_s. Then $|V(G)|$ = 12 and $|E(G)|$ = 21.

Case 1. Duplication of vertex u_k, where $k = 1,2,3,4,5,6,10$.

We define the vertex labeling $f : V(G) \to \{1,2,...,|V(G)|\}$ as follows.

$f(v) = 1$, $f(u_3) = 11$, $f(u_8) = 4$, $f(u_k') = 12$,

$f(u_i) = i$; $i = 9,10$.

$f(u_i) = i + 1$; $i = 1,2,4,5,6,7$.

Case 2. Duplication of vertex u_9.

$f(v) = 9$, $f(u_4) = 7$, $f(u_8) = 4$, $f(u_k') = 12$,

$f(u_i) = i$; $i = 1, 2, 3, 5, 6$.

$f(u_i) = i + 1$; $i = 7, 9, 10$.

From the above two cases, we have $|e_f(0) - e_f(1)| \leq 1$.

Thus G is a sum divisor cordial graph.

Example 2.3. The sum divisor cordial labeling of the duplication of the vertex u_{10} in H_s is shown in Figure 4.

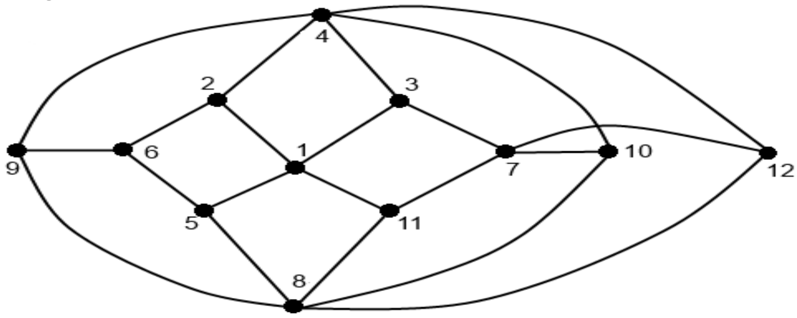

Figure 4: Sum divisor cordial labeling of the duplication of the vertex u_{10}

Theorem 2.4. The switching of a central vertex v in the Herschel graph H_s is a sum divisor cordial graph.

Proof: Let H_s be the Herschel graph with $|V(H_s)|$= 11 and $|E(H_s)|$= 18. Let v be the central vertex and G be the new graph obtained by switching the central vertex v. Then$|V(G)|$= 11 and $|E(G)|$= 20.

We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$f(v) = 1$,

$f(u_3) = 11$,

$f(u_8) = 6$,

$f(u_i) = i$; $i = 4, 5, 9, 10$.

$f(u_i) = i + 1$; $i = 1, 2, 6, 7$.

From the above labeling pattern, we observe that

$e_f(0) = e_f(1) = 10$.

Hence $|e_f(0) - e_f(1)| \leq 1$.

Thus G is a sum divisor cordial graph.

Example 2.4. The sum divisor cordial labeling of switching of a central vertex v in H_s is shown in Figure 5.

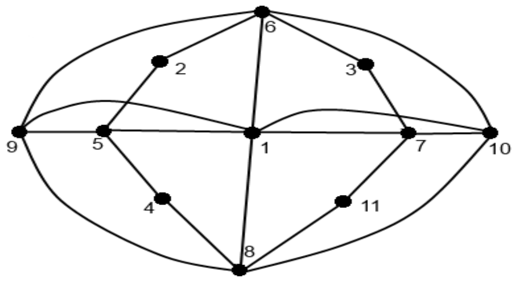

Figure 5: Sum divisor cordial labeling of switching of a central vertex v in H_s.

Theorem 2.5. The graph obtained by path union of two copies of Herschel graphs H_s is a sum divisor cordial graph.

Proof: Consider two copies of Herschel graphs H_s^1 and H_s^2 respectively. Let $V(H_s^1)=\{u,u_i:1\le i\le 10\}$ and let $V(H_s^2)=\{v,v_i:1\le i\le 10\}$. Then $|V(H_s^1)|=11$ and $|E(H_s^1)|=18$ and $|V(H_s^2)|=11$ and $|E(H_s^2)|=18$. Let G be the graph obtained by the path union of two copies of Herschel graphs H_s^1 and H_s^2. Then $V(G)=V(H_s^1)\cup V(H_s^2)$ and $E(G)=E(H_s^1)\cup E(H_s^2)\cup\{u_8v_8\}$. Note that G has 22 vertices and 37 edges.

We define the vertex labeling $f:V(G)\to\{1,2,...,|V(G)|\}$ as follows.

Labeling of H_s^1:

$f(u)=1$, $f(u_3)=9$, $f(u_5)=11$, $f(u_8)=6$, $f(u_9)=4$, $f(u_{10})=10$,

$f(u_i)=i+1$; $i=1,2,4,6,7$.

Labeling of H_s^2:

$f(v)=12$,

$f(v_3)=19$,

$f(v_5)=15$,

$f(v_7)=20$,

$f(v_8)=17$,

$f(v_9)=22$, $f(v_{10})=21$,

$f(v_i)=i+12$; $i=1,2,4,6$.

From the above labeling pattern, we observe that $e_f(0)=18$ and $e_f(1)=19$.

Hence $|e_f(0)-e_f(1)|\le 1$.

Thus G is a sum divisor cordial graph.

Example 2.5. The sum divisor cordial graph of the path union of H_s^1 and H_s^2 is shown in Figure 6.

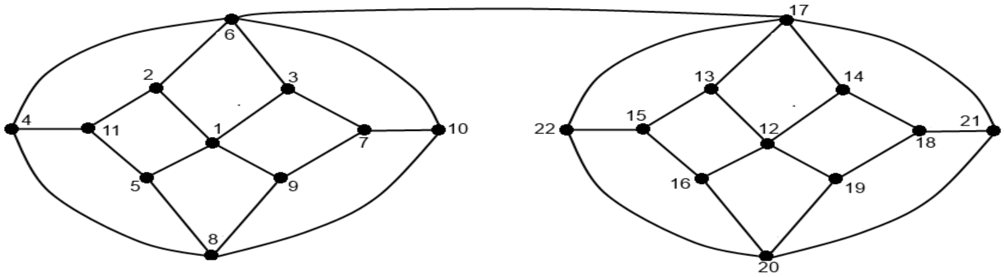

Figure 6: The sum divisor cordial graph of the path union of H_s^1 and H_s^2.

3. Conclusion

In this paper, we have investigated the sum divisor cordiality on special graph namely Herschel graph and proved that the Herschel graph H_s, fusion of any two vertices of degree 3 in H_s, duplication of any vertex of degree 3 in H_s, switching of central vertex in H_s and path union of two copies of H_s are sum divisor cordial graphs.

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