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Solving Fully Fuzzy Linear Systems with Trapezoidal Fuzzy Number Matrices by Partitioning the Block Matrices

N.Jayanth Karthik[1]* and *E.Chandrasekaran[2]

[1]Department of Mathematics, D.G.Vaishnav College, Chennai 600106, India
Email : n.jayanthkarthik@yahoo.com
[2]Department of Mathematics, Presidency College, Chennai 600005, India
Email:e_chandrasekaran@yahoo.com

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Abstract. In this paper, a*n x n* fully fuzzy linear system $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A}$ is a fuzzy matrix$\tilde{x}$ and $\tilde{b}$ are fuzzy vectors with trapezoidal fuzzy numbers is solved by converting theminto block matrices and there by partitioning into smaller sub-matrices. The method is illustrated with a numerical example.

Keywords: Fully fuzzy linear system, trapezoidal fuzzy number matrices, block matrices

AMS Mathematics Subject Classification (2010): 08A72, 15B15

1. Introduction

Linear system of equation has applications in many areas of science, engineering, finance and economics. Fuzzy linear system whose coefficient matrix is crisp and right hand side column is an arbitrary fuzzy number was first proposed by Friedman et al [5].

A linear system is called a fully fuzzy linear system (FFLS) if all coefficients in the system are all fuzzy numbers. Several methods based on numerical algorithms were used for solving fuzzy linear systems have been introduced by many authors [1,3,4,8]. Nasseri [9] investigated linear system of equations with trapezoidal fuzzy numbers using embedding approach. Kumar [2] solved FFLS with trapezoidal fuzzy numbers using row reduced echelon form. In [7,10] the solution of FFLS was obtained by converting the FFLS into a crisp block matrix.

In this paper a*n x n* FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A}$ is a fuzzy matrix $\tilde{x}$ and $\tilde{b}$ are fuzzy vectors with trapezoidal fuzzy numbers is solved by converting them into block matrices and there by partitioning into smaller sub-matrices.

This paper is organized as follows. Some basic definitions and results on fuzzy sets and trapezoidal fuzzy numbers are given in section 2. In section 3, the method of finding inverses of partitioned matricesis presented. Section 4 introduces the method to solve FFLS using trapezoidal fuzzy number matrices by partitioning the block matrices. Illustration with a numerical example is given in section 5. Section 6 ends this paper with conclusion.

2. Preliminaries

1. A fuzzy subset $\tilde{A}$ of R is defined by its membership function $\mu_{\tilde{A}}: R \to [0,1]$ which

assigns a real number $\mu_{\tilde{A}}$in the interval $[0,1]$ to each element $x \in R$ where the value of $\mu_{\tilde{A}}$ shows grade membership of x in Ã.

2. A trapezoidal fuzzy number denoted by Ã=(m, n, α, β)has the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \le \alpha \\ \dfrac{x-\alpha}{m-\alpha} & \alpha \le x \le m \\ 1 & m \le x \le n \\ \dfrac{\beta - x}{\beta - n} & n \le x \le \beta \\ 0 & x \ge \beta \end{cases}$$

3. A fuzzy number Ã is called positive (negative) denoted by Ã>0 (Ã<0), if its membership function $\mu_{\tilde{A}}$(x) satisfies $\mu_{\tilde{A}}$= 0, $\forall x \le 0 (\forall x \ge 0)$.

4. A Trapezoidal fuzzy number Ã = (m, n, α, β) is said to be zero trapezoidal fuzzy number if and only if m= 0, n =0, α =0, β=0.

5. Two fuzzy numbers $M = (m, n, \alpha, \beta)$and N =$(x, y, \gamma, \delta)$are equal if and only if . $m = x, n = y, \alpha = \gamma, \beta = \delta$.

6. For two fuzzy numbers $M = (m, n, \alpha, \beta)$and N =$(x, y, \gamma, \delta)$the operations extended addition, extended opposite and extended multiplication are

6.1 $M \oplus N = (m, n, \alpha, \beta) \oplus (x, y, \gamma, \delta) = (m + x, n + y, \alpha + \gamma, \beta + \delta)$

6.2 $-M = -(m, n, \alpha, \beta) = (-m, -n, \beta, \alpha)$

6.3 If M>0,N>0 then

$$M \otimes N = (m, n, \alpha, \beta) \otimes (x, y, \gamma, \delta) = (mx, ny, m\gamma + x\alpha, n\delta + y\beta)$$

6.4 For scalar multiplication

$$\lambda \otimes M = \lambda \otimes (m, n, \alpha, \beta) = \begin{cases} \lambda m, \lambda n, \lambda\alpha, \lambda\beta & \lambda \ge 0 \\ \lambda m, \lambda n, -\lambda\alpha, -\lambda\beta & \lambda < 0 \end{cases}$$

7. A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix if each element of $\tilde{A}$ is a fuzzy number. A fuzzy matrix $\tilde{A}$ is positive denoted by $\tilde{A}$>0 if each element of $\tilde{A}$ is positive. Fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})$ which is $n \ x \ n$ matrix can be represented such that $\tilde{a}_{ij} = (a_{ij}, b_{ij}, m_{ij}, n_{ij})$ where $\tilde{A} = (A, B, M, N)$ where $A = (a_{ij})$ $B = (b_{ij})$ $M = (m_{ij})$ $N = (n_{ij})$aren $x \ n$ crisp matrices.

8. Consider n x n fuzzy linear system of equations

$$(\tilde{a}_{11} \otimes \tilde{x}_1) \oplus (\tilde{a}_{12} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{1n} \otimes \tilde{x}_n) = \tilde{b}_1$$
$$(\tilde{a}_{21} \otimes \tilde{x}_1) \oplus (\tilde{a}_{22} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{2n} \otimes \tilde{x}_n) = \tilde{b}_2$$

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$$(\tilde{a}_{n1} \otimes \tilde{x}_1) \oplus (\tilde{a}_{n2} \otimes \tilde{x}_2) \oplus \dots \dots \dots \oplus (\tilde{a}_{nn} \otimes \tilde{x}_n) = \tilde{b}_n$$

The matrix of the above equation is $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$ where $1 \leq i, j \leq n$ is a n x n fuzzy matrix and $\tilde{x}_j$, $\tilde{b}_j \in F(R)$. This system is called fully fuzzy linear system. (FFLS).

3. Inverses of partitioned matrices.

In this paper, square partitioned matrices with two row blocks and two column blocks in the form $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ where A, B, C and D are square sub-matrices of same order are considered.

A square matrix partitioned such that its diagonal elements are square matrices, while all other elements are zero's is a called a block diagonal matrix.

A block diagonal matrix with two row blocks and two column blocks is of the form $\begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}$ where A and D are square sub-matrices.

Theorem 3.1. [6] If A and D are non singular then $\begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} & 0 \\ 0 & D^{-1} \end{bmatrix}$

Proof: Let $\begin{bmatrix} P & Q \\ R & S \end{bmatrix}$ be the inverse of given matrix $\begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}$ where all the sub matrices are of order $n\ x\ n$.

Then $\begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}\begin{bmatrix} P & Q \\ R & S \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ D & I_n \end{bmatrix}$

By matrix multiplication and comparing$= I_n$; $Q = 0$; $DR = 0$; $DS = I_n$

Since A and D are non singular $P = A^{-1}$; $= 0$; $R = 0$; $S = D^{-1}$.

$\begin{bmatrix} P & Q \\ R & S \end{bmatrix} = \begin{bmatrix} A^{-1} & 0 \\ 0 & D^{-1} \end{bmatrix}$

Theorem 3.2. [6] If C is non singular then $\begin{bmatrix} C & 0 \\ D & C \end{bmatrix}^{-1} = \begin{bmatrix} C^{-1} & 0 \\ -C^{-1}DC^{-1} & C^{-1} \end{bmatrix}$

Proof: Let $\begin{bmatrix} P & 0 \\ Q & S \end{bmatrix}$ be the inverse of given matrix $\begin{bmatrix} C & 0 \\ D & C \end{bmatrix}$ where all the sub matrices are of order $n\ x\ n$.

Then $\begin{bmatrix} C & 0 \\ D & C \end{bmatrix}\begin{bmatrix} P & 0 \\ Q & P \end{bmatrix} = \begin{bmatrix} I_n & 0 \\ D & I_n \end{bmatrix}$

By matrix multiplication and comparing $CP = I_n$; $DP + CQ = 0$

Since C is non singular $P = C^{-1}$

$$CQ = -DP = -DC^{-1} \Longrightarrow Q = -C^{-1}DC^{-1}$$

Hence $\begin{bmatrix} C & 0 \\ D & C \end{bmatrix}^{-1} = \begin{bmatrix} C^{-1} & 0 \\ -C^{-1}DC^{-1} & C^{-1} \end{bmatrix}$.

4. Solving FFLS using trapezoidal fuzzy number matrices by partitioning the block matrices.

For solving $n\ x\ n$ FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ where $\tilde{A} = (A, B, M, N)$, $\tilde{x} = (x, y, z, w)$ and $\tilde{b} = (b, g, h, k)$ are trapezoidal fuzzy number matrices.

$(A, B, M, N) \otimes (x, y, z, w) = (b, g, h, k)$

$(Ax, By, Az + Mx, Bw + Ny) = (b, g, h, k)$ (by 6.3)

$Ax = b$ (4.1)

$By = g$ (4.2)
$Az + Mx = h$ (4.3)
$Bw + Ny = k.$ (4.4)

The above system of linear equations can be written as $4n$ x $4n$crisp block matrix.

$$\begin{bmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ M & 0 & A & 0 \\ 0 & N & 0 & B \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} b \\ g \\ h \\ k \end{bmatrix} \qquad (4.5)$$

where $A, B, M,$ and N are n x n crisp matrices and x, y, z, w, b, g, h and karecrisp column vectors.

Let $C = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$, $D = \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix}$, $u = \begin{bmatrix} x \\ y \end{bmatrix}$, $v = \begin{bmatrix} z \\ w \end{bmatrix}$, $r = \begin{bmatrix} b \\ g \end{bmatrix}$ $and\ s = \begin{bmatrix} h \\ k \end{bmatrix}$. Then (4.5) may be partitioned and written as

$$\left(\begin{array}{cc|cc} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ \hline M & 0 & A & 0 \\ 0 & N & 0 & B \end{array}\right) \left(\begin{array}{c} x \\ y \\ \hline z \\ w \end{array}\right) = \left(\begin{array}{c} b \\ g \\ \hline h \\ k \end{array}\right) \Longrightarrow \begin{bmatrix} C & 0 \\ D & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} r \\ s \end{bmatrix} \Longrightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} C & 0 \\ D & C \end{bmatrix}^{-1} \begin{bmatrix} r \\ s \end{bmatrix}$$

$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} C^{-1} & 0 \\ -C^{-1}DC^{-1} & C^{-1} \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix}$(By theorem 3.2)

Now, $C^{-1} = \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix}$(By theorem 3.1)

$$-C^{-1}DC^{-1} = -\begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix} \begin{bmatrix} M & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix} = -\begin{bmatrix} A^{-1}MA^{-1} & 0 \\ 0 & B^{-1}NB^{-1} \end{bmatrix}$$

Hence,

$$\left(\begin{array}{c} x \\ y \\ \hline z \\ w \end{array}\right) = \left(\begin{array}{cc|cc} A^{-1} & 0 & 0 & 0 \\ 0 & B^{-1} & 0 & 0 \\ \hline -A^{-1}MA^{-1} & 0 & A^{-1} & 0 \\ 0 & -B^{-1}NB^{-1} & 0 & B^{-1} \end{array}\right) \left(\begin{array}{c} b \\ g \\ \hline h \\ k \end{array}\right) \qquad .(4.6)$$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} A^{-1}b \\ B^{-1}g \\ -A^{-1}MA^{-1}b + A^{-1}h \\ -B^{-1}NB^{-1}g + B^{-1}k \end{pmatrix}$$

Another approach for solving FFLS using trapezoidal fuzzy number matrices.

From (4.1) and (4.3)

$$\begin{bmatrix} A & 0 \\ M & A \end{bmatrix}\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} b \\ h \end{bmatrix} \tag{4.7}$$

From (4.2) and (4.4)

$$\begin{bmatrix} B & 0 \\ N & B \end{bmatrix}\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} g \\ k \end{bmatrix} \tag{4.8}$$

where $A, B, M,$ and N are n x n crisp matrices and x, y, z, w, b, g, h and kare crisp column vectors.

(4.7) and (4.8) are *2n* x *2n* crisp block matrices and can be solved by partitioning

$$.\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} A & 0 \\ M & A \end{bmatrix}^{-1}\begin{bmatrix} b \\ h \end{bmatrix}$$

$$\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} A^{-1} & 0 \\ -A^{-1}MA^{-1} & A^{-1} \end{bmatrix}\begin{bmatrix} b \\ h \end{bmatrix} \quad \text{(using theorem 3.2)}$$

$$= \begin{bmatrix} A^{-1}b \\ -A^{-1}MA^{-1}b + A^{-1}h \end{bmatrix} \tag{4.9}$$

$$\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} B & 0 \\ N & B \end{bmatrix}^{-1}\begin{bmatrix} g \\ k \end{bmatrix}$$

$$\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} B^{-1} & 0 \\ -B^{-1}NB^{-1} & B^{-1} \end{bmatrix}\begin{bmatrix} g \\ k \end{bmatrix} \quad \text{(using theorem 3.2)}$$

$$= \begin{bmatrix} B^{-1}g \\ -B^{-1}NB^{-1}g + B^{-1}k \end{bmatrix} \tag{4.10}$$

Combining (4.9) and (4.10) the solution of (4.5) is

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} A^{-1}b \\ B^{-1}g \\ -A^{-1}MA^{-1}b + A^{-1}h \\ -B^{-1}NB^{-1}g + B^{-1}k \end{pmatrix}$$

5. Numerical example

Example 5.1. Consider the FFLS

$$\begin{bmatrix} (2,3,1,5) & (3,4,9,1) \\ (5,4,4,2) & (7,5,2,6) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (25,42,62,56) \\ (60,54,59,74) \end{bmatrix}$$

Solution: The given fully fuzzy linear system can be written as

$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix}$, $M = \begin{bmatrix} 1 & 9 \\ 4 & 2 \end{bmatrix}$, $N = \begin{bmatrix} 5 & 1 \\ 2 & 6 \end{bmatrix}$;

$b = \begin{bmatrix} 25 \\ 60 \end{bmatrix}$, $g = \begin{bmatrix} 42 \\ 54 \end{bmatrix}$, $h = \begin{bmatrix} 62 \\ 59 \end{bmatrix}$, $k = \begin{bmatrix} 56 \\ 74 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$; $B^{-1} = \begin{bmatrix} -5 & 4 \\ 4 & -3 \end{bmatrix}$

$$-A^{-1}MA^{-1} = -\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 1 & 9 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} -320 & -129 \\ -226 & 91 \end{bmatrix}$$

$$-B^{-1}NB^{-1} = -\begin{bmatrix} -5 & 4 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} -5 & 4 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} -161 & 125 \\ 126 & -98 \end{bmatrix}$$

$$\begin{pmatrix} x \\ y \\ \hline z \\ w \end{pmatrix} = \left(\begin{array}{c:c|c:c} A^{-1} & 0 & 0 & 0 \\ \hdashline 0 & B^{-1} & 0 & 0 \\ \hline -A^{-1}MA^{-1} & 0 & A^{-1} & 0 \\ \hdashline 0 & -B^{-1}NB^{-1} & 0 & B^{-1} \end{array}\right)\begin{pmatrix} b \\ g \\ \hline h \\ k \end{pmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ z_1 \\ z_2 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -7 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -5 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -3 & 0 & 0 & 0 & 0 \\ 320 & -129 & 0 & 0 & -7 & 3 & 0 & 0 \\ -226 & 91 & 0 & 0 & 5 & -2 & 0 & 0 \\ 0 & 0 & -161 & 125 & 0 & 0 & -5 & 4 \\ 0 & 0 & 126 & -98 & 0 & 0 & 4 & -3 \end{bmatrix}\begin{bmatrix} 25 \\ 60 \\ 42 \\ 54 \\ 62 \\ 59 \\ 56 \\ 74 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 6 \\ 6 \\ 3 \\ 2 \\ 4 \\ 2 \end{bmatrix}$$

Hence $\tilde{x} = \begin{bmatrix} (5,6,3,4) \\ (5,6,2,2) \end{bmatrix}$.

Solving by another approach

$$\begin{bmatrix} A & 0 \\ M & A \end{bmatrix}\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} b \\ h \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} A & 0 \\ M & A \end{bmatrix}^{-1}\begin{bmatrix} b \\ h \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} A^{-1} & 0 \\ -A^{-1}MA^{-1} & A^{-1} \end{bmatrix}\begin{bmatrix} b \\ h \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -7 & 3 & 0 & 0 \\ 5 & -2 & 0 & 0 \\ -320 & -129 & -7 & 3 \\ -226 & 91 & 5 & -2 \end{bmatrix}\begin{bmatrix} 25 \\ 60 \\ 62 \\ 59 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} B & 0 \\ N & B \end{bmatrix}\begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} g \\ k \end{bmatrix} \Rightarrow \begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} B & 0 \\ N & B \end{bmatrix}^{-1}\begin{bmatrix} g \\ k \end{bmatrix} \Rightarrow \begin{bmatrix} y \\ w \end{bmatrix} = \begin{bmatrix} B^{-1} & 0 \\ -B^{-1}NB^{-1} & B^{-1} \end{bmatrix}\begin{bmatrix} g \\ k \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -5 & 4 & 0 & 0 \\ 4 & -3 & 0 & 0 \\ -161 & 125 & -5 & 4 \\ 126 & -98 & 4 & -3 \end{bmatrix} \begin{bmatrix} 42 \\ 54 \\ 56 \\ 74 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 4 \\ 2 \end{bmatrix}$$

Hence $\tilde{x} = \begin{bmatrix} (5,6,3,4) \\ (5,6,2,2) \end{bmatrix}$.

6. Conclusion

In this paper an n x n FFLS $\tilde{A} \otimes \tilde{x} = \tilde{b}$ is transformed to a $4n$ x $4n$ crisp block matrix or two $2n$ x $2n$ crisp block matrices and solution is obtained bypartitioning the block matrices. When $n$is large, computing inverse for such cases become complicated and impractical because they consume much computer time and space. By partitioning, the present method is easier to implement as the order of the system reduces to sub-matrices of order n x n .

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