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Strong Non-Split Domination in Directed Graphs

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Abstract. The concept of connectedness plays an important role in many networks. Digraphs are considered as an excellent modeling tool and are used to model many types of relations amongst any physical situations. In this paper the concept of strong non-split domination in directed graph D has been introduced by considering the dominating set S is a strong non-split dominating set if the complement of S is complete A dominating set S of a directed graph D is a strong non-split dominating set if the induced subdigraph <V-S> is complete. The minimum cardinality of strong non-split dominating set is denoted by $\gamma_{sns}(D)$. In this paper, the domination parameters corresponding to strong non-split domination in digraphs has been analyzed in various types of digraphs and obtained several results on these parameters.

1. Introduction

Kulli and Janakiram introduced the concept of strong non-split domination number of a graph [10]. Throughout this paper D=(V,A) is a finite directed graph with neither loops nor multiple arcs(but pairs of arcs are allowed) and G=(V,E) is a undirected graph with neither loops nor multiple edges. For basic terminology on graphs and digraphs, we refer to Chartrand and Lesniak [2]

Let G=(V,E) be a graph. A subset D of V is called dominating set of G if every vertex in V-D is adjacent to at least one vertex in D. The minimum cardinality of dominating set of G is called domination number of G and is denoted by $\gamma(G)$ [6].

A dominating set $D \subseteq V$ of a graph G is non- split dominating set if the induced subgraph <V-D> is complete. The strong non-split domination number $\gamma_{sns}(G)$ is the minimum cardinality of a strong non-split dominating set [10].

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Let D=(V,A) be a digraph. A subset S of V is called a dominating set of D if for every vertex $v \in$ V-S there exists a vertex $u \in$ S such that (u, v) $\in$ A. The domination number $\gamma(D)$ is the minimum cardinality of dominating set D [5].

Let D=(V,A) be a digraph. For any vertex $u \in$ V, the sets O(u)={v/(u,v) $\in$ A} and I(u)={v/(v, u) $\in$ A} are called outset and inset of u. The in degree and out degree of u are defined by id(u)=$|I(u)|$ and od(u)=$|O(u)|$. The minimum in degree , the minimum out degree, the maximum in degree and maximum out degree of D are denoted by $\delta^-, \delta^+, \Delta^-$, and Δ^+ respectively. [1]

An out- domination set of digraph D is a set S^+ of vertices of D such that every vertex of V-S^+ is adjacent from some vertex of S. The minimum cardinality of out-domination set for D is the out-domination number $\gamma^+(D)$ [3,5].

The in- domination number $\gamma^-(D)$ is defined as expected.

Although domination and other related concepts have been extensively studied for undirected graphs ,the respective analogue on digraphs have not received much attention.

γ - set is the set of all vertices in dominating set with # $\gamma(D)$

γ^+- set is the set of all vertices in out- dominating set with # $\gamma^+(D)$

γ^- - set is the set of all vertices in in-dominating set with #$\gamma^-(D)$

γ_{sns} - set is set of all vertices in strong non-split dominating set with # $\gamma_{ns}(D)$

γ_{sns}^+ - set is set of all vertices in strong non-split out dominating set with # $\gamma_{ns}^+(D)$

γ_{sns}^- - set is set of all vertices in strong non-split in dominating set with # $\gamma_{ns}^-(D)$

Throughout this paper the out-domination number of digraphs have been analyzed.

2. Preliminaries
Definition 1. A dominating set S of digraph D is a strong non-split dominating set if the induced sub-digraph $< V - S >$ is complete. The minimum cardinality of strong non-split domination number is denoted by $\gamma_{sns}(D)$.

Definition 2. A digraph D is strongly connected or strong if for every pair x, y of distinctvertices in D, there exists an (x, y) path and (y, x) path [7].

Definition 3. An oriented graph is a digraph with no cycle of length two [7].

Definition 4. A tournament is transitive if (u, w) is an arc whenever (u, v) and (v, w) both are arcs [7].

3. Results and observations
For directed path P_n , $n \geq 2$

$$\gamma_{sns}^+(D) = n - 1.$$

For directed cycle C_n, $n \geq 3$

$$\gamma_{sns}^{+}(D) = n - 1.$$

For complete digraph $\gamma_{sns}^{+}(D) = n - \Delta^{+}$

For directed tree, $\gamma_{sns}^{+}(D) = n - 1$

For directed star, $\gamma_{sns}^{+}(D) = 0$

$$\gamma_{sns}^{-}(D) = 1.$$

Theorem 3.1. For any digraph D

$$\gamma^{+}(D) \leq \gamma_{ns}^{+}(D) \leq \gamma_{sns}^{+}(D).$$

Proof: Let S^{+} be the strong non-split dominating set of digraph D. Every strong non-split dominating set of D is a non-split dominating set and every non-split dominating set is a dominating set.

Theorem 3.2. If T is a directed tree which is not a directed star, then

$$\gamma_{sns}^{+}(T) = \gamma_{ns}^{+}(T) = n - 1.$$

Proof: Since T is not a directed star, where directed star is a connected directed tree. Let S^{+} be the strong non-split dominating set which has a root vertex and cut vertices with one end vertex. The other end vertex which is in $< V - S^{+} >$ is complete.

Theorem 3.3. For any digraph D which is directed cycle then $\gamma_{sns}^{+}(D) = n - \Delta^{+}$

Proof: Suppose $\gamma_{sns}^{+}(D) > n - \Delta^{+}$

Then it must be $\gamma_{sns}^{+}(D) \geq n - \Delta^{+} + 1$

That is $n - \Delta^{+} + 1 \leq \gamma_{sns}^{+}(D) < n - \Delta^{+}$

which is a contradiction

Hence $\gamma_{sns}^{+}(D) = n - \Delta^{+}$.

Theorem 3.4. Let D be the digraph having cut vertices then γ_{sns}^{+}- set of D would contain all cut vertices.

Proof: Let v be a cut vertex and S^{+} is a γ_{sns}^{+} - set of D. If $v \notin S^{+}$ then $D - v$ has exactly two components D_1 *and* D_2 such that at least one of the digraphs $H_1 = < D_1 \cup \{v\} >$ or $H_2 = < D_2 \cup \{v\} >$ is a path or directed cycle.

Theorem 3.5. Let $K_{1,p}$ be a directed star in which od(v$_i$)=p and id(v)=1, $1 \leq i \leq p$

Then $\gamma_{sns}^{+}(D) = 0$.

Proof: As $<V - S^+>$ contains the vertices v_1, v_2, v_3 and $S^+ = \{v\}$ then $<V - S^+>$ is not connected and complete. Hence there is no strong non-split domination number as well as non-split domination number.

$$\therefore \ \gamma_{sns}^{\ +}(D) = \gamma_{ns}^{\ +}(D) = 0$$

Theorem 3.6. Let $K_{1,p}$ be a directed star in which od(v_i)=1 , $1 \le i \le p$ and id(v)=p

Then $\gamma_{sns}^{\ +}(D) = 1$

Proof: Let $S^+ = p = \{v_1, v_2, v_3\}$ and $<V - S^+> = 1$ which is connected and complete, then $\gamma_{sns}^{\ +}(D) = \{v\} = 1$.

Theorem 3.7. Let $l(D)$ denotes the length of largest directed path then

$\gamma_{sns}^{\ +}(D) = l(D)$.

Proof: Let P=$\{v_0, v_1, ...v_k\}$ be a largest path in D with k= $l(D)$. Let S be the strong non-split dominating set of D.

As $V - S$ has only one vertex with od=0 and id=1.

Therefore $V - S$ is complete. Hence S has all the vertices except that vertex which has od=0 and id=1 then $\gamma_{sns}^{\ +}(D) = l(D)$.

Theorem 3.8. If D is the tournament digraph then $\gamma_{sns}^{\ +}(D) = n - \Delta^+$.

Proof: Suppose D is a tournament digraph which has vertices n=4

Case (i) : Let $V - S$ =\{u,v,w\} be complete , S has only one vertex 'x' which dominates u,v,and w, then the maximum out degree of D is 3. Hence $\gamma_{sns}^{\ +}(D) = n - \Delta^+$.

Case (ii) : Let $V - S$ =\{u,v\} be complete, S =\{x,w\} x dominates u and v or w dominates u and v. then the maximum out degree of D is 2. Hence $\gamma_{sns}^{\ +}(D) = n - \Delta^+$.

After analyzing both the cases, for n=4.we can conclude in general for any tournament digraph $\gamma_{sns}^{\ +}(D) = n - \Delta^+$.

4. Conclusion

In this paper, we have introduced the parameter strong non-split domination in directed graphs. Some interested results related with the above are proved. Further, the authors proposed to introduce new dominating parameters in directed graphs using the MATLAB codes. In the last decade, we have seen an impressively increasing number of neuroscience studies using digraph theory and networks. Neural networks are considered as directed graphs that allows a broad range application of analytical tools from digraph theory.

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