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A Comparative Study of Vertex Deleted Centrality Measures

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Abstract. The centrality of a node or identification of more central nodes than others is very important in network analysis. In this paper we present a comparative study of two vertex deleted centrality measures viz. Laplacian centrality and algebraic centrality. Here we apply these measures on two classic synthetic network *viz*, Barabási-Albert network (BA-network), Watts-Strogatz network (WS-network) and four real network data sets selected from four different categories namely, infrastructural, animal social, biological, and communication network.

Keywords: Laplacian centrality, algebraic connectivity centrality, ANI (Airport Network of India), dolphins network, amino acid network, email-eu-core network.

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1. Introduction

In Network analysis, the centrality of a node or identification of more central nodes than others plays a vital role. The findings of such important nodes with high centralities to characterize the properties on the networks have significant uses in analyzing the structure and dynamics of the network. These include the synchronization transition, epidemic spreading, identification of most sensitive nodes for vulnerability analysis and transmission of information. Various centrality measures have been proposed viz. degree centrality, closeness centrality, betweenness centrality [7], eigenvector centrality [2] and subgraph centrality [4]. Some of these measures are also extended to apply to groups and classes [5]. Several attempts were made to generalize degree centrality, betweenness centrality and closeness centrality measures to weighted networks [13].

In 2012, Qi et al. [16] proposed a novel node centrality measure based on Laplacian Energy which captures the effect of removal of the node on Laplacian Energy. Let G be an undirected graph (or undirected network) with the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$containing n vertex and the edge set E containing m edges. Then the matrix $L(G) = D(G) - A(G)$ is called the Laplacian matrix of the network G where $D(G)$ is the diagonal matrix of G and $A(G)$ is the adjacency matrix of G. The Laplacian matrix is symmetric, singular and positive definite. The eigenvalues of the Laplacian matrix are all real and non-negative, and the smallest eigenvalues of the matrix is 0. The multiplicity of the eigenvalues 0 is equal to the number of connected component of the network G. If $\mu_1, \mu_2, \dots, \mu_n$are the eigenvalues of the Laplacian matrix of a network G, then the Laplacian energy [10] of G is defined as

$$E(G) = \sum_{i=1}^{n} \mu_i{}^2$$

A survey work on Laplacian energy is found in [14].

The other vertex deleted centrality measure, considered in this work is defined based on algebraic connectivity of the network. Algebraic connectivity of a network G, is the second smallest eigenvalues of the Laplacian matrix of G. It is denoted by $\alpha(G)$. And G is connected if and only if $\alpha(G) > 0$.

In this paper, we present a comparative study of two node deleted centrality measures viz. Laplacian centrality (LC) and algebraic centrality (AC) applying on classic synthetic and real network data sets.

The rest of this paper is organized as follows. In section 2, we present few definitions relevant to this study. In section 3, a comparative studyof LC and AC centrality measuresin synthetic network environment like Barabási-Albert Network (BA-network), Watts-Strogatz network (WS-network) and in case of real networks like dolphins network, amino acid network, airport network of India (ANI) and email-eu-core network will be presented. Conclusions are made in section 4.

2. Some definitions

In this section, we present the formal definition of the two centrality measures under study and the relevant notation, which are going to be expedited in the rest of the paper.

Laplacian centrality [16]: Let $G = (V, E, W)$ is a network with n nodes $\{v_1, v_2, \dots, v_n\}$. Let us also consider that $G_i$be the network obtained by deleting v_i from G. The LC, $LC(v_i, G)$ of a node v_i is defined as follows

$$LC(v_i, G) = \frac{E(G) - E(G_i)}{E(G)} = \frac{(\Delta E)_i}{E}$$

Since $(\Delta E)_i = E(G) - E(G_i)$ is always a non-negative quantity, because of the interlacing property [8] of the eigenvalues of Laplacian matrix. Clearly, $0 \le LC(v_i, G) \le 1, \forall i$.

In [6], algebraic connectivity is defined as $\alpha(G) = min\{u^T L(G) u | u^T = 0, ||u|| = 1\}$. Now, let G be a connected graph and let $G - v$ be the subgraph obtained from G by deleting a node$v$and all edges incident on it. Then $\alpha(G)$ and $\alpha(G - V)$ are the algebraic connectivity of $G$and $G - v$ respectively.

Algebraic centrality [9]: *Steve Kirkland* in his seminal paper [9] considers a function $AC(v) = \alpha(G) - \alpha(G - v)$ which can be used as vertex deleted centrality measure. We will call it algebraic centrality (AC).

Clearly, if the graph is disconnected it is always equal to zero as a node is a disconnected graph will lie in one of the components and removing such node can only result in more number of components. On the other hand if the graph is connected but the vertex deleted graph is disconnected, then it will be equal to the algebraic connectivity of the graph. So the main advantage of LC over AC is that in case of disconnected network also calculation of LC can give us some information about the network.

3. Comparative study

3.1. Synthetic networks

In this section, we apply the node deleted centrality measures LC and AC on two synthetic networks, *viz,* WS-network [15] and BA-network [3] of different sizes, seeds and rewiring probabilities. We know that WS-network and BA-network are probabilistic networks so to minimize the effect of randomness, we consider the average result of five networks generated for each models, and also the values are normalized for the sake of graphical comparison. In WS- network, we take 300 nodes with rewiring probability 0.01. Here we observed that there are huge differences of values of nodes for these two centralities and most of the nodes have algebraic connectivity 0 as shown in figure 1. In BA-network, we take 300 nodes with size of seeds and average degree 10. Here we observed that both Laplacian and algebraic centrality follow power law degree distribution as shown in figure 2.

3.2. Real networks

In this section, we apply centrality measures on four real networks viz. Airport Network of India (ANI), dolphins network [12], amino acid network and email-eu-core network [11]. These four real networks are selected from three different categories namely, infrastructural, animal social, biological and communication network respectively.

- **Airport Network of India (ANI):** In this network airports are the nodes and two nodes are connected by an edge if there is a direct flight between them. The number of nodes in the network is 79 and number of undirected edges is 248.
- **Dolphin Network:** In this network, two nodes, the Doubtful Sound bottlenose dolphins, are connected by an edge if they were seen together more than expected by chance. The number of nodes is 62 and number of edges is 159.
- **Amino Acid Network (AAN):** In this network, nodes are amino acids and edge is defined between a pair of amino acids on the basis of a distance matrix [1]. It is an unweighted network. The number of nodes in the network is 20 and the number of edges is 69.
- **Email-Eu-core Network:** This network was generated using email data from a large European research institution. There is an edge (u, v) in the network if person u sent person v at least one email. The number of nodes in the network is 1005 and the number of edges is 25,571.

From fig 3-6, we observe a subtle change in the rankings of nodes on the basis of the two centrality measures. We also observe few advantages of LC over AC, which are as follows:

- AC is failed to distinguish among many nodes and ultimately assigned the same rank to all those nodes but that is not the case for LC, which is amain drawback of AC as per the ranking is concerned.
- Most of the values of AC are zero or equal to algebraic connectivity, which may be attributed to the fact the AC only measures the connectivity of the graph or network, which is a global characteristic but LC captures both local and global topology of the network.

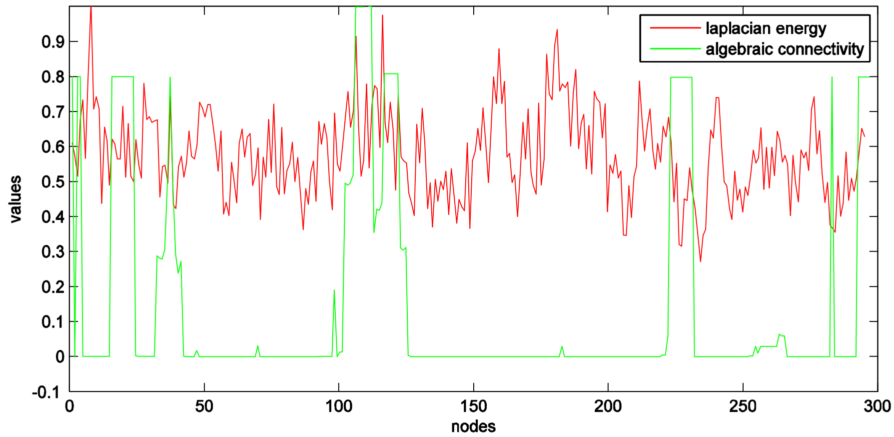

Figure 1: WS-network with n=300, p=0.01

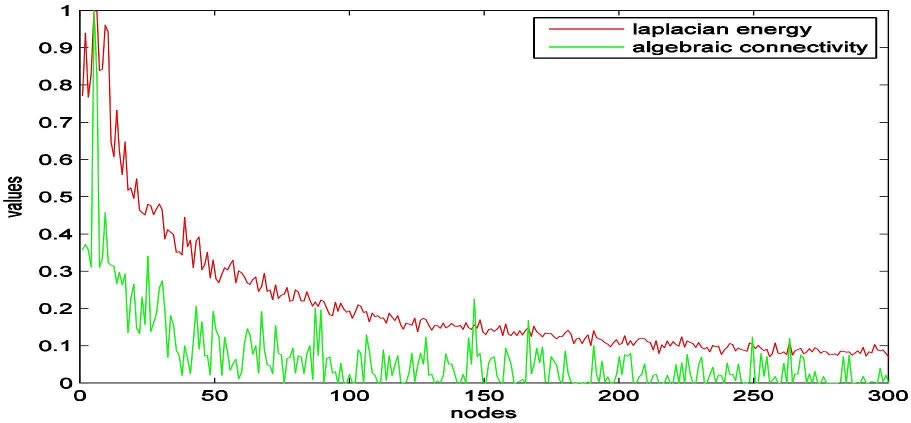

Figure 2: BA-network with n=300, m=m_0=10

Few advantages of AC over LC are

- It is based on the second eigenvalue of the Laplacian, so we need to have to calculate the other eigenvalues, which is required to calculate LC as it is based on all the Laplacian eigenvalues.
- If the size of the graph or network is small AC is a useful tool, as it is easy to calculate and being a small network the local effect is rarely significant in the study of such network.

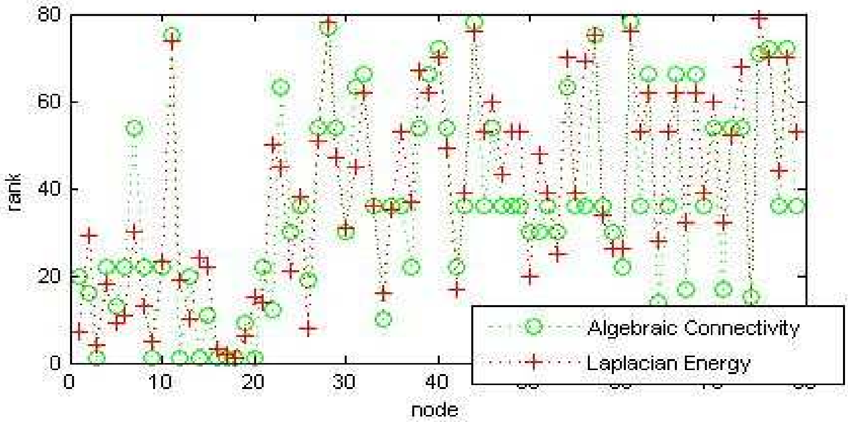

Figure 3: Airport Network of India (n=79, m=248)

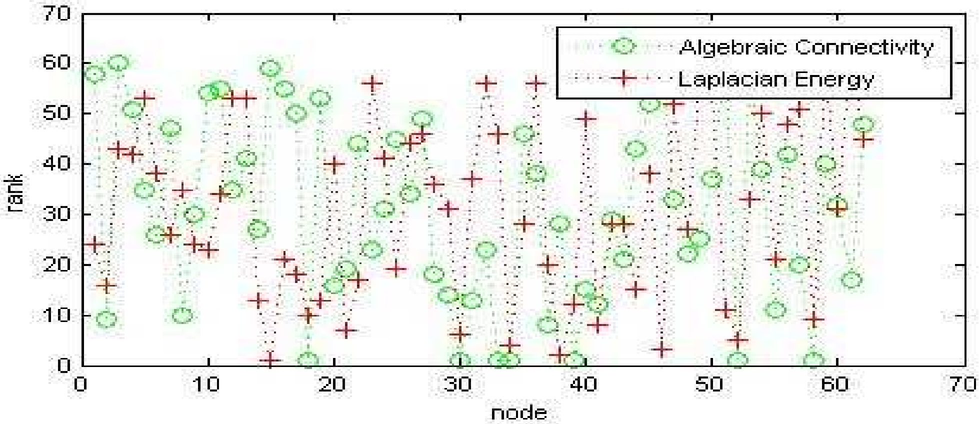

Figure 4: Dolphins Network (n=62, m=159)

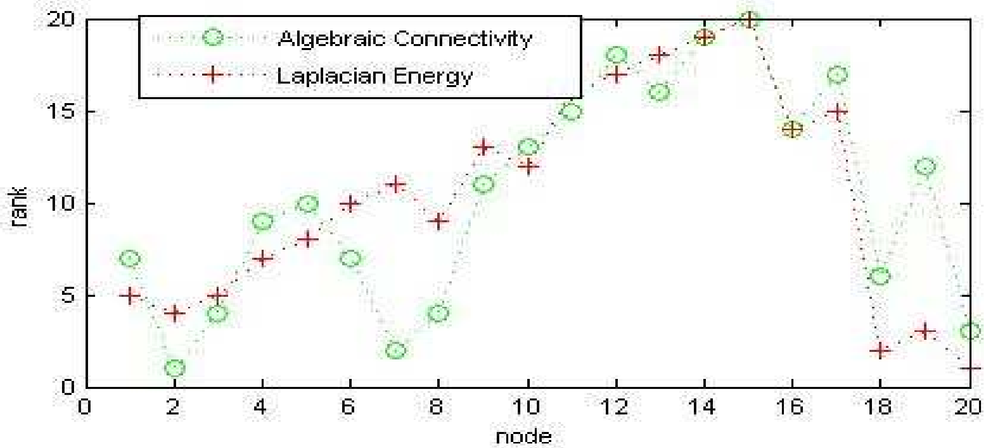

Figure 5: Amino Acid Network (n=20, m=69)

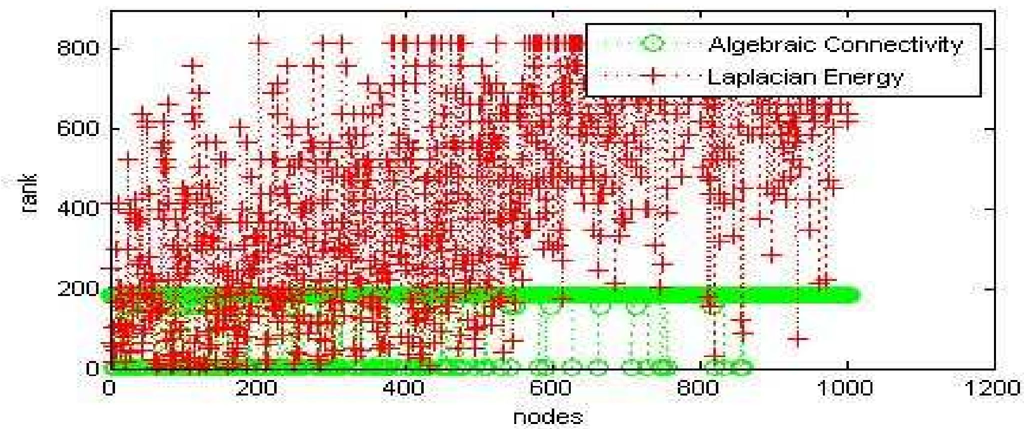

Figure 6: Email-Eu-core Network (n=1005, m=25,571)

4. Conclusion

In this paper, we present a comparative study of two popular vertex deleted centrality measures namely, Laplacian centrality and algebraic centrality. There we can observe that AC is easier to compute than LC, whereas LC can capture more information about the role of the nodes. For large network LC should be considered above AC because it is capable of distinguishing one node from the other as gives a better picture of ranking of nodes. If the analysis is based on the connectivity of a network then one may prefer AC over LC, because it is specially defined to capture connectivity of a network.

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