Annals of Pure and Applied Mathematics
Vol. 13, No. 2, 2017, 249-255
ISSN: 2279-087X (P), 2279-0888(online)
Published on 9 May 2017
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v13n2a12

Annals of
Pure and Applied Mathematics

Some Remarks on Intuitionistic L- T_2 Spaces

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Received 11 April 2017; accepted 23 April 2017

Abstract. We defined and studied four notions of T_2 space in intuitionistic L-topological spaces in this paper. We showed some implications among them. Also, we studied some other properties of these concepts.

Keywords: Intuitionistic L-fuzzy sets, Intuitionistic L-fuzzy point, Intuitionistic L-topology, Intuitionistic L-fuzzy open sets, Intuitionistic topology.

AMS Mathematics Subject Classification (2010): 54A40, 03F55

1. Introduction

The notion of fuzzy sets was initially introduced by Zadeh [20] in 1965. After then in 1984, intuitionistic fuzzy sets were first published by Attanassov [2] and many works by the same author and his colleagues appeared in the literature [3, 4, 5]. Later, this concept was generalized to 'intuitionistic L-fuzzy set' by Atanassov and Stoeva [6]. Here, we introduced 'intuitionistic L-topology' by using 'intuitionistic L-fuzzy set' in Chang sense [7]. Moreover, we defined possible four notions, investigated some properties and features of T_2 space in intuitionistic L-topological spaces.

2. Basic definitions and preliminaries

We recall some definitions and known results in intuitionistic L-fuzzy sets and intuitionistic L-topological spaces.

Definition 2.1. [20] Let X be a non-empty set and $I = [0, 1]$. A fuzzy set in X is a function $u: X \to I$ which assigns to each element $x \in X$, a degree of membership $u(x) \in I$.

Definition 2.2. [16] Let $f: X \to Y$ be a function and u be fuzzy set in X. Then the image $f(u)$ is a fuzzy set in Y which membership function is defined by

$(f(u))(y) = \{sup\, (u\,(x)) \big| f(x) = y\}$ if $f^{-1}(y) \neq \emptyset, x \in X$

$(f(u))(y) = 0$ if $f^{-1}\,(y) = \emptyset, x \in X$.

Definition 2.3. [14] Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. An L-fuzzy set in X is a function $\alpha: X \to L$ which assigns to each element $x \in X$, a degree of membership, $\alpha(x) \in L$.

Remark 2.4. Throughout this paper we consider the complete distributive lattice $L = \{0, 0.1, 0.2, \dots, 1\}$ and from above definitions we show that every L-fuzzy set is also a fuzzy set but converse is not true in general.

Example 2.4.1. Let $X = \{a, b, c\}$ and $L = \{0, 0.1, 0.2, \dots, 1\}$. A function $\alpha: X \to L$ is defined by $\alpha(a) = 0.2, \alpha(b) = 0.5, \alpha(c) = 0$ which is L-fuzzy set and also a fuzzy set.

Example 2.4.2. Let $X = \{a, b, c\}$ and $I = [0, 1]$. A function $u: X \to I$ is defined by $u(a) = 0.25, u(b) = 0.55, u(c) = 0$ which is fuzzy set but not an L-fuzzy set because $0.25, 0.55 \notin L$.

Definition 2.5. [6] Let X be a non-empty set and L be a complete distributive lattice with 0 and 1. An intuitionistic L-fuzzy set (ILFS for short) A in X is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)): x \in X\}$. Where the functions $\mu_A: X \to L$ and $\gamma_A: X \to L$ denote the degree of membership (namely $\mu_A(x)$) and the degree of none membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Let $L(X)$ denote the set of all intuitionistic L-fuzzy set in X. Obviously every L-fuzzy set $\mu_A(x)$ in X is an intuitionistic L-fuzzy set of the form $(\mu_A, 1 - \mu_A)$.
Throughout this paper we use the simpler notation $A = (\mu_A, \gamma_A)$ instead of $A = \{(x, \mu_A(x), \gamma_A(x)): x \in X\}$.

Definition 2.6. [6]) Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic L-fuzzy sets in X. Then

(1) $A \subseteq B$ if and only if $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(3) $A^c = (\gamma_A, \mu_A)$
(4) $A \cap B = (\mu_A \cap \mu_B; \gamma_A \cup \gamma_B)$
(5) $A \cup B = (\mu_A \cup \mu_B; \gamma_A \cap \gamma_B)$
(6) $0_\sim = (0^\sim, 1^\sim)$ and $1_\sim = (1^\sim, 0^\sim)$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an ILFS of X and $B = (\mu_B, \gamma_B)$ be an ILFS of Y. Then $f^{-1}(B)$ is an ILFS of X defined by $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ and $f(A)$is an ILFS of Y defined by $f(A) = \big(f(\mu_A), 1 - f(1 - \gamma_A)\big)$.

Definition 2.7. [11] An intuitionistic topology (IT for short) on a nonempty set X is a family t of IS's in X satisfies the following axioms:

(i) $\emptyset_\sim, X_\sim \in t$.
(ii) If $G_1, G_2 \in t$ then $G_1 \cap G_2 \in t$.
(iii) If $G_i \in t$ for each $i \in \Lambda$ then $\cup_{i \in \Lambda} G_i \in t$.

Then the pair (X,t) is called an intuitionistic topological space (ITS for short) and the members of t are called intuitionistic open sets (IOS for short).

Definition 2.8. [12] An ITS (X,t) is called $I-T_2$ space if for all $x,y \in X, x \neq y, \exists$ IOS $A=(A_1,A_2), B=(B_1,B_2) \in t$ such that $x \in A_1, y \in B_1$ and $A \cap B = \emptyset_{\sim}$.

Definition 2.9. Let $p,q \in L = \{0, 0.1, 0.2, \dots, 1\}$ and $p+q \leq 1$. An intuitionistic L-fuzzy point (ILFP for short) $x_{(p,q)}$of X is an ILFS of X defined by

$$x_{(p,q)}(y) = \begin{cases} (p,q) \; if \; y = x, \\ (0,1) if \; y \neq x \end{cases}$$

In this case, x is called the support of $x_{(p,q)}$ and p and q are called the value and none value of $x_{(p,q)}$, respectively. The set of all ILFP of X we denoted it by $S(X)$.

An ILFP $x_{(p,q)}$ is said to belong to an ILFS $A=(\mu_A,\gamma_A)$ of X denoted by $x_{(p,q)} \in A$, if and only if $p \leq \mu_A(x)$ and $q \geq \gamma_A(x)$ but $x_{(p,q)} \notin A$ if and only if $p \geq \mu_A(x)$ and $q \leq \gamma_A(x)$.

Definition 2.10. If A is an ILFS and $x_{(p,q)}$ is an ILFP then the intersection between ILFS and ILFP is defined as $x_{(p,q)} \cap A = \left(p \cap \mu_A(x); q \cup \gamma_A(x)\right)$.

Definition 2.11. An intuitionistic L-topology (ILT for short) on X is a family τ of ILFSs in X which satisfies the following conditions:

(i) $0_{\sim}, 1_{\sim} \in \tau$.

(ii) If $A_1, A_2 \in \tau$ then $A_1 \cap A_2 \in \tau$.

(iii) If $A_i \in \tau$ for each $i \in \Lambda$ then $\cup_{i\in\Lambda} A_i \in \tau$.

Then the pair (X,τ) is called an intuitionistic L-topological space (ILTS for short) and the members of τ are called intuitionistic L-fuzzy open sets (ILFOS for short). An intuitionistic L-fuzzy set B is called an intuitionistic L-fuzzy closed set (ILFC for short) if $1-B \in \tau$.

Definition 2.12. Let (X,τ) and (Y,s) be two ILTSs. Then a map $f: X \to Y$ is said to be

(i) Continuous if $f^{-1}(B)$ is an ILFOS of X for each ILFOS B of Y, or equivalently, $f^{-1}(B)$ is an ILFCS of X for each ILFCS B of Y,

(ii) Open if $f(A)$ is an ILFOS of Y for each ILFOS A of X,

(iii) Closed if $f(A)$ is an ILFCS of Y for each ILFCS A of X,

(iv) A homeomorphism if f is bijective, continuous and open.

3. Definition and properties of ILF-T_2 Spaces

Definition 3.1. An ILTS (X,τ) is called

(a) $IL-T_2(i)$ if for all $x,y \in X, x \neq y, \exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B = 0_{\sim}$.

(b) $IL-T_2(ii)$ if for any pair of distinct ILFP $x_{(p,q)}, y_{(r,s)} \in S(X), \exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B) \in \tau$ such that $x_{(p,q)} \in A, y_{(r,s)} \in B$ and $A \cap B = (0_{\sim}, \alpha_{\sim})$ where $\alpha \in L-\{0\}$.

(c) $IL - T_2(iii)$ if for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) > 0, \mu_B(y) > 0$ and $A \cap B = 0_{\sim}$.

(d) $IL - T_2(iv)$ if for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y), \gamma_A(x) = 0 = \gamma_B(y)$ and $A \subseteq B^c$ where B^c is the complement of B.

Theorem 3.2. Let (X, τ) be an ILTS. Then we have the following implications:

$$\begin{tikzcd}
 & IL - T_1(iii) \arrow[dd] & \\
IL - T_2(i) \arrow[ur] \arrow[dr] & & IL - T_2(ii) \arrow[ul] \arrow[dl] \\
 & IL - T_2(iv) &
\end{tikzcd}$$

Proof: $\boldsymbol{IL - T_2(i) \Rightarrow IL - T_2(iii)}$**and** $\boldsymbol{IL - T_2(i) \Rightarrow IL - T_2(iv)}$: Suppose (X, τ) is an $IL - T_2(i)$. Then we have by definition, for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B = 0_{\sim}$.

(1) ……… $\Rightarrow \mu_A(x) > 0, \mu_B(y) > 0$ and $A \cap B = 0_{\sim}$.

(2) ……… $\Rightarrow \begin{cases} \mu_A(x) = 1 = \mu_B(y) \\ \gamma_A(x) = 0 = \gamma_B(y) \text{ and } A \subseteq B^c. \end{cases}$

From (1) and (2) we see that $IL - T_2(i) \Rightarrow IL - T_2(iii)$and $IL - T_2(i) \Rightarrow IL - T_2(iv)$.

$\boldsymbol{IL - T_2(ii) \Rightarrow IL - T_2(iii)}$ **and** $\boldsymbol{IL - T_2(ii) \Rightarrow IL - T_2(iv)}$: Suppose $(X, \tau)$is an $IL - T_2(ii)$. Then we have by definition, if for any pair of distinct ILFP $x_{(p,q)}$, $y_{(r,s)} \in S(X), \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $x_{(p,q)} \in A, y_{(r,s)} \in B$ and $A \cap B = (0_{\sim}, \alpha_{\sim})$ where $\alpha \in L - \{0\}$.

(3) ………… $\Rightarrow \begin{cases} p \leq \mu_A(x), q \geq \gamma_A(x) \\ r \leq \mu_B(y), s \geq \gamma_B(y) \text{ and } A \cap B = (0_{\sim}, \alpha_{\sim}). \end{cases}$

Since $p, q, r, s \in L = \{0, 0.1, 0.2, \ldots, 1\}$ and $\alpha \in L - \{0\}$, we have from (3)

(4) ……… $\Rightarrow \{\mu_A(x) > 0, \mu_B(y) > 0$ and $A \cap B = 0_{\sim}$. And

(5) ……… $\Rightarrow \{\mu_A(x) = 1 = \mu_B(y); \gamma_A(x) = 0 = \gamma_B(y)$ and $A \subseteq B^c$

From (4) and (5) which shows that $IL - T_2(ii) \Rightarrow IL - T_2(iii)$ and $IL - T_2(ii) \Rightarrow IL - T_2(iv)$.

$\boldsymbol{IL - T_2(iii) \Rightarrow IL - T_2(iv)}$: Suppose (X, τ) is an $IL - T_2(iii)$. Then we have by definition, if for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) > 0, \mu_B(y) > 0$ and $A \cap B = 0_{\sim}$.

$\Rightarrow \begin{cases} \mu_A(x) = 1, \gamma_A(x) = 0; \mu_A(y) = 0, \gamma_A(y) = 1 \text{ and} \\ \mu_B(y) = 1, \gamma_B(y) = 0; \mu_B(x) = 0, \gamma_B(x) = 1. \end{cases}$

(6) ……… $\Rightarrow \begin{cases} \mu_A(x) = 1 = \mu_B(y) \\ \gamma_A(x) = 0 = \gamma_B(y) \text{ and } A \subseteq B^c \end{cases}$

This is $IL - T_2(iii) \Rightarrow IL - T_2(iv)$.

None of the reverse implications is true in general which can be seen from the following counter examples:

Example 3.2.1. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \ldots, 1\}$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 0.5, 0), (y, 0, 1)\}$ and $B = \{(x, 0, 1), (y, 0.4, 0)\}$. Hence we see that (X, τ) is an $IL - T_2(iii)$ but not $IL - T_2(i)$and $IL - T_2(ii)$.

Example 3.2.2. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 1, 0), (y, 0, 1)\}$ and $B = \{(x, 0, 1), (y, 1, 0)\}$. Hence we see that (X, τ) is an $IL - T_2(iv)$ but not $IL - T_2(i), IL - T_2(ii)$ and $IL - T_2(iii)$.

Theorem 3.3. Let (X, τ) be an ILTS and (X, t) be an ITS. Then we have the following implications:

$$\begin{tikzcd} & IL - T_2(iii) & \\ IL - T_2(i) \arrow[r, leftrightarrow] & I - T_2 \arrow[u] \arrow[r] \arrow[d] & IL - T_2(ii) \\ & IL - T_2(iv) & \end{tikzcd}$$

Proof: Suppose (X, t) is $I - T_2$. We prove that (X, τ) is $IL - T_2(i)$. Since (X, t) is $I - T_2$, $\forall\, x, y \in X, x \neq y, \exists\, A = (A_1, A_2), B = (B_1, B_2) \in t$ such that $x \in A_1, y \in B_1$and $A \cap B = \emptyset_{\sim}$. $\Rightarrow 1_{A_1}(x) = 1, 1_{B_1}(y) = 1$and $A \cap B = \emptyset_{\sim}$. Let $1_{A_1} = \mu_A$ and $1_{B_1} = \mu_B$. Then $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B = 0_{\sim}$ which is $IL - T_2(i)$. Conversely suppose that (X, τ) is $IL - T_2(i)$. We prove that (X, t) is $I - T_2$. Since (X, τ) is $IL - T_2(i)$, we have by definition, for all $x, y \in X, x \neq y, \exists$ ILOS $A = (\mu_A, \gamma_A), B = (\mu_B, \gamma_B) \in \tau$ such that $\mu_A(x) = 1 = \mu_B(y)$ and $A \cap B = 0_{\sim}$. Let $A_1 = \mu_A^{-1}\{1\}$ and $B_1 = \mu_B^{-1}\{1\}$. Then we have $x \in A_1, y \in B_1$and $A \cap B = \emptyset_{\sim}$ which is $I - T_2$. Therefore $I - T_2 \Leftrightarrow IL - T_2(i)$. Furthermore it can be shown that $I - T_2 \Rightarrow IL - T_2(ii), I - T_2 \Rightarrow IL - T_2(iii)$and $I - T_2 \Rightarrow IL - T_2(iv)$.

None of the reverse implications is true in general which can be seen from the following counter examples:

Example 3.3.1. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}, x_{(0.2,0.3)}, y_{(0.3,0.4)} \in S(X)$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 0.5, 0), (y, 0, 0.5)\}$ and $B = \{(x, 0, 0.5), (y, 0.5, 0)\}$. Hence we see that (X, τ) is an $IL - T_2(ii)$ but not $I - T_2$.

Example 3.3.2. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 0.5, 0), (y, 0, 1)\}$ and $B = \{(x, 0, 1), (y, 0.4, 0)\}$. Hence we see that (X, τ) is an $IL - T_2(iii)$ but not $I - T_2$.

Example 3.3.3. Let $X = \{x, y\}, L = \{0, 0.1, 0.2, \dots, 1\}$ and τ be an ILT on X generated by $\{A, B\}$ where $A = \{(x, 1, 0), (y, 0, 1)\}$ and $B = \{(x, 0, 1), (y, 1, 0)\}$. Hence we see that (X, τ) is an $IL - T_2(iv)$ but not $I - T_2$.

4. Hereditary properties of ILF –T_2(j) for (j= i, ii, iii, iv)

Definition 4.1. Let (X, τ) be an ILTS and $A \subseteq X$. we define $\tau_A = \{u|A : u \in \tau\}$ the subspace ILT's on A induced by τ. Then (A, τ_A) is called the subspace of (X, τ) with the underlying set A.

An IL-topological property 'P' is called hereditary if each subspace of an IL-topological space with property 'P' also has property 'P'.

Theorem 4.2. Let (X, τ) be an ILTS, $U \subseteq X$ and $\tau_U = \{A|U : A \in \tau\}$. Then

(a) (X, τ) is $IL - T_2(i) \Rightarrow (U, \tau_U)$ is $IL - T_2(i)$.

(b) (X, τ) is $IL - T_2(ii) \Rightarrow (U, \tau_U)$ is $IL - T_2(ii)$.

(c) (X,τ) is $IL-T_2(iii) \Longrightarrow (U,\tau_U)$ is $IL-T_2(iii)$.
(d) (X,τ) is $IL-T_2(iv) \Longrightarrow (U,\tau_U)$ is $IL-T_2(iv)$.

Proof: We prove only (a). Suppose (X,τ) is $IL-T_2(i)$, we prove that (U,τ_U) is $IL-T_2(i)$. Let $x,y \in U, x \neq y$. Then $x,y \in X, x \neq y$ as $U \subseteq X$. Since (X,τ) is $IL-T_2(i)$, we have for all $x,y \in X, x \neq y, \exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B) \in \tau$ such that $\mu_A(x)=1=\mu_B(y)$ and $A \cap B = 0_\sim$. For $U \subseteq X$, we find ILOS $A|U=\left(\mu_{A|U},\gamma_{A|U}\right), B|U=\left(\mu_{B|U},\gamma_{B|U}\right) \in \tau_U$ such that $\mu_{A|U}(x)=1=\mu_{B|U}(y)$ and $A|U \cap B|U=(A\cap B)|U=0_\sim$ as $A \cap B = 0_\sim$. Hence (U,τ_U) is $IL-T_2(i)$. Similarly (b), (c), (d) can be proved.

We observe here that ILF-T_2(j), (j = i, ii, iii, iv) concepts are preserved under continuous, one-one and open maps.

Theorem 4.3. Let (X,τ) and (Y,s) be two ILTS, $f:(X,\tau)\to(Y,s)$ be one-one, onto and continuous map. Then
(a) (X,τ) is $IL-T_2(i) \Leftrightarrow (Y,s)$ is $IL-T_2(i)$
(b) (X,τ) is $IL-T_2(ii) \Leftrightarrow (Y,s)$ is $IL-T_2(ii)$
(c) (X,τ) is $IL-T_2(iii) \Leftrightarrow (Y,s)$ is $IL-T_2(iii)$
(d) (X,τ) is $IL-T_2(iv) \Leftrightarrow (Y,s)$ is $IL-T_2(iv)$.

Proof: We prove only (a). Suppose (X,τ) is $IL-T_2(i)$, we prove that (Y,s) is $IL-T_2(i)$. Let $y_1,y_2 \in Y$ with $y_1 \neq y_2$. Since f is onto, $\exists\, x_1,x_2 \in X$, such that $f(x_1)=y_1, f(x_2)=y_2$ and $x_1 \neq x_2$ as f is one-one. Again since (X,τ) is $IL-T_2(i)$, we have for all $x_1,x_2 \in X, x_1 \neq x_2, \exists$ an ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B) \in \tau$ such that $\mu_A(x_1)=1=\mu_B(x_2)$ and $A\cap B=0_\sim$. Now $\exists$ ILOS $f(A)=\left(f(\mu_A),1-f(1-\gamma_A)\right), f(B)=(f(\mu_B),1-f(1-\gamma_B)) \in s$ such that $f(\mu_A)(y_1)=\{\sup \mu_A(x_1): f(x_1)=y_1\}=1$
$f(\mu_B)(y_2)=\{\sup \mu_B(x_2): f(x_2)=y_2\}=1$and $f(A)\cap f(B)=f(A\cap B)=0_\sim$as
$A\cap B=0_\sim$. Thus we have $f(x_1), f(x_2) \in Y$ with $f(x_1)\neq f(x_2)$ $\exists$ ILOS $f(A)=\left(f(\mu_A),1-f(1-\gamma_A)\right), f(B)=(f(\mu_B),1-f(1-\gamma_B)) \in s$ such that $f(\mu_A)(y_1)=1=f(\mu_B)(y_2)$ and $f(A)\cap f(B)=0_\sim$. Hence (Y,s) is $IL-T_2(i)$.
Conversely suppose that (Y,s) is $IL-T_2(i)$. We prove that (X,τ) is $IL-T_2(i)$. Let $x_1,x_2 \in X$ with $x_1 \neq x_2 \Longrightarrow f(x_1)\neq f(x_2)$ as f is one-one. Put $f(x_1)=y_1$, and $f(x_2)=y_2$ then $y_1 \neq y_2$. Since (Y,s) is $IL-T_2(i), \exists$ ILOS $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B) \in s$ such that $\mu_A(y_1)=1=\mu_B(y_2)$ and $A\cap B=0_\sim$.
$\Longrightarrow \{\mu_A f(x_1)=1=\mu_B f(x_2)$ and $f^{-1}(A\cap B)=0_\sim$
$\Longrightarrow \{f^{-1}\mu_A(x_1)=1=f^{-1}\mu_B(x_2)$ and $f^{-1}(A)\cap f^{-1}(B)=0_\sim$.
Since $A=(\mu_A,\gamma_A), B=(\mu_B,\gamma_B) \in s, f^{-1}(A)=(f^{-1}(\mu_A), f^{-1}(\gamma_A)), f^{-1}(B)=(f^{-1}(\mu_B), f^{-1}(\gamma_B)) \in \tau$. Hence it is clear that, $x_1,x_2 \in X, x_1 \neq x_2$ $\exists$ ILOS $f^{-1}(A)=(f^{-1}(\mu_A), f^{-1}(\gamma_A)), f^{-1}(B)=(f^{-1}(\mu_B), f^{-1}(\gamma_B)) \in \tau$ such that
$f^{-1}\mu_A(x_1)=1=f^{-1}\mu_B(x_2)$ and $f^{-1}(A)\cap f^{-1}(B)=0_\sim$. Hence (X,τ) is also $IL-T_2(i)$. Similarly, (b), (c) and (d) can be proved.

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