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Optimal Solution of an Intuitionistic Fuzzy Transportation Problem

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Abstract. The emerging area of research in fuzzy set theory in this decade is intuitionistic fuzzy set (IFS). IFS is an extension of fuzzy set. IFS has many applications in various areas. This paper deals with the optimal solution of an intuitionistic fuzzy transportation problem whose quantities are intuitionistic triangular fuzzy numbers.

Keywords: Intuitionistic fuzzy set, intuitionistic triangular fuzzy number

AMS Mathematics Subject Classification (2010): 03E72, 03F55, 90B06

1. Introduction

Intuitionistic fuzzy sets have been introduced by Atanassov (1983) as an extension of Zadeh's notion of fuzzy set, which itself extends the classical notion of a set. In recent years researchers have become interested to deal with the complexity of uncertain data. Semantic representation of intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness.

Intuitionistic fuzzy set is a tool in modelling real life problems like sale analysis, new product marketing, financial services, negotiation process, psychological investigations etc., since there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object. Intuitionistic fuzzy set has greater influence in solving transportation problem to find the optimal solution in which the cost, supply and demand are fuzzy numbers.

Paul et al. [8] proposed a new method for solving transportation problem using triangular intuitionistic fuzzy number. Gani et al. [2] introduced revised distribution method for solving intuitionistic fuzzy transportation problem. Geetharamani et al., [5] introduced an innovative method to solve fuzzy transportation problem via. Robust ranking. Kumar et al., [9] solved balanced intuitionistic fuzzy assignment problem. Kumar et al., [6] gave systematic approach for solving mixed intuitionistic fuzzy transportation problem. Gani et al., [7] proposed a new method for solving intuitionistic fuzzy transportation problem. Hussain et al., [3] discussed algorithmic approach for solving intuitionistic fuzzy transportation problem.

The purpose of this paper is to find the least transportation cost of some commodities through a capacitated network, when the quantities are intuitionistic triangular fuzzy numbers. The methodology proposed by Geetharamani et al., [5] to solve fuzzy transportation problem is used to find the optimal cost. A numerical example is given.

2. Preliminaries

Definition 2.1. Fuzzy set. Let A be a classical set, $\mu_{\bar{A}}(x)$ be a function from A to [0,1]. A fuzzy set $\bar{A}$ with the membership function $\mu_{\bar{A}}(x)$ is defined by

$$\bar{A} = \{(x, \mu_{\bar{A}}(x)); x \in A, \mu_{\bar{A}}(x) \in [0,1]\}$$

Definition 2.2. Intuitionistic fuzzy set. Let X denote universe of discourse, then an intuitionistic fuzzy set $\bar{A}^I$ in X is given by $\bar{A}^I = \{(x, \mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x)) / x \in X\}$ where $\mu_{\bar{A}^I}(x)$, $\vartheta_{\bar{A}^I}(x)$: $X \to [0,1]$ are functions such that $0 \le \mu_{\bar{A}^I}(x) + \vartheta_{\bar{A}^I}(x) \le 1$ for all x∈ X. For each x the member ship function $\mu_{\bar{A}^I}(x)$ and $\vartheta_{\bar{A}^I}(x)$) represent the degree of membership and non-membership of the element x∈ X to A ⊂ X respectively.

Definition 2.3. Intuitionistic fuzzy number. An intuitionistic fuzzy subset $\bar{A}^I = \{(x, \mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x)) / x \in X\}$ of the real line R is called an intuitionistic fuzzy number if the following holds:

(i) There exists m∈ R, $\mu_{\bar{A}^I}(m)$ =1 and $\vartheta_{\bar{A}^I}(m)$ = 0, m is called the mean value of $\bar{A}^I$.

(ii) $\mu_{\bar{A}^I}$ is a continuous mapping from R to the closed interval [0,1] and for all x∈ R, the relation $0 \le \mu_{\bar{A}^I} + \vartheta_{\bar{A}^I} \le 1$ holds.

The membership and non-membership function of $\bar{A}^I$ is of the following form

$$\mu_{\bar{A}^I}(x) = \begin{cases} 0 & -\infty \prec x \le m-\alpha \\ f_1(x) & x \in [m-\alpha, m] \\ 1 & x = m \\ h_1(x) & x \in [m, m+\beta] \\ 0 & m+\beta \le x \le \infty \end{cases}$$

$$\vartheta_{\bar{A}^I}(x) = \begin{cases} 1 & -\infty \prec x \le m-\alpha' \\ f_2(x) & x \in [m-\alpha', m], 0 \le f_1(x) + f_2(x) \le 1 \\ 0 & x = m \\ h_2(x) & x \in [m, m+\beta'], 0 \le h_1(x) + h_2(x) \le 1 \\ 1 & m+\beta' \le x \le \infty \end{cases}$$

Here m is the mean value of $\overline{A}^I$, α and β are called left and right spreads of membership function $\mu_{\overline{A}^I}(x)$ respectively. α', β' represent left and right spreads of non -membership function $\vartheta_{\overline{A}^I}(x)$ respectively.

Definition 2.4. Triangular intuitionistic fuzzy number. A triangular intuitionistic fuzzy number $\overline{a}^I$ is an intuitionistic fuzzy subset in R with the following membership function $\mu_{\overline{a}^I}(x)$ and non-membership function $\vartheta_{\overline{a}^I}(x)$.

$$\mu_{\overline{a}^I}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2} & a_2 \le x \le a_3 \\ 0 & otherwise \end{cases} \qquad \vartheta_{\overline{a}^I}(x) = \begin{cases} \dfrac{a_2-x}{a_2-a_1'} & a_1' \le x \le a_2 \\ \dfrac{x-a_2}{a_3'-a_2} & a_2 \le x \le a_3' \\ 1 & otherwise \end{cases}$$

where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $\mu_{\overline{a}^I}(x)$, $\vartheta_{\overline{a}^I}(x) \le 0.5$ for $\mu_{\overline{a}^I}(x) = \vartheta_{\overline{a}^I}(x)$ for all x∈ X. Triangular intuitionistic fuzzy number $\overline{a}^I$ is denoted by $(a_{1,} a_2, a_3; a_1', a_2', a_3')$.

Operations on triangular intuitionistic fuzzy number

Let $\overline{a}^I$ = $(a_{1,} a_2, a_3; a_1', a_2, a_3')$ and $\overline{b}^I$ = $(b_{1,} b_2, b_3; b_1', b_2, b_3')$ be two triangular intuitionistic fuzzy numbers the arithmetic operations on $\overline{a}^I$ and $\overline{b}^I$ is given below:

Addition: $(a_{1,} a_2, a_3; a_1', a_2, a_3') + (b_{1,} b_2, b_3; b_1', b_2, b_3')$ =

$$(a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, a_3' + b_3')$$

Subtraction: $(a_{1,} a_2, a_3; a_1', a_2, a_3')$ - $(b_{1,} b_2, b_3; b_1', b_2, b_3')$ =

$$(a_1 - b_3, a_2 - b_2, a_3 - b_1; a_1' - b_3', a_2 - b_2, a_3' - b_1')$$

Multiplication: $(a_{1,} a_2, a_3; a_1', a_2, a_3')$ X $(b_{1,} b_2, b_3; b_1', b_2, b_3')$ =

$$(a_1b_1, a_2b_2, a_3b_3; a_1'b_1', a_2b_2, a_3'b_3')$$

Scalar Multiplication: k$(a_{1,} a_2, a_3; a_1', a_2, a_3')$ = $(ka_{1,} ka_2, ka_3; ka_1', ka_2, ka_3')$ if k > 0

= $(ka_3, ka_2, ka_1; ka_3', ka_2, ka_1')$ if k < 0

Defuzzification

We define a accuracy function H($\overline{a}^I$) = $\dfrac{(a_1 + 2a_2 + a_3) + (a_1' + 2a_2 + a_3')}{8}$ to defuzzify a given triangular intuitionistic fuzzy number.

3. Intuitionistic fuzzy transportation problem (IFTP)

Consider a transportation problem with m intuitionistic fuzzy (IF) origins and n IF destination. Let C_{ij} (i=1,2,…,m, j=1,2,…,n) be the cost of transporting one unit of the product from ith origin to jth destination .Let $\overline{a}_i^I$ (i= 1,2,..,m) be the quantity of commodity available at IF origin i. Let $\bar{b}_j^{\,I}$ (j= 1,2,..n) be the quantity of commodity needed of IF destination j. Let X_{ij} (i=1,2,…,m, j=1,2,…,n) is quantity transported from ith IF origin to jth IF destination.

Mathematical Model of Intuitionistic Fuzzy Transportation Problem is

$$\text{Minimize } \bar{Z}^I = \sum_{i=1}^{m}\sum_{j=1}^{n} \bar{x}^I_{ij}\bar{c}^I_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} \bar{x}^I_{ij} = \bar{a}_i^{\,I} \qquad i = 1,2,\ldots\ldots\ldots m$$

$$\sum_{i=1}^{m} \bar{x}^I_{ij} = \bar{b}_j^{\,I} \qquad j=1, 2,\ldots\ldots\ldots n$$

$$\bar{x}^I_{ij} \geq 0 \text{ for all i,j.}$$

Algorithm

Step 1: In the IFTP defuzzify the quantities of the problem and if any of the values are not integers, round off into integers.

Step 2: Select the minimum odd cost from all cost in the matrix. Suppose all the costs are even, multiply each column by 1/2.

Step 3: Subtract selected least odd cost only from odd cost in the matrix. Now there will be at least one zero and remaining all cost become even.

Step 4: Allocate minimum of supply / demand at the place of zero.

Step 5: After the allotment, multiply each column by 1/2.

Step 6: Again select minimum odd cost in the remaining column except zeros in that column.

Step 7: Go to step 3 and repeat step 4 and 5 till optimal solution are obtained.

Step 8: Finally total minimum cost is calculated as sum of the product of the cost and corresponding allocated value of supply/demand.

4. Numerical example

Consider an intuitionistic fuzzy transportation problem whose quantities are triangular intuitionistic fuzzy number.

	D1	D2	D3	D4	Supply
O1	(14,16,18; 12,16,20)	(0,1,2; -1,1,3)	(7,8,9; 6,8,10)	(11,13,15; 10,13,16)	(2,4,6; 1,4,7)
O2	(8,11,14; 7,11,15)	(3,4,5; 2,4,6)	(5,7,9; 4,7,10)	(8,10,12; 6,10,14)	(5,6,7; 4,6,8)
O3	(6,8,10; 5,8,11)	(13,15,17; 12,15,18)	(7,9,11; 6,9,12)	(1,2,3; 0,2,4)	(7,8,9; 5,8,11)
O4	(5,6,7; 4,6,8)	(11,12,13; 10,12,14)	(3,5,7; 1,5,9)	(12,14,16; 11,14,17)	(8,10,12; 6,10,14)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

By defuzzifying the quantities we get, $a_{11=}$ 16, a_{12} =1, a_{13} =8, a_{14} =13, a_{21} =11, a_{22} =4, a_{23} =7, a_{24} =10, a_{31} =8, a_{32} =15, a_{33} =9, a_{34} =2, a_{41} =6, a_{42} =12, a_{43} =5, a_{44} =14. Hence

	D1	D2	D3	D4	Supply
O1	16	1	8	13	(2,4,6;1,4,7)
O2	11	4	7	10	(5,6,7;4,6,8)
O3	8	15	9	2	(7,8,9;5,8,11)
O4	6	12	5	14	(8,10,12; 6,10,14)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

Since the minimum odd cost in the odd matrix is 1, subtract 1 from all the odd costs and allocate minimum of supply or demand to the cell where there is zero cost then delete the row or column.

	D1	D2	D3	D4	Supply
O1	16	0**(2,4,6; 1,4,7)**	8	12	(2,4,6; 1,4,7)
O2	10	4	6	10	(5,6,7; 4,6,8)
O3	8	14	8	2	(7,8,9; 5,8,11)
O4	6	12	4	14	(8,10,12; 6,10,14)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

Now all the cost is even, hence multiply all the cost by ½ and subtract the minimum of odd cost from all the odd cost.

	D1	D2	D3	D4	Supply
O2	5	2	3	5	(5,6,7; 4,6,8)
O3	4	7	4	1	(7,8,9; 5,8,11)
O4	3	6	2	7	(8,10,12; 6,10,14)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

	D1	D2	D3	D4	Supply
O2	4	2	2	4	(5,6,7; 4,6,8)
O3	4	6	4	0**(6,7,8; 5,7,9)**	(7,8,9; 5,8,11)
O4	2	6	2	6	(8,10,12; 6,10,14)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

Proceeding like this, we get

	D1	D2	D3	D4	Supply
O1	16	1**(2,4,6; 1,4,7)**	8	13	(2,4,6; 1,4,7)
O2	11	4	7**(5,6,7; 4,6,8)**	10	(5,6,7; 4,6,8)
O3	8	15**(-1,1,3; -4,1,6)**	9	2**(6,7,8; 5,7,9)**	(7,8,9; 5,8,11)
O4	6**(3,4,5; 2,4,6)**	12	5**(3,6,9; 0,6,12)**	14	(8,10,12; 6,10,14)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

	D1	D2	D3	D4	Supply
O1	(14,16,18; 12,16,20)	(0,1,2; -1,1,3) **(2,4,6; 1,4,7)**	(7,8,9; 6,8,10)	(11,13,15; 10,13,16)	(2,4,6; 1,4,7)
O2	(8,11,14; 7,11,15)	(3,4,5; 2,4,6)	(5,7,9; 4,7,10) **(5,6,7; 4,6, 8)**	(8,10,12; 6,10,14)	(5,6,7; 4,6,8)
O3	(6,8,10; 5,8,11)	(13,15,17; 12,15,18) **(-1,1,3; -4,1,6)**	(7,9,11; 6,9,12)	(1,2,3; 0,2,4) **(6,7,8; 5,7,9)**	(7,8,9; 5,8,11)
O4	(5,6,7; 4,6,8) **(3,4,5; 2,4,6)**	(11,12,13; 10,12,14)	(3,5,7; 1,5,9) **(3,6,9; 0,6,12)**	(12,14,16; 11,14,17)	(8,10,12; 6,10,14)
Demand	(3,4,5; 2,4,6)	(3,5,7; 1,5,9)	(10,12,14; 8,12,16)	(6,7,8; 5,7,9)	

Intuitionistic fuzzy optimum cost =

Min $\overline{Z}^I$ = (0,1,2;-1,1,3) x (2,4,6;1,4,7) +(5,7,9; 4,7,10) x (5,6,7;4,6,8)
+ (13,15,17; 12,15,18) x (-1,1,3; -4,1,6) + (1,2,3; 0,2,4) x (6,7,8;5,7,9)
+ (5,6,7; 4,6,8) x (3,4,5;2,4,6) + (3,5,7; 1,5,9) x (3,6,9;0,6,12)
= (42,129,248 ; -25,129,401).

5. Conclusion

In this paper, cost, supply and demand of an IFTP are considered as intuitionistic triangular fuzzy number. By defuzzifying, the costs are converted to crisp values and the optimum solution is obtained by the given methodology. This method is very helpful for the decision makers, since the methodology is very simple and takes less number of iterations. This method can also be applied for mixed intuitionistic fuzzy transportation problem.

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