Annals of Pure and Applied Mathematics
Vol. 8, No. 2, 2014, 59-65
ISSN: 2279-087X (P), 2279-0888(online)
Published on 17 December 2014
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Antimagic Total Labeling of r-Copies of Harary Graphs

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Received 10 September 2014; accepted 21 November 2014

Abstract. Let G = (V,E) be a simple (p,q)-graph. An (*a*,*d*) - *vertex antimagic total labeling* (VATL) of G is a bijection *f* : V(G) ∪ E(G) → { 1,2,… *p* +*q* } so that the set of vertex weights in G is given by W = {*w*(*v*): *v*∈ V } = {*a*,*a*+*d*,*a*+2*d*,…*a*+(q-1)*d* } where *a*, *d* are two fixed positive integers. In this paper, we examine the existence of the vertex antimagic total labeling of the Harary graph C_p^t and hence prove that *r* C_p^t admits antimagic total labeling as well as (*a*,*d*)-*vertex antimagic total labeling* where *r* > 0 and *t* > 0 are finite integers.

Keywords: Graph labeling, vertex antimagic total labeling, harary graphs

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

Graph theory is one of the famous branches in Mathematics which has attracted many research Scholars, by its rapid growth and wide range of applications. It is the study of discrete structures called graphs. A graph G = (V(G) , E(G)) consists of a non-empty set of vertices called the vertex set V(G) and a set of edges which are ordered or unordered pair of elements of V(G) called the edge set E(G) where each edge *e* in E(G) is assigned with a pair of vertices. Graphs can be used to represent almost every physical problem involving discrete objects and a relationship among them. For introduction of Graph theory and its basic concepts, there are various textbooks, among which Harary [3] and West [2] gives the fundamental concepts in a detailed manner.

Graph Labeling

The concept of labeling of graphs was introduced in the late 1960's by A.Rosa which is now a famous research topic in Graph theory. Graph Labelings are widely used in Coding theory, Wireless networks, Circuit designs, Communication technology, Radar, X-Ray Crystallography etc. Gallian [5] has made a complete dynamic survey of graph labelings.

Definition 1. Graph labeling is an assignment of integers to the vertices or edges or both under certain conditions. The three major classes of labelings are vertex labeling, edge labeling and total labeling. The vertex labeling of a graph G assigns labels to the vertices which induces a label for each edge. Similarly, edge labeling of a graph assigns labels to the edges which induces a label for each vertex while the total labeling assigns labels to both vertices and edges.

Definition 2. Let G be a connected graph with q edges. If the edges of G are labeled with distinct positive integers, such that for each vertex v , the sum of the labels of all edges incident with v is the same for all v, then such a labeling is called ***magic labeling*** and the graph G is called a magic graph. A general study of magic graphs has been given in [8].

Definition 3. Let G = (V,E) be a simple (p,q)-graph. An (***a,d***) - ***vertex antimagic total labeling*** (VATL) of G is a bijection $f : V(G) \cup E(G) \rightarrow \{ 1,2,\dots p+q \}$ so that the set of vertex weights in G is given by $W = \{w(v): v\in V \} = \{ a, a+d, a+2d, \dots a+(q-1)d \}$ where a, d are two fixed positive integers .
If $w(v)$ is distinct for all $v\in V$, then the labeling is called **anti magic total labeling**. Some significant antimagic labelings can be seen in [6] and [7].

Definition 4. For $t \geq 2$ and $p \geq 4$, a **Harary graph** C_p^t is a graph constructed from a cycle C_p by joining any two vertices at distance t in C_p.

2. Main results

In this section we examine the existence of the vertex antimagic total labeling of the Harary graph C_p^t and show that $r\,C_p^t$ admits antimagic total labeling as well as (a,d) - *vertex antimagic total labeling* where $r > 0$ and $t > 0$ are finite integers.
Vertex antimagic total labeling on Harary graghs was studied by C.Balbuena [1].

Theorem 1. [4] For any odd $p \geq 5$ and $t \geq 2$, C_p^t admits a $\left(\frac{13p+9}{2}, 3\right)$ - *vertex antimagic total labeling* .

Theorem 2. For any odd $p \geq 5$,$t \geq 2$ and $r > 0$, $r\,C_p^t$ is anti magic.

Proof: Let $|V(G)| = p$ and $|E(G)| = q$, then the vertex and edge sets of C_p^t are denoted by $V = \{ v_i : 1 \leq i \leq p \}$, $E = \{ v_i v_{i+1} : 1 \leq i \leq p \} \cup \{ v_i v_{i+t} : 1 \leq i \leq p \}$ where all indices are taken mod p.
Define the labeling $f : V \cup E \rightarrow \{ 1,2,\dots p+q \}$ as follows:

$f(v_i) = 2i - 1$, $1 \leq i \leq p$

$f(v_i v_{i+t}) = 2p - 2(i-1)$, $1 \leq i \leq p$

$$f(v_i v_{i+1}) = \begin{cases} \frac{1}{2}(5p+t-i+1), & for\ \ i = t, t+2, t+4, \dots, p-1 \\ 3p + \frac{1}{2}(t-i+1), & for\ \ i = t+1, t+3, t+5, \dots, p \end{cases}$$

For $i = 1,2,3,\dots,t-1$, $f(v_i v_{i+1})$ is defined in two cases:

Case 1. If t is even

$$f(v_i v_{i+1}) = \begin{cases} 2p + \frac{1}{2}(t-i+1), & for\ \ i = 1,3,5, \dots, t-1 \\ \frac{1}{2}(5p+t-i+1), & for\ \ i = 2,4,6, \dots, t-2 \end{cases}$$

Case 2. If t is odd

$$f(v_i v_{i+1}) = \begin{cases} 2p + \frac{1}{2}(t-i+1), & for\ \ i = 2,4,6,\dots,t-1 \\ \frac{1}{2}(5p+t-i+1), & for\ \ i = 1,3,5,\dots,t-2 \end{cases}$$

With this labeling f, we get that the vertex weights are given by

W = $\{w(v): v \in V\}$ = $\left\{\frac{13p+9}{2}, \frac{13p+15}{2}, \dots, \frac{19p+3}{2}\right\}$ with $a = \frac{13p+9}{2}$ and $d = 3$.

Thus f is a $\left(\frac{13p+9}{2}, 3\right)$ - *vertex antimagic total labeling*.

Now define the labeling g_1 as follows:

g_1^k: V($r\,C_p^t$) ∪ E($r\,C_p^t$) → { 1,2,… $r(p+q)$ } for the k th graph (copy) of $r\,C_p^t$ is given by

$g_1^k(v_i) = f(v_i) + 3p(r-k)$

$g_1^k(v_i v_{i+1}) = f(v_i v_{i+1}) + 3p(k-1)$

$g_1^k(v_i v_{i+t}) = f(v_i v_{i+t}) + 3p(k-1)$

Then we get that the vertex weights for the k th graph are given by

$$W_k = \left\{\frac{(6r+18k-11)p+9}{2}, \frac{(6r+18k-11)p+9}{2} + 3, \frac{(6r+18k-11)p+9}{2} + 6, \dots \frac{(6r+18k-11)p+9}{2} + 3(p-1)\right\}$$

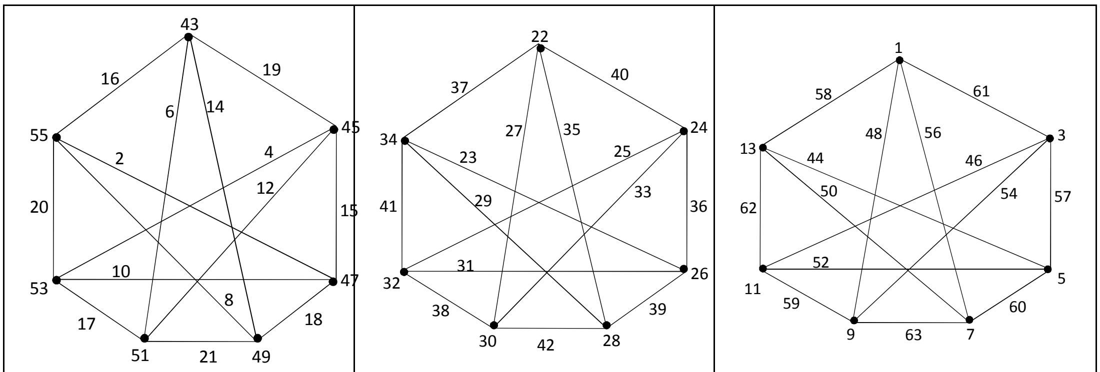
i.e, when this k th graph is considered separately as an individual graph, it has an $\left(\frac{(6r+18k-11)p+9}{2}, 3\right)$ *vertex antimagic total labeling* for $k = 1,2,3,\dots,r$.

Hence for any two distinct vertices v_i and v_j of $r\,C_p^t$, $w(v_i) \neq w(v_j)$.

Thus $r\,C_p^t$ is anti magic.

Example 1. Antimagic total labeling of 3 C_7^3

Theorem 3. For any odd $p \geq 5$, $t \geq 2$ and $r > 0$, $r\,C_p^t$ admits a $\left(\frac{(12r+1)p+9}{2}, 3\right)$ - *vertex antimagic total labeling* .



W_1 = { 92,95,98,…,110 } = $\left(\frac{25p+9}{2}, 3\right)$ VATL	**W_2 = { 155,158,161,…,173 } = $\left(\frac{43p+9}{2}, 3\right)$ VATL**	**W_3 = { 218,221,221,…,236 } = $\left(\frac{61p+9}{2}, 3\right)$ VATL**

Figure 1: 3 C_7^3

Proof: Consider the labeling f on C_p^t given in theorem 2.
Now define the labeling g_2 as follows:
g_2^k: V($r\ C_p^t$) $\cup$ E($r\ C_p^t$) $\rightarrow$ { 1,2,… $r(p+q)$ } for the k th graph (copy) of $r\ C_p^t$ is given by
$g_2^k(v_i) = f(v_i) + 3p(k-1)$, $g_2^k(v_i\, v_{i+t}) = f(v_i\, v_{i+t}) + 3p(k-1)$, $g_2^k(v_i\, v_{i+1}) = f(v_i\, v_{i+1}) + 3p(r-k)$
Then we get that the vertex weights for $r\ C_p^t$ are given by

$$W = \left\{\frac{((12r+1)p+9)}{2}, \frac{((12r+1)p+9)}{2} + 3, \frac{((12r+1)p+9)}{2} + 6, \ldots \frac{((12r+1)p+9)}{2} + 3(p-1)\right\}.$$

Thus $r\ C_p^t$ admits a $\left(\frac{(12r+1)p+9}{2}, 3\right)$ - *vertex antimagic total labeling* .

Example 2. $\left(\frac{\mathbf{61p+9}}{\mathbf{2}}, \mathbf{3}\right)$ - vertex antimagic total labeling of 5 C_5^2

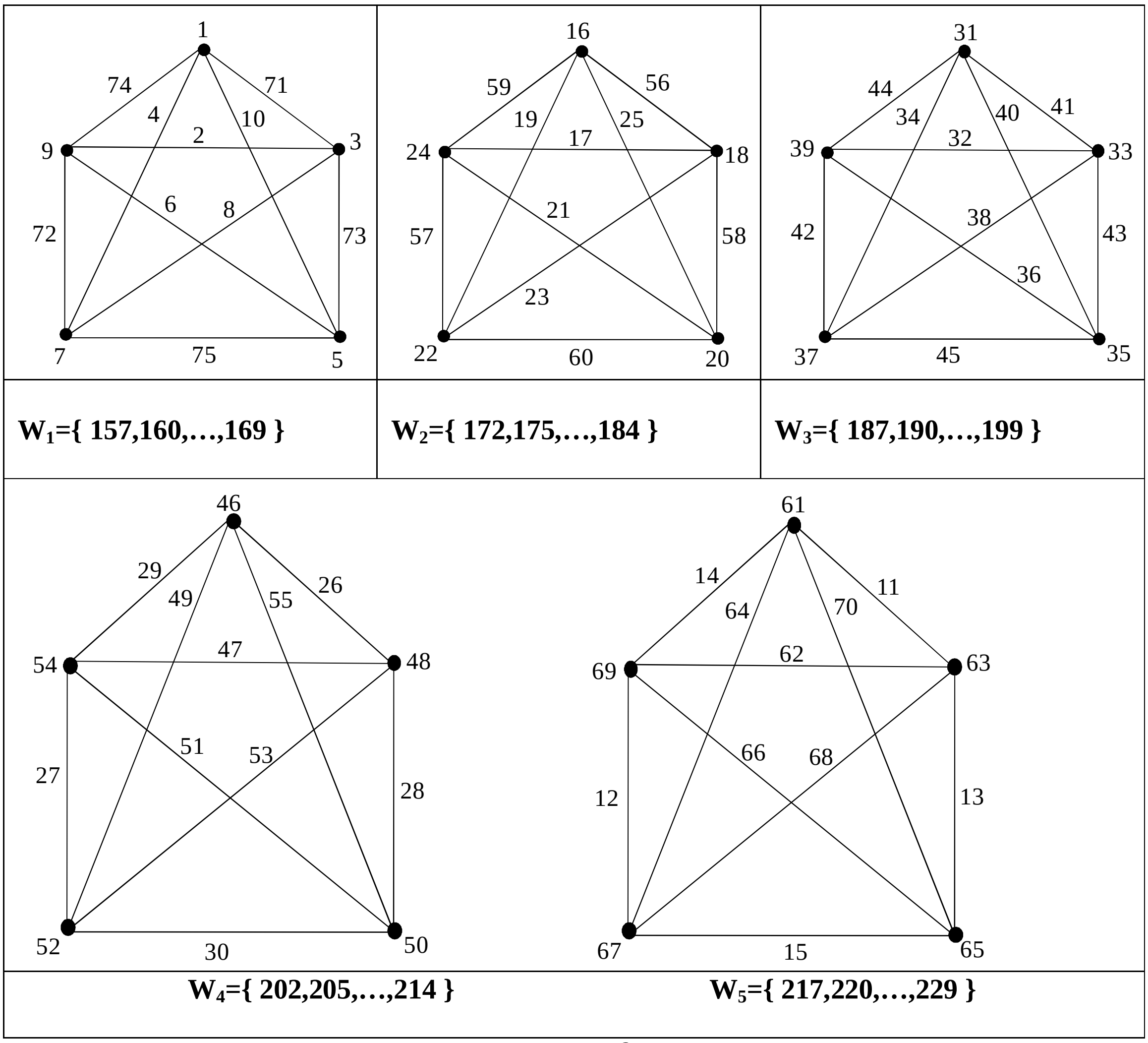


Figure 2: 5 C_5^2

Here $W = W_1 \cup W_2 \cup W_3 \cup W_4 \cup W_5 = \{157,160,\ldots,229\} = \left(\frac{61p+9}{2}, 3\right)$ -VATL

Theorem 4. [4] For $p \geq 5$ and $t \geq 2$, C_p^t admits a $(7p+3,1)$ - *vertex antimagic total labeling* provided that $p \neq 2t$.

Theorem 5. For $p \geq 5$, $t \geq 2$ and $r > 0$, rC_p^t is anti magic provided that $p \neq 2t$ with vertex weights given by $W_k = \{ (9k+3r-5)p+3, (9k+3r-5)p+4, \ldots, (9k+3r-5)p+p+2 \}$ for each k^{th} copy where $k = 1,2,3,\ldots,r$.

Proof: Let $|V(G)| = p$ and $|E(G)| = q$, then the vertex and edge sets of C_p^t are denoted by $V = \{ v_i : 1 \leq i \leq p \}$, $E = \{ v_i v_{i+1} : 1 \leq i \leq p \} \cup \{ v_i v_{i+t} : 1 \leq i \leq p \}$ where all indices are taken mod p.
Define the labeling $h : V \cup E \to \{ 1,2,\ldots p+q \}$ as follows:

$h(v_i) = p+i, \quad 1 \leq i \leq p$
$h(v_i v_{i+t}) = p+1-i, \quad 1 \leq i \leq p$
$$h(v_i v_{i+1}) = \begin{cases} 3p-t+i, & for\ 1 \leq i \leq t \\ 2p-t+i, & for\ t+1 \leq i \leq p \end{cases}$$
With this labeling h, we can observe that the vertex weights are given by
$W = \{w(v): v \in V\} = \{ 7p+3, 7p+4, 7p+5, \ldots, 8p+2\}$ with $a = 7p+3$ and $d = 1$.
Thus h is a $(7p+3,1)$ - *vertex antimagic total labeling* .
Now define the labeling g_3 as follows:
$g_3^k : V(r\,C_p^t) \cup E(r\,C_p^t) \to \{ 1,2,\ldots r(p+q) \}$ for the k^{th} graph (copy) of $r\,C_p^t$ is given by
$g_3^k(v_i) = h(v_i) + 3p(r-k)$
$g_3^k(v_i v_{i+1}) = h(v_i v_{i+1}) + 3p(k-1)$
$g_3^k(v_i v_{i+t}) = h(v_i v_{i+t}) + 3p(k-1)$
Then we get that the vertex weights for the k^{th} graph are given by
$W_k = \{ (9k+3r-5)p+3, (9k+3r-5)p+4, (9k+3r-5)p+5, \ldots (9k+3r-5)p+p+2\}$
i.e, when this k^{th} graph is considered separately as an individual graph, it has an
$((9k+3r-5)p+3, 1)$ - *vertex antimagic total labeling* for $k = 1,2,3,\ldots,r$.
Hence for any two distinct vertices v_i and v_j of $r\,C_p^t$, $w(v_i) \neq w(v_j)$.
Thus $r\,C_p^t$ is anti magic.

Theorem 6. For $p \geq 5$, $t \geq 2$ and $r > 0$, rC_p^t is anti magic provided that $p \neq 2t$ with vertex weights given by $W_k = \{(3k+6r-2)p+3, (3k+6r-2)p+4, \ldots, (3k+6r-2)p+p+2\}$ for each k^{th} copy where $k = 1,2,3,\ldots,r$.

Proof: Consider the labeling h on C_p^t given in theorem 5.
Now define the labeling g_4 as follows:
$g_4^k : V(r\,C_p^t) \cup E(r\,C_p^t) \to \{ 1,2,\ldots r(p+q) \}$ for the k^{th} graph (copy) of $r\,C_p^t$ is given by
$g_4^k(v_i) = h(v_i) + 3p(k-1)$
$g_4^k(v_i v_{i+t}) = h(v_i v_{i+t}) + 3p(k-1)$
$g_4^k(v_i v_{i+1}) = h(v_i v_{i+1}) + 3p(r-k)$
Then we get that the vertex weights for the k^{th} graph are given by
$W_k = \{ (3k+6r-2)p+3, (3k+6r-2)p+4, (3k+6r-2)p+5, \ldots (3k+6r-2)p+p+2 \}$

i.e. when this k^{th} graph is considered separately as an individual graph, it has an ($(3k+6r-2)p+3$, 1) - *vertex antimagic total labeling* for k = 1,2,3,…,r.
Hence for any two distinct vertices v_i and v_j of $r\ C_p^t$, $w(v_i) \neq w(v_j)$.
Thus $r\ C_p^t$ is anti magic.

Example 3. Antimagic total labeling of 4 C_7^3

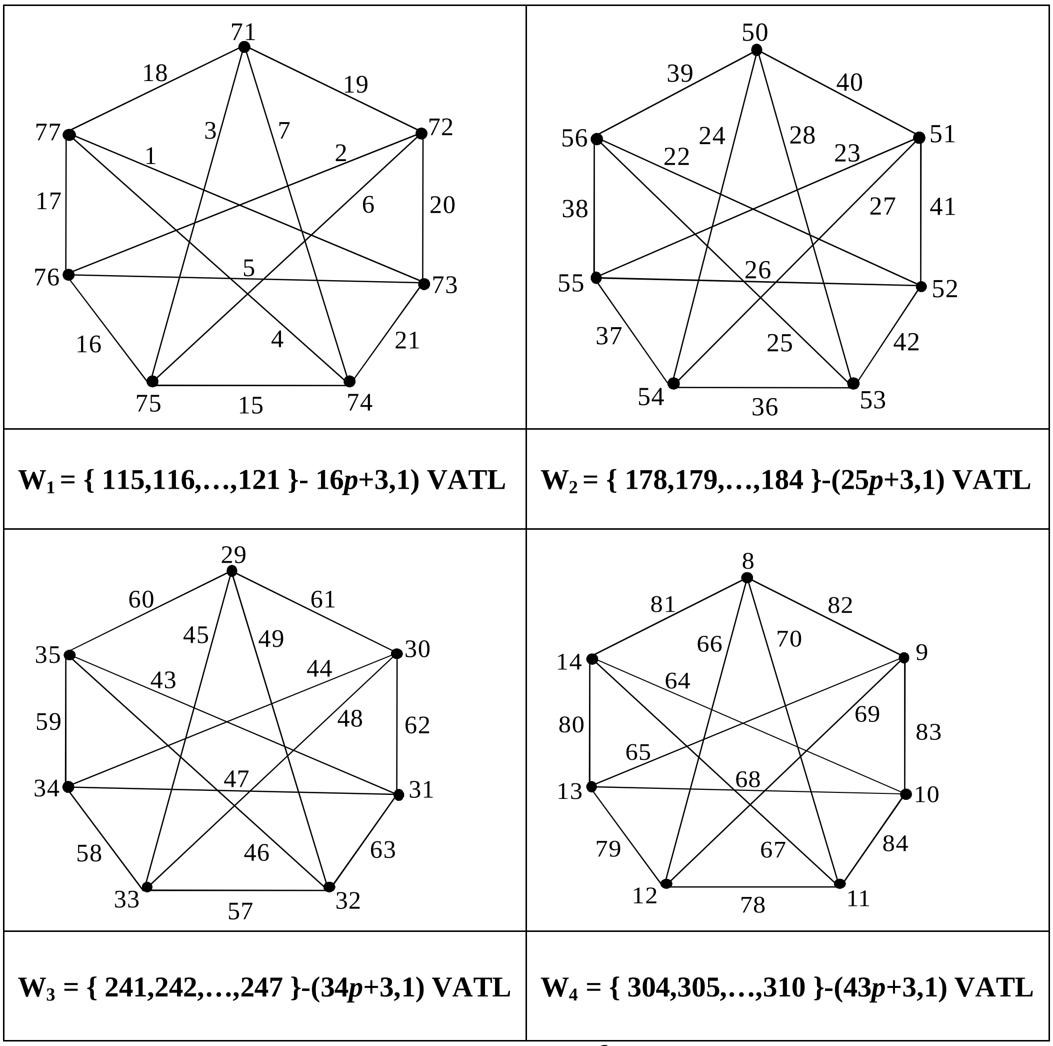
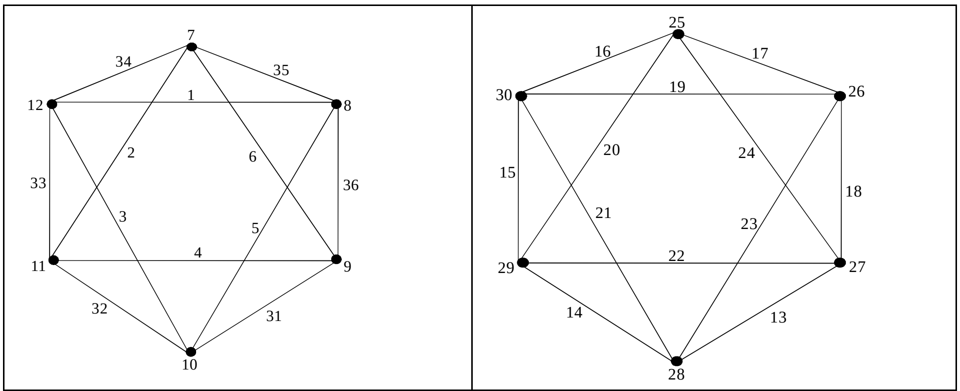


Figure 3: 4 C_7^3

Example 4. Antimagic total labeling of 2 C_6^2



W_1={81,82,…,86}–(13*p*+3,1) VATL	**W_2={99,100,…,104}–(16*p*+3,1) VATL**

Figure 4: 2 C_6^2

3. Conclusion

We have studied some anti magic labelings of *r* copies of Harary graphs. There are many other open problems in this area which we are working with.

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