

Variants of Cyclic Three Dimensional Matching Problems

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Abstract. The variants of Cyclic Three Dimensional Matching problem is introduced in this paper, which studies about matching elements of three sets of elements. The problem is solved using solid assignment technique, considering the Preference value of members. The Reduction method is applied to the assignment table and the findings were discussed with the real life problems for better understanding of the readers.

Keywords: Preference value, Preference value Matrix, Solid assignment table, Reduction Method.

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1. Introduction

An instance of the Stable Marriage problem(SM) [1] consists of n men and n women each of whom has a list that ranks, in strict order of preference, those individuals of the opposite sex who are acceptable to him/her as a partner [2]. A matching M for the instance is a set of mutually acceptable man-woman pairs such that every individual appears in at most one pair. If $(m,w) \in M$, then we say that m and w are partners in M . A man m' and a woman w' are said to form a blocking pair for M , if each of them either (i) is unmatched in M and finds the other acceptable, or (ii) prefers the other over his/her partner in M . If no blocking pair exists for M , then M is called a Stable matching. A well-known result by Gale and Shapley states that every instance of SM has a stable matching [3, 4].

A number of variations of SM have been discussed in [4]. A solution to matching a elements of two sets using SMA algorithm is explained in [6]. A variations of matching problems, solution by SMA, is discussed in [7]. Matching elements of three sets (Cyclic Three Dimensional Matching Problem) using reduction method is discussed in [8]. The variants of CTDM is introduced in this study.

2. Cyclic 3-dimensional matching problem

The classical cyclic Three-dimensional matching problem (CTDM) instance consists of

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three finite equal sized set of members, namely set A, set B and set C. Each members of set A prefer members of set B in strict order, forming its preference list. Similarly each members of set B prefer members of set C in strict order, forming its preference list and each members of set C prefer members of set A in strict order, forming its preference list. The members of set A are matched with the members of set C through members of set B by applying Reduction method. In real life problems, the preference list may not be strict and complete. The Variations in the preference list are incomplete lists, ties in the list and both. The solution to variants of CTDM is explained with real life problem in the next section.

The Related terminologies, Preference value, Preference value matrix are discussed in [6] and Solid assignment problem and Reduction method are discussed in [5].

Example 1. Consider CTDM instance with three applicants a_1, a_2, a_3 three Qualifications q_1, q_2, q_3 and three Posts p_1, p_2, p_3 . The three Applicants ranks three posts, Qualification for three posts and Applicant for three Qualifications are ranked and preference lists are given below in the order of preference.

Posts for Applicant:

a_1 : p_2, p_3
 a_2 : $(p_1, p_2), p_3$
 a_3 : p_2, p_1, p_3

Qualification for Posts:

p_1 : q_1, q_3, q_2
 p_2 : q_2, q_1
 p_3 : $q_3, (q_2, q_1)$

Applicant for Qualification:

q_1 : $(a_1, a_3), a_2$
 q_2 : a_1, a_2, a_3
 q_3 : a_3, a_1

The Satisfactory Value Matrix (Posts for Applicant) is

$$SM_{M(P-A)} = \begin{matrix} & \begin{matrix} p_1 & p_2 & p_3 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} & \begin{pmatrix} - & \frac{3}{3} & \frac{2}{3} \\ \frac{10}{3} & \frac{10}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{3}{3} & \frac{1}{3} \end{pmatrix} \end{matrix}$$

The Satisfactory Value Matrix (Qualification for Posts) is

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$$SM_{M(Q-P)} = \begin{matrix} & q_1 & q_2 & q_3 \\ p_1 & \left(\begin{array}{ccc} \frac{3}{3} & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{3}{3} & - \\ \frac{1}{2} & \frac{1}{2} & \frac{3}{3} \end{array} \right) \\ p_2 & \\ p_3 & \end{matrix}$$

The Satisfactory Value Matrix (Applicant for Qualification) is

$$SM_{M(A-Q)} = \begin{matrix} & a_1 & a_2 & a_3 \\ q_1 & \left(\begin{array}{ccc} \frac{10}{3} & \frac{1}{3} & \frac{10}{3} \\ \frac{3}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & - & \frac{3}{3} \end{array} \right) \\ q_2 & \\ q_3 & \end{matrix}$$

The assignment table is

Qualification	q ₁			q ₁			q ₁		
		q ₂			q ₂			q ₂	
			q ₃			q ₃			q ₃
Applicant/Posts	p ₁			p ₂			p ₃		
a ₁	13/3	4/3	4/3	5	3	5/3	9/2	13/6	7/3
a ₂	14/3	13/3	4	13/3	5	10/3	7/6	3/2	4/3
a ₃	5	4/3	7/3	5	7/3	2	25/6	7/6	7/3

The Reduction method is applied to the above assignment table and matching is found.

The matching is $a_1 \xrightarrow{q_2} p_2$, $a_2 \xrightarrow{q_3} p_1$ and $a_3 \xrightarrow{q_1} p_3$ and the total maximum assignment cost is 23/3. The satisfactory level of Applicant, Posts and Qualification is 26.9%, 35.8% and 37.3 % respectively.

3. Conclusion

In this paper, Variants of classical Cyclic Three Dimensional Matching problem is introduced. An algorithm namely, Reduction method is applied to Cyclic Three Dimensional Matching problem. The proposed method can help decision makers in the matching problem related issues of real life problems by aiding them in the decision making process and providing an optimal solution in a simple and effective manner.

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