Annals of Pure and Applied Mathematics
Vol. 12, No. 2, 2016, 129-136
ISSN: 2279-087X (P), 2279-0888(online)
Published on 19 October 2016
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

On Semi Prime Filters in Lattices

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Received 19 September 2016; accepted 30 September 2016

Abstract. Recently Yehuda Rav has given the concept of Semi prime ideals in a general lattice by generalizing the notion of 0-distributive lattices. In this paper we study several properties of these filters and include some of their characterizations. We give some results regarding maximal ideals and include a number of Separation properties in a general lattice with respect to the annihilator filter containing a semi prime filter. Here we prove that a filter J is Semi prime if and only if every maximal ideal of a lattice L disjoint with J is prime.

Keywords: 0-distributive lattice, semi prime ideal, annihilator ideal, maximal filter

AMS Mathematics Subject Classification (2010): 06A12, 06A99, 06B10

1. Introduction

In generalizing the notion of pseudo complemented lattice, Varlet [9] introduced the notion of 0-distributive lattices. Then [3] have given several characterizations of these lattices. On the other hand, [6] have studied them in meet semi lattices. A lattice L with 0 is called a *0-distributive* lattice if for all $a,b,c \in L$ with $a \wedge b = 0 = a \wedge c$ imply $a \wedge (b \vee c) = 0$. Of course every distributive lattice is 0-distributive. 0-distributive lattice L can be characterized by the fact that the set of all elements disjoint to $a \in L$ forms an ideal. So every pseudo complemented lattice is 0-distributive. An algebra $< L; \wedge, \vee, 0, 1 >$ is called a lattice with *pseudo complementation* if $< L; \wedge, \vee, 0, 1 >$ is bounded lattice and for all $a \in L,$ there exists a^* such that $a \wedge a^* = 0$ and $a \wedge x = 0$ implies that $x \wedge a^* = 0,$ for all $x \in L.$ Let L be a bounded distributive lattice and $a \in L,$ an element $a^* \in L$ is called *pseudo complement* of a in L if $a \wedge a^* = 0$ and $a \wedge x = 0$ implies $x \le a^*$ for all $x \in L.$ A bounded lattice L is called *pseudo complemented* if its every element has a pseudo complement.

Similarly, let L be a bounded distributive lattice and $a \in L,$ an element $a^+ \in L$ is called a *dual pseudo complement* of a in L if $a \vee a^+ = 0$ and $a \vee x = 0$ implies $x \le a^+$ for all $x \in L.$ A bounded lattice L is called a *dual pseudo complemented* if its every element has a dual pseudo complement. A lattice L with 1 is called a *1-*

distributive lattice if for all $a,b,c \in L$ with $a \vee b = 1 = a \vee c$ imply $a \vee (b \wedge c) = 1$. It is easy to see that for each $a \in L$, $\{x \in L : a \vee x = 1\}$ is a filter. Thus every dual pseudo complemented lattice is 1-distributive.

Recently, Rav [7] has generalized this concept and gave the definition of semi prime filters in a lattice. For a non-empty subset F of L, F is called an *up set* if for $a \in F$ and $x \geq a$ imply $x \in F$. Moreover an up set F is a *filter* if $a \wedge b \in F$ for all $a,b \in F$. Similarly, a non-empty subset I of L is called a *down set* if $a \in I$ and $x \leq a$ implies $x \in I$. A down set I is called an *ideal* if $x \vee y \in I$ for all $x, y \in I$. A proper ideal I is called a *prime ideal* if $a \wedge b \in I$ $(a,b \in L)$ implies either $a \in I$ *or* $b \in I$. A proper ideal I is called a *maximal ideal* if for any $M \supseteq I$ implies $M = I$ or $M = L$. A prime filter P is called *minimal prime filter* if it does not contain any other prime filter. A proper filter Q is called a *prime filter* if $a \vee b \in Q$ ($a,b \in L$) implies either $a \in Q$ or $b \in Q$. It is very easy to check that F is a filter of L if and only if $L - F$ is a prime down set. Moreover, F is a prime filter if and only if $L - F$ is a prime ideal.

An ideal I of a lattice L is called a Semi prime ideal if for $x, y, z \in L$, $x \wedge y \in I$, $x \wedge z \in I$ imply $x \wedge (y \vee z) \in I$. Recently [1,2,4,5,10] studied the Semi prime ideals in lattices and meet semi lattices. A filter F of a lattice L is called *semi prime filter* if for all $x, y, z \in L$, $x \vee y \in F$ and $x \vee z \in F$ imply $x \vee (y \wedge z) \in F$. Thus, for a lattice L with 1, L is called *1-distributive* if and only if [1) is a semi prime filter. In a distributive lattice L, every filter is a semi prime filter. Moreover, every prime filter is semi prime. In a pentagonal lattice $\{0, a, b, c, 1; a < b\}$, [1) is semi prime but not prime. Here $[a)$ and $[c)$ are prime, but $[b)$ is not even semi prime. Again in $M_3 = \{0, a, b, c, 1;\ a \wedge b = b \wedge c = a \wedge c = 0; a \vee b, b \vee c, a \vee c = 1\}$, [1), $[a)$, $[c)$ are not semi prime. In this paper we study the semi prime filters and included several characterizations.

2. Main results

To obtain the main results of the paper, we need to prove the following Lemmas:

Lemma 1. Non empty intersection of all prime (semi prime) filters of a lattice is a semi-prime filter.

Proof: Let $a,b,c \in L$ and $F = \bigcap\{Q : Q$ is a prime (semi prime) filter$\}$ and F is nonempty. Let $a \vee b \in F$ and $a \vee c \in F$. Then $a \vee b \in Q$ and $a \vee c \in Q$ for all Q. Since each Q is prime (semi prime), so $a \vee (b \wedge c) \in Q$. Hence $a \vee (b \wedge c) \in F$, and so F is semi prime.

Corollary 2. Intersection of two prime (semi prime) filters is a semi prime filter.

Lemma 3. Every ideal disjoint from a filter F is contained in a maximal ideal disjoint from F.

Proof: Let I be an ideal in L disjoint from F. Let P be the set of all ideals containing I and disjoint from F. Then P is nonempty as $I \in P$. Let C be a chain in P and let $M = \bigcup(X : X \in C)$. We claim that M is a ideal. Let $x \in M$ and $y \leq x$. Then $x \in X$ for some $X \in C$. Hence $y \in X$ as X is an ideal. Therefore, $y \in M$. Let $x, y \in M$. Then $x \in X$ and $y \in Y$ for some $X, Y \in C$. Since C is a chain, either $X \subseteq Y$ or $Y \subseteq X$. Suppose $X \subseteq Y$. So $x, y \in Y$. Then $x \vee y \in Y$ and so $x \vee y \in M$. Thus M is an ideal. Moreover, $M \cap F = \phi$ and $M \supseteq I$. So M is a maximum element of C. Then by Zorn's Lemma, P has a maximal element.

Let L be a lattice with 0. For $A \subseteq L$, we define $A^{\perp} = \{x \in L : x \wedge a = 0$ *for all* $a \notin A\}$. Let L be a lattice with 1. For $A \subseteq L$, we define $A^{\perp^d} = \{x \in L : x \vee a = 1 \text{ for all } a \in A\}$. $A^{\perp^d}$ is always an up set of L. Moreover, it is convex but it is not necessarily a filter. Let $A \subseteq L$ and J be a filter of L. We define $A^{\perp^d{}_J} = \{x \in L : x \vee a \in J \text{ for all } a \in A\}$.

Theorem 4. Let A be a non-empty subset of a lattice L and J be a filter of L. Then $A^{\perp^d{}_J} = \bigcap(P$ is a minimal prime up set containing J but not containing A).

Proof: Suppose $X = \bigcap(P : A \not\subset P, P \text{ is a minimal prime up set containing } J)$. Let $x \in A^{\perp^d{}_J}$. Then $x \vee a \in J$ for all $a \in A$. Choose any P of right hand expression. Since $A \not\subset P$, there exists $z \in A$ but $z \notin P$. Then $x \vee z \in J \subseteq P$. So $x \in P$, as P is prime. Hence $x \in X$.

Conversely, let $x \in X$. If $x \notin A^{\perp^d{}_J}$, then $x \vee b \notin J$ for some $b \in A$. Let $D = (x \vee b]$. Hence D is an ideal disjoint from J. Then by Lemma 3, there is a maximal ideal $M \supseteq D$ but disjoint from J. Then $L - M$ is a minimal prime up set containing J. Now $x \notin L - M$ as $x \in D$ implies $x \in M$. Moreover $A \nsubseteq L - M$ as $b \in A$, but $b \in M$ implies $b \notin L - M$, which is a contradiction to $x \in X$. Hence $x \in A^{\perp^d{}_J}$.

Lemma 5. Let F be a filter of a lattice L. An ideal I disjoint from F is a maximal ideal disjoint from F if and only if for all $a \notin I$, there exists $b \in I$ such that $a \vee b \in F$.

Proof: Let I be maximal and disjoint from F and $a \notin I$. Let $a \vee b \notin F$ for all $b \in I$. Consider $M = \{y \in L : y \leq a \vee b, b \in I\}$. Clearly M is an ideal. For any $b \in I, b \leq a \vee b$ implies $b \in M$. So $M \supseteq I$. Also $M \cap F = \phi$. For if not, let $x \in M \cap F$. This implies $x \in F$ and $x \leq a \vee b$ for some $b \in I$. Hence $a \vee b \in F$, which is a contradiction. Hence $M \cap F = \phi$. Now $M \supset I$ because $a \notin I$ but $a \in M$. This contradicts the maximality of I. Hence there exists $b \in I$ such that $a \vee b \in F$.

Conversely, if I is not maximal ideal disjoint from F, then there exists a ideal N containing I with disjoint F. Now let $a \in N - I$ by the given condition there exists

$b \in I$ such that $a \vee b \in F$. Hence $a, b \in N$ implies $a \vee b \in F \cap N$, which is a contradiction. Hence I must be a maximal ideal disjoint with F.

Theorem 6. Let L be a dual pseudo complemented lattice. Then for $A \subseteq L$, $A^{\perp^d} = \{x \in L : x \vee a = 1 \ for\ all\ a \in A\}$ is a semi prime filter.

Proof: We have already mentioned that $A^{\perp^d}$ is an up set of L. Since L is dual pseudo complemented so it is 1-distributive. Now let $x, y \in A^{\perp^d}$. Then $x \vee a = 1 = y \vee a$ for all $a \in L$. Hence $a \vee (x \wedge y) = 1$ for all $a \in A$. This implies $x \wedge y \in A^{\perp^d}$ and so $A^{\perp^d}$ is a filter. Now let $x \vee y \in A^{\perp^d}$ and $x \vee z \in A^{\perp^d}$. Then $x \vee y \vee a = 1 = x \vee z \vee a$ for all $a \in A$. This implies $y \geq (x \vee a)^+$, $z \geq (x \vee a)^+$ and so $y \wedge z \geq (x \vee a)^+$ and this implies $x \vee a \vee (y \wedge z) = 1$ for all $a \in L$. Hence $x \vee (y \wedge z) \in A^{\perp^d}$ and so $A^{\perp^d}$ is a semi prime filter.

Let $A \subseteq L$ and J be a filter of L.
We define $A^{\perp^{d_J}} = \{x \in L : x \vee a \in J \ for\ all\ a \in A\}$. This is clearly an up set containing J. In presence of distributivity, this is a filter. $A^{\perp^{d_J}}$ is called a dual annihilator of A relative to J. We denote $F_J(L)$, by the set of all filters containing J. Of course $F_J(L)$ is a bounded lattice with J and L as the smallest and the largest elements. If $A \in F_J(L)$, and $A^{\perp^{d_J}}$ is a filter, then $A^{\perp^{d_J}}$ is called an *annihilator filter* and it is the dual pseudo complement of A in $F_J(L)$.
Following Theorem gives some nice characterizations semi prime filters.

Theorem 7. Let L be a lattice and J be a filter of L. The following conditions are equivalent:
(i) J is semi prime.
(ii) $\{a\}^{\perp^{d_J}} = \{x \in L : x \vee a \in J\}$ is a semi prime filter containing J.
(iii) $A^{\perp^{d_J}} = \{x \in L : x \vee a \in J \ for\ all\ a \in A\}$ is a semi prime filter containing J.
(iv) $F_J(L)$ is pseudo complemented.
(v) $F_J(L)$ is a 0-distributive lattice.
(vi) Every maximal ideal disjoint from J is prime.

Proof: (i)$\Rightarrow$(ii). $\{a\}^{\perp^{d_J}}$ is clearly an up set containing J. Now let $x, y \in \{a\}^{\perp^{d_J}}$. Then $x \vee a \in J$, $y \vee a \in J$. Since J is semi prime, so $a \vee (x \wedge y) \in J$. Thus $x \wedge y \in \{a\}^{\perp^{d_J}}$. This implies $\{a\}^{\perp^{d_J}}$ is a filter containing J. Now let $x \vee y \in \{a\}^{\perp^{d_J}}$ and $x \vee z \in \{a\}^{\perp^{d_J}}$. Then $x \vee y \vee a \in J$ and $x \vee z \vee a \in J$. Thus, $(x \vee a) \vee y \in J$

and $(x \vee a) \vee z \in J$. Then $(x \vee a) \vee (y \wedge z) \in J$, as J is semi prime. This implies $x \vee (y \wedge z) \in \{a\}^{\perp^{d_J}}$, and so $\{a\}^{\perp^{d_J}}$ is semi prime.

(ii)$\Rightarrow$(iii). This is trivial by Lemma 1, as $A^{\perp^{d_J}} = \bigcap(\{a\}^{\perp^{d_J}}; a \in A)$.

(iii)$\Rightarrow$(iv). Since for any $A \in F_J(L)$, $A^{\perp^{d_J}}$ is a filter, it is the pseudo complement of A in $F_J(L)$, so $F_J(L)$ is the pseudo complemented.

(iv)$\Rightarrow$(v). This is trivial as every pseudo complemented lattice is 0-distributive.

(v)$\Rightarrow$(vi). Let $F_J(L)$ is 0-distributive. Suppose I is a maximal ideal disjoint from J. Suppose $f, g \notin I$. By Lemma 5, there exist $a, b \in I$ such that $a \vee f \in J, b \vee g \in J$. Then $f \vee a \vee b \in J$, $g \vee a \vee b \in J$.

Hence $[f) \cap [a \vee b) \subseteq J$ *and* $[g) \cap [a \vee b) \subseteq J$.

Since $F_J(L)$ is 0-distributive, so, $[a \vee b) \cap ([f) \vee [g)) \subseteq J$.

Thus, $[a \vee b) \cap [f \vee g) = [(a \vee b) \vee (f \wedge g)) \subseteq J$. Hence $(f \wedge g) \vee (a \vee b) \in J$. This implies $f \wedge g \notin I$ as $I \cap J = \varphi$, and so I is prime.

(vi)$\Rightarrow$(i). Let (vi) holds. Suppose $a, b, c \in L$ with $a \vee b \in J, a \vee c \in J$. If $a \vee (b \wedge c) \notin J$, then $(a \vee (b \wedge c)] \cap J = \varphi$. Then by Lemma 3, there exists a maximal ideal $I \supseteq (a \vee (b \wedge c)]$ and disjoint from J. Then $a \in I, b \wedge c \in I$. By (vi) I is prime. Hence either $a \vee b \in I$ or $a \vee c \in I$. In any case $J \cap I \neq \phi$, which gives a contradiction. Hence $a \vee (b \wedge c) \in J$, and so J is semi prime.

Corollary 8. In a lattice L, every ideal disjoint to a semi prime filter J is contained in a prime ideal.

Proof: This immediately follows from Lemma 3 and theorem 7.

Theorem 9. If J is a semi prime filter of a lattice L and $J \neq A = \bigcap\{J_\lambda : J_\lambda$ is a filter containing $J\}$, Then $A^{\perp^{d_J}} = \{x \in L : \{x\}^{\perp^{d_J}} \neq J\}$.

Proof: Let $x \in A^{\perp^{d_J}}$. Then $x \vee a \in J$ for all $a \in A$. So $a \in \{x\}^{\perp^{d_J}}$ for all $a \in A$. Then $A \subseteq \{x\}^{\perp^{d_J}}$ and so $\{x\}^{\perp^{d_J}} \neq J$. Conversely, let $x \in L$ such that $\{x\}^{\perp^{d_J}} \neq J$. Since J is semi prime, so $\{x\}^{\perp^{d_j}}$ is a filter containing J. Then $A \subseteq \{x\}^{\perp^{d_J}}$, and so $A^{\perp^{d_J}} \supseteq \{x\}^{\perp^{d_J}\perp^{d_J}}$. This implies $x \in A^{\perp^{d_J}}$, which completes the proof.

In [8], a series of characterizations of 1-distributive lattices have been provided. Here we give some results on semi prime filters related to their results.

Theorem 10. Let L be a lattice and J be a filter. Then the following conditions are equivalent.

(i) J is semi-prime.

(ii) Every maximal ideal of L disjoint with J is prime

(iii) Every minimal prime up set containing J is a minimal prime filter containing J
(iv) Every ideal disjoint with J is disjoint from a minimal prime filter containing J.
(v) For each element $a \notin J$, there is a minimal prime filter containing J but not containing a.
(vi) Each $a \notin J$ is contained in a prime ideal disjoint to J.

Proof: (i) $\Leftrightarrow$ (ii) follows from Theorem 7.

$(ii) \Rightarrow (iii)$. Let A be a minimal prime up set containing J. Then $L - A$ is a maximal ideal disjoint with J. Then by (ii), $L - A$ is a prime ideal and so A is a minimal prime filter.

$(iii) \Rightarrow (ii)$. Let P be a maximal ideal disjoint with J. Then $L - P$ is a minimal prime up set containing J. Thus by (iii), $L - P$ is a minimal prime filter and so P is a prime ideal.

$(i) \Rightarrow (iv)$. Let P be an ideal of L disjoint from J. Then there exists a maximal ideal $Q \supseteq P$ disjoint to J. By theorem 7, Q is a prime ideal and so $L - Q$ is a minimal prime filter containing J and so $P \cap (L - Q) = \phi.$

$(iv) \Rightarrow (v)$. Let $a \in L$, $a \notin J$. Then $[a) \cap J = \varphi$. Then by (iv) there exists a minimal prime filter A containing J disjoint from $[a)$. Thus $a \notin A$. $(v) \Rightarrow (vi)$. Let $a \in L$, $a \notin J$. Then by (v) there exists a minimal prime filter A containing J such that $a \notin A$. Implies $a \in L - A$ and $L - A$ is a prime ideal disjoint to J.

$(vi) \Rightarrow (i)$. Suppose J is not semi-prime. Then there exists $a, b, c \in L$ such that $a \vee b \in J$, $a \vee c \in J$ but $a \vee (b \wedge c) \notin J$. Then by (vi) there exists a prime ideal I disjoint from J and $a \vee (b \wedge c) \in I$. Now $a \vee (b \wedge c) \in I$ implies $a \in I$, $b \wedge c \in I$. Since I is prime so either $a \vee b \in I$ or $a \vee c \in I$, which gives a contradiction to the fact that $I \cap J = \varphi$. Therefore, $a \vee (b \wedge c) \in J$ and so J is semi prime.

Now we give another characterization of semi prime filters with the help of Prime Separation Theorem using annihilator filters.

Theorem 11. Let J be a filter in a lattice L. J is semi prime if and only if for all ideals I disjoint to $\{x\}^{\perp^{d_J}}$, there is a prime ideal containing I disjoint to $\{x\}^{\perp^{d_J}}$.

Proof: Suppose J is semi prime. Let I be an ideal disjoint to $\{x\}^{\perp^{d_J}}$. Using Zorn's Lemma we can easily find a maximal ideal P containing I and disjoint to $\{x\}^{\perp^{d_J}}$. We claim that $x \in P$. If not, then $P \vee (x] \supset P.$ By maximality of $P,$

$(P \vee (x]) \cap \{x\}^{\perp^{d_J}} \neq \phi$. If $t \in (P \vee (x]) \cap \{x\}^{\perp^{d_J}}$, then $t \leq p \vee x$ for some $p \in P$ and $t \vee x \in J$. This implies $p \vee x \in J$ and so $p \in \{x\}^{\perp^{d_J}}$ gives a contradiction. Hence $x \in P.$ Now let $z \notin P.$ Then $(P \vee (z]) \cap \{x\}^{\perp^{d_J}} \neq \phi.$ Suppose $y \in (P \vee (z]) \cap \{x\}^{\perp^{d_J}}$ then $y \leq p_1 \vee z$ *and* $y \vee x \in J$ for some $p_1 \in P.$ This implies $p_1 \vee x \vee z \in J$ and

$p_1 \vee x \in P$. Hence by Lemma 5, P is a maximal ideal disjoint to $\{x\}^{\perp^d J}$. Then by Theorem 7, P is prime.

Conversely, let $x \vee y \in J$, $x \vee z \in J$. If $x \vee (y \wedge z) \notin J$, then $y \wedge z \notin \{x\}^{\perp^d J}$. Thus $(y \wedge z] \cap \{x\}^{\perp^d J} = \phi$. So there exists a prime ideal P containing $(y \wedge z]$ and disjoint from $\{x\}^{\perp^d J}$. As $y, z \in \{x\}^{\perp^d J}$, *so* $y, z \notin P$. Thus $y \wedge z \notin P$, as P is prime. This gives a contradiction. Hence $x \vee (y \wedge z) \in J$, and so J is semi prime.

We conclude the paper with the following characterization of semi prime filters.

Theorem 12. Let J be a semi prime filter of a lattice L and $x \in L$. Then a prime filter Q containing $\{x\}^{\perp^d J}$ is a minimal prime filter containing $\{x\}^{\perp^d J}$ if and only if for $p \in Q$, there exists $q \in L - Q$ such that $p \vee q \in \{x\}^{\perp^d J}$.

Proof: Let Q be a prime filter containing $\{x\}^{\perp^d J}$ such that the given condition holds. Let K be a prime filter containing $\{x\}^{\perp^d J}$ such that $K \subseteq Q$. Let $p \in Q$. Then there is $q \in L - Q$ such that $p \vee q \in \{x\}^{\perp^d J}$. Hence $p \vee q \in K$. Since K is prime and $q \notin K$, so $p \in K$. Thus, $Q \subseteq K$ and so $K = Q$. Therefore, Q must be a minimal prime filter containing $\{x\}^{\perp^d J}$.

Conversely, let Q be a minimal prime filter containing $\{x\}^{\perp^d J}$. Let $p \in Q$. Suppose for all $q \in L - Q$, $p \vee q \notin \{x\}^{\perp^d J}$. Let $I = (L - Q) \vee (p]$. We claim that $\{x\}^{\perp^d J} \cap I = \varphi$. If not, let $y \in \{x\}^{\perp^d J} \cap I$. Then $y \in \{x\}^{\perp^d J}$ and $y \leq p \vee q$. Thus $p \vee q \in \{x\}^{\perp^d J}$, which is a contradiction to the assumption. Then by Theorem 11, there exists a maximal (prime) ideal $P \supseteq I$ and disjoint to $\{x\}^{\perp^d J}$. Let $M = L - P$. Then M is a prime filter containing $\{x\}^{\perp^d J}$. Now $M \cap I = \varphi$.

This implies $M \cap (L - Q) = \varphi$ and hence $M \subseteq Q$. Also $M \neq Q$, because $p \in I$ implies $p \notin M$ but $p \in Q$. Hence M is a prime filter containing $\{x\}^{\perp^d J}$ which is properly contained in Q. This gives a contradiction to the minimal property of Q. Therefore the given condition holds.

3. Conclusion

The results of this paper can be extended for the filters in a join semi lattice directed below. Using these results, one can study the semi prime filters in join semi lattices by proving several characterizations and prime separation theorem.

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