*Annals of Pure and Applied Mathematics*
*Vol. 8, No. 2, 2014, 251-260*
*ISSN: 2279-087X (P), 2279-0888(online)*
*Published on 17 December 2014*
www.researchmathsci.org

Annals of
Pure and Applied
Mathematics

# Fuzzy Chromatic Number of Line, Total and Middle Graphs of Fuzzy Complete Graphs

***S. Kavitha*[1] and *S. Lavanya*[2]**

[1]Research and Development Centre, Bharathiar University, Coimbatore – 641046 and Department of Mathematics, Chevalier T. Thomas Elizabeth College for Women Perambur, Chennai-600011,Tamilnadu, India. e-mail:kavithactte25@yahoo.com

[2]Department of Mathematics, D.G. Vaishnav College, Arumbakkam, Chennai-600 106 Tamilnadu, India. e-mail: lavanyaprasad1@gmail.com

*Received 2 October 2014; accepted 21 November 2014*

***Abstract.***In this paper vertex coloring in fuzzy graphs is defined as a family offuz zy sets satisfying some conditions. The fuzzy chromatic number for complete graphs, its fuzzy Line graph, middle fuzzy graph and total fuzzy graphs are foundand the results are summarized.

***Keywords:*** Fuzzy graph, Fuzzy complete graph, Fuzzy line graph, middle fuzzy graph, total fuzzy graph, Fuzzy chromatic number.

***AMS Mathematics Subject Classification (2010):*** 05C78

## 1. Introduction

Graph theory is rapidly moving into the mainstream of mathematics mainly because of its applications in diverse fields which include biochemistry (DNA double helix and SNP assembly problem), Chemistry (model chemical compounds) electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling) [2,3].

Graph coloring is one of the most important concepts in graph theory and is used in many real time applications like Job scheduling, Aircraft scheduling, computer network security, Map coloring and GSM mobile phone networks, Automatic channel allocation for small wireless local area networks[5].

The proper coloring of a graph is the coloring of the vertices with minimal number of colors such that no two adjacent vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph.

We know that graphs are simply model of relation. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. In many real world problems, we get partial information about that problem. So there is vagueness in the description of the objects or in its relationships or in both. To describe this type of relation, we need to design fuzzy graph model.

Fuzzy graph coloring is one of the most important problems of fuzzy graph theory. It uses in combinatorial optimization like traffic light control, exam-scheduling, register allocation etc. [12]. The first definition of a fuzzy graph was by Kaufmann in 1973, based on Zadeh's fuzzy relations [18]. But it was Rosenfeld [16] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. The fuzzy vertex coloring of a fuzzy graph was defined by the authors Eslahchi and Onagh [4] as family of fuzzy sets satisfying some conditions. Pal, Samanta and Rashmanlou did several works on fuzzy graphs and related its related graphs [10-28]..

In our paper, we consider a fuzzy graph G whose edge and vertices both are fuzzy. We study about the fuzzy chromatic number of the Total and middle fuzzy graphs is denoted by $\overline{\chi}_F(T(G))$ .

**2. Basic concepts**

In this section, we define the basic concepts of fuzzy set and fuzzy graphs.

**Definition 2.1.** A fuzzy set *A* defined on a non-empty set *X* is the family $A = \{(x, \mu_A(x))/x \in X\}$ where $\mu_A: x \to I$is the membership function. In fuzzy set theory the set *I* is usually defined as the interval [0,1] such that $\mu_A(x)=0$ if *x* does not belong to *A*, $\mu_A(x) = 1$ if *x* strictly belongs to *A* and any intermediate value represents the degree in which *x* could belong to *A*. The set I could be discrete set of the form $I = \{0,1,\ldots k\}$ where $\mu_A(x) < \mu_A(x_1)$ indicates that the degree of membership of *x* to *A* is lower than the degree of membership of $x_1$[18,19].

**Definition 2.2.** Fuzzy graph using their membership value of vertices and edges. Let *V* be a finite nonempty set. The triple G= (*V*, σ, μ) is called a fuzzy graph on *V* where μ and σ are fuzzy sets on *V* and *E* (*V* × *V*), respectively, such that $\mu(\{u, v\}) \le \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V$.

Note that a fuzzy graph is a generalization of crisp graph in which

$$\mu(v) = \begin{cases} 1 \text{ for all v } \in \text{ V and } \rho\text{ (i, j) } = \text{ 1 if (i, j) } \in \text{ E} \\ 0 \text{ otherwise} \end{cases}$$

so all the crisp graph are fuzzy graph but all fuzzy graph are not crisp graph.[7,11]

**Definition 2.3.** Two vertices *u* and *v* for any are called adjacent if $\mu(uv)>0$. An edge uv is said to be effective if $(1/2)\min\{\sigma(u), \sigma(v)\} \le \mu(uv) \le \min\{\sigma(u), \sigma(v)\}$[13,14,15]

**Definition 2.4.** Two edges *vivj*and *vjvk*are said to be incident if
$2 \min\{\mu(vivj), \mu(vjvk),\} \le \sigma(vj)$ for $j = 1,2\ldots|v|$ , $1 \le i, k \le |v|$.[13,14,15]

**Definition 2.5.** A family $\Gamma = \{\gamma_1, \ldots ,\gamma_k\}$ of fuzzy sets on *X* is called a *k*-fuzzy coloring of $G = (X,\sigma,\mu)$ if
(a) $\vee\Gamma = \sigma$,
(b) $\gamma i \wedge \gamma j = 0$,
(c) For every effective edge *xy*of *G*, $\min\{\gamma i(x),\gamma i(y)\} = 0$ $(1 \le i \le k)$.
The least value of *k* for which *G* has a *k*-fuzzy coloring, denoted by $\chi_F(G)$, is called the fuzzy chromatic numberof *G*[1].

The maximum value of *k* for which *G* has a *k*-fuzzy complete coloring, denoted by $\psi_F$(G), is called the fuzzy achromatic numberof *G*.

**Definition 2.6.** In fuzzy graph theory, a fuzzy graph G:(σ,μ) is complete if μ(uv) = σ(u) ∧σ(v) for all u,v∈ V where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ[8].

**Definition 2.7.** A family Γ = {γ1, γ2, … γk} of fuzzy sets on E(V× $V$) is called a k-fuzzy edge coloring of G = (μ,σ) if
(a) maxi γi(v) = μ (uv) for all uv∈ E
(b) γi∧γj = 0
(c) for every incident edges min{γi(vjvk)/vjvkare set of incident edges from the vertex vj, j =1, 2,… |v|}=0.i=1,2,….k
The least value of k for which G has a k-fuzzy edge coloring, denoted by $\chi_F^E$(G), is called the fuzzy edge chromatic number of G[9].

## 3. Line graph middle graph and total graph

**Definition 3.1. Fuzzy line graph.**
Given a graph G, its line graph L(G) is a graph L(G) is a graph G such that

- Each vertex of L (G) represents an edge of G; and
- Two vertices of L(G) are adjacent if and only if their corresponding edges share a common end point in G

That is, it is the intersection graph of the edges of G, representing each edge by the set of its two endpoints [17].

**Definition 3.2. Middle fuzzy graph.**
Let G: (σ, μ) be a fuzzy graph with vertex set V. M( G ) has vertex set V∪E.

$\sigma_M(u) = \sigma(u)$ if u∈ V ;
$\sigma_M(e) = \mu(e)$ if e∈ E
$\mu_M(v_i,v_j) = 0$ if $v_i,v_j \in V$
$\mu_M(e_i,e_j) = \mu(e_i) \wedge \mu(e_j)$ if $e_i$ and $e_j$ have a node in common between them.
$\mu_M(e_i,e_j) = 0$ otherwise.
$\mu_M(v_i,e_j) = \mu_M(e_j)$ if $v_i$ lies on the edge $e_j$
= 0 otherwise.
M(G): ($\sigma_M$, $\mu_M$) is a middle fuzzy graph of G[17].

**Definition 3.3. Total fuzzy graph.**
Let G: ( σ, μ ) be a fuzzy graph with vertex set V. T(G) has vertex set V∪E.

$\sigma_T(u) = \sigma(u)$ if u∈ V ;
$\sigma_T(e) = \mu(e)$ if e∈ E
$\mu_T(v_i,v_j) = \mu(v_i,v_j)$ if $v_i,v_j \in V$
$\mu_T(e_i,e_j) = \mu(e_i) \wedge \mu(e_j)$ if $e_i$ and $e_j$ have a node in common between them.
$\mu_T(e_i,e_j) = 0$ otherwise.
$\mu_T(v_i,e_j) = \sigma(v_i) \wedge \mu(e_j)$ if $v_i$ lies on the edge $e_j$

= 0 otherwise.

T(G): ($\sigma_{M,}$ $\mu_M$) is a total fuzzy graph of G[17].

## 4. Fuzzy chromatic number for complete fuzzy graphs, itsLINE, MIDDLE ANDtotal fuzzy graphs

In 1941 Brook's gave an upper bound for the chromatic number in case of crisp graphs and stated that 'For a connected graph which is neither an odd cycle nor a complete graph, the chromatic number $\chi(G) \leq \Delta(G)$[6].

**Theorem 4.1.** For a fuzzy graph G(σ,μ), the fuzzy chromatic number $\chi F(G) \leq \Delta^s(G)+1$ where $\Delta^s(G)$ is the maximum vertex strong degree of G[10].

**Corollary 4.1.1.** The fuzzy chromatic number of a fuzzy complete graph is n where n is the number of vertices of G, i.e.,$\chi_F(G)$= n[10].

(Since $\Delta^S$ =n-1 is the maximum vertex strong degree of the complete fuzzy graphG, $\chi_F(G)= \Delta^S+1$)

**Proof:** Since $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all x,y ∈ V, every pair of vertices are strongly adjacent and strong degree of each vertex is n-1. By (c) of definition 2.5min{γi(x),γi(y)} = 0 for strongly adjacent vertices x,y. Since all vertices are strongly adjacent, every member of the family defining fuzzy coloring have value for only one vertex and 0 for all other vertices. By (b) of definition2.5γi∧γj =0. So γi(xi)= σ(xi) i=1,2…n where n=|V|. Thus $\chi_F(G)$= n.

**Theorem 4.2.** For a fuzzy graph G(σ,μ), the edge chromatic number of complete fuzzy graph on n vertices is n, if n is odd and n-1, if n is even [10].

**Theorem 4.3.** $L_F(G) = (\lambda, \omega)$ is a fuzzy line graph of G(σ,μ) if and only if (Supp(λ), Supp($\omega$)) is a line graph and $\forall (u, v) \in Supp(\omega), \omega(u, v) = \lambda(u) \wedge \lambda(v)$ [10].

**Corollary 4.3.1.** For a fuzzy line graph $L_F(G)= (\lambda, \omega)$ the fuzzy chromatic number $\chi_F(L_G(k_n))$ =n, if n is odd and $\chi_F(L_G(k_n))$ =n-1, if n is even

**Proof:** By theorem 4.3, we have $, \omega(u, v) = \lambda(u) \wedge \lambda(v)$. for $uv \in w$ so for all $uv \in w$ with $\mu(u, v) > 0$ the vertices u and v are strongly adjacent. Thus the definition of fuzzy vertex coloring can be applied to the fuzzy line graph to obtain the fuzzy chromatic number which is nothing but the fuzzy edge chromatic number of the fuzzy graph G.

**Theorem 4.4.** The fuzzy chromatic number of a middle graph of a fuzzy complete graph is n where n is the number of vertices of G, i.e.,$\chi_F(M_G(k_n))$= n.

**Proof:** In a middle graph M(G), $\mu_M$ ( $v_{i,}v_{j,}$) = 0 if $v_{i,}v_j \in$ V.

Since there is no edge between any two vertices of the corresponding completegraph G, for each u∈ $V$, $\sigma_M$(u) can be included in the same member of the family Γ defining fuzzy coloring.

For any edge $e_i \in E_,$ there are two end nodes. The edges incident to those nodes and the edge $e_i$cannot be included in the same member ofΓ and for a complete graph there are $\frac{n-1}{2}$ edges and n-1 edges are incident to any vertex. So, the common $e_i$and n-2 edges incident to both the end nodes (ie) 2(n-2) = 2n-4 cannot be included in the same member of Γ.

Now, the remaining edges are $\frac{n(n-1)}{2} - [(2n-4)+1] = \frac{n^2-n-4n+6}{2} = \frac{n-5n+6}{2} = \frac{(n-2)(n-3)}{2}$

Then one of the remaining $\mu_M$ ($e_{j,}$) can be included in the same member of Γ as that of $e_i$.

This process is continued for n-1 times( Since it is the maximum degree). So there will be n-1 members to color all the vertices of M(G) corresponding to the edges of the graph G.

Therefore, these n-1 members of Γ of $\mu_M$ ($e_{i,}$) for all $e_i \in E$ together with the member of Γ that includes $\sigma_M$(u) for all u$\in V$ gives n-1+1, (ie) n members of Γ = $\{\gamma_1, \ldots, \gamma_n\}$ defining n- fuzzy coloring of M(G).

Hence $\chi_F$($M_G(k_n)$)= n.

**Theorem 4.5.** The fuzzy chromatic number of a Total graph of a fuzzy complete graph is 2n-1 where n is the number of vertices of G, i.e., $\chi_F$($T_G(k_n)$)= n+ $\Delta^S$

(Since $\Delta^S$ =n-1 is the maximum vertex strong degree of the complete fuzzy graph G, $\chi_F$($T_G(k_n)$)= 2n-1)

**Proof:** In a Total graph T(G), $\mu_T$ ( $v_{i,}v_{j,}$) = μ ( $v_{i,}v_{j,}$) if $v_{i,}v_j \in$ V.

Since there exist an edge between any two vertices of the corresponding complete graph G, for each u$\in V$, $\sigma_M$(u) cannot be included in the same member of the family Γ defining k- fuzzy coloring. So, there exist n members of Γ for coloring all n vertices.

For any edge $e_i \in E$, there are two end nodes. The edges incident to those nodes and the edge $e_i$cannot be included in the same member of Γ and for a complete graph there are $\frac{n-1}{2}$ edges and n-1 edges are incident to any vertex. So, the common $e_i$and n-2 edges incident to both the end nodes (ie) 2(n-2) = 2n-4 cannot be included in the same member of Γ.

Now, the remaining edges are $\frac{n(n-1)}{2} - [(2n-4)+1] = \frac{n^2-n-4n+6}{2} = \frac{n-5n+6}{2} = \frac{(n-2)(n-3)}{2}$

Then one of the remaining $\mu_M$ ($e_{j,}$) can be included in the same member of Γ as that of $e_i$. This process is continued for n-1 times( Since it is the maximum degree). So there will be n-1 members to color all the vertices of M(G) corresponding to the edges of the graph G.

Therefore, these n-1 members of Γ of $\mu_M$ ($e_{i,}$) for all $e_i \in E$ together with the n members of Γ that includes $\sigma_M$(u) for all u$\in V$ gives n-1+n, (ie) 2n-1 members of Γ = $\{\gamma_1, \ldots, \gamma_{2n-1}\}$ defining 2n- 1 fuzzy coloring of T(G).

Hence, $\chi_F$(($T_G(k_n)$)= 2n – 1.

## 5. Illustrations

### 5.1. Fuzzy chromatic number of complete fuzzy graph

Consider the fuzzy complete graph in fig -5.1.1 the fuzzy chromatic number of *G*, $\chi_F(k_4)$=4. Since there exists a family Γ = $\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ satisfying the definition of fuzzy coloring for fuzzy graph G is shown in table 5.1.2.

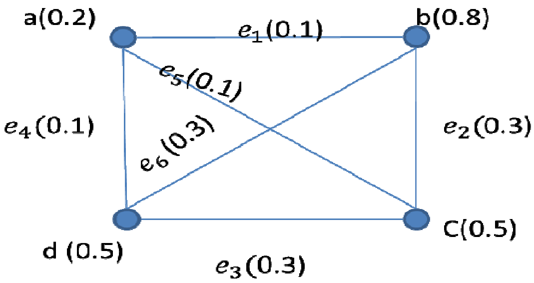


**Figure 5.1.1:**

| Vertices | | | | | Max |
|---|---|---|---|---|---|
| a | 0.2 | 0 | 0 | 0 | 0.2 |
| b | 0 | 0.8 | 0 | 0 | 0.8 |
| c | 0 | 0 | 0.5 | 0 | 0.5 |
| d | 0 | 0 | 0 | 0.5 | 0.5 |

**Table 5.1.2:**

## 5.2. Fuzzy chromatic number of fuzzy line graph

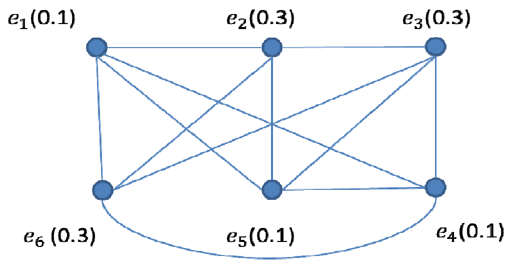


**Figure 5.2.1:**

Consider the fuzzy line graph of in fig - 5.2.1 the fuzzy chromatic number $\chi_F(L_G( \ ))$ =3 Since there exists a family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$ satisfying the definition of fuzzy coloring for fuzzy graph L( G) is shown in table 5.2.2.

| Vertices | | | | Max |
|---|---|---|---|---|
| $e_1$ | 0.1 | 0 | 0 | 0.1 |
| $e_2$ | 0 | 0.3 | 0 | 0.3 |
| $e_3$ | 0.3 | 0 | 0 | 0.3 |
| $e_4$ | 0 | 0.1 | 0 | 0.1 |
| $e_5$ | 0 | 0 | 0.1 | 0.1 |
| $e_6$ | 0 | 0 | 0.3 | 0.3 |

**Table 5.2.2:**

**5.3. Fuzzy chromatic number of middle fuzzy graph**

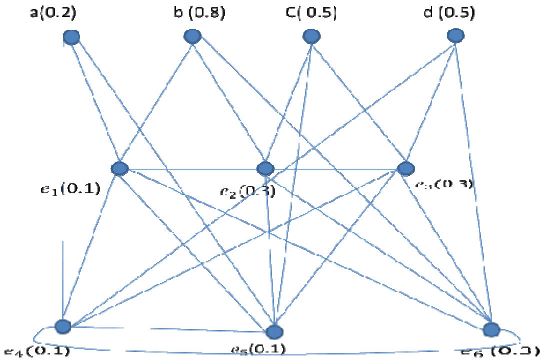


**Fig 5.3.1:**

Consider the middle fuzzy graph of in fig-5.3.1 the fuzzy chromatic number $(M_G( ))= 4$. Since there exists a family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3 \}$ satisfying the definition of fuzzy coloring for fuzzy graph M( G) is shown in table 5.3.2.

| Vertices | | | | | Max |
|---|---|---|---|---|---|
| a | 0.2 | 0 | 0 | 0 | 0.2 |
| b | 0.8 | 0 | 0 | 0 | 0.8 |
| c | 0.5 | 0 | 0 | 0 | 0.5 |
| d | 0.5 | 0 | 0 | 0 | 0.5 |
| $e_1$ | 0 | 0.1 | 0 | 0 | 0.1 |
| $e_2$ | 0 | 0 | 0.3 | 0 | 0.3 |
| $e_3$ | 0 | 0.3 | 0 | 0 | 0.3 |
| $e_4$ | 0 | 0 | 0.1 | 0 | 0.1 |
| $e_5$ | 0 | 0 | 0 | 0.1 | 0.1 |
| $e_6$ | 0 | 0 | 0 | 0.3 | 0.3 |

**Table 5.3.2:**

**5.4. Fuzzy chromatic number of total fuzzy graph**

Consider the Total fuzzy graph of in fig - 5.4.1 the fuzzy chromatic number $((T_G( ))=7$Since there exists a family $\Gamma = \{\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 \}$ satisfying the definition of fuzzy coloring for fuzzy graph T(G) ) is shown in table 5.4.2.

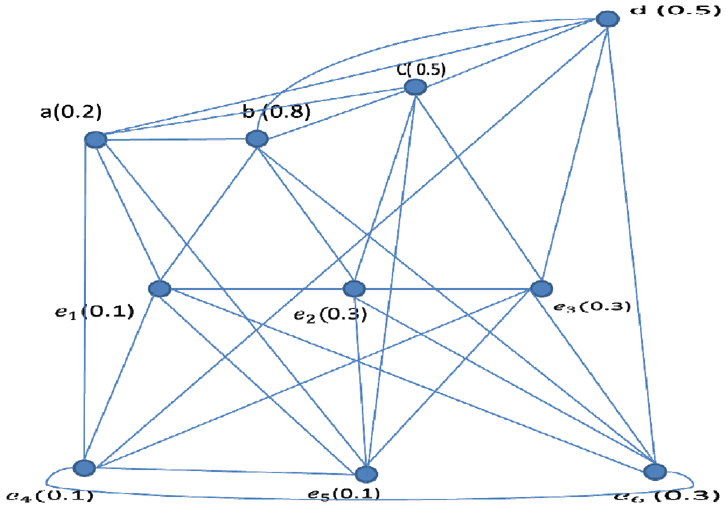


**Figure 5.4.1:**

| Vertices | | | | | | | | Max |
|---|---|---|---|---|---|---|---|---|
| A | 0.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0.2 |
| B | 0 | 0.8 | 0 | 0 | 0 | 0 | 0 | 0.8 |
| C | 0 | 0 | 0.5 | 0 | 0 | 0 | 0 | 0.5 |
| D | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0.5 |
| $e_1$ | 0 | 0 | 0 | 0 | 0.1 | 0 | 0 | 0.1 |
| $e_2$ | 0 | 0 | 0 | 0 | 0 | 0.3 | 0 | 0.3 |
| $e_3$ | 0 | 0 | 0 | 0 | 0.3 | 0 | 0 | 0.3 |
| $e_4$ | 0 | 0 | 0 | 0 | 0 | 0.1 | 0 | 0.1 |
| $e_5$ | 0 | 0 | 0 | 0 | 0 | 0 | 0.1 | 0.1 |
| $e_6$ | 0 | 0 | 0 | 0 | 0 | 0 | 0.3 | 0.3 |

**Table 4.2.3:**

**The results obtained**

**(i)** $\chi_F(L_G(\quad))$ =n, if n is odd and $\chi_F(L_G(\quad))$ =n-1, if n is even

**(i)** $(M_G(\quad))$= n.

**(ii)** $((T_G(\quad))$= 2n – 1

## REFERENCES

1. Anjaly Kishore and M.S.Sunitha, Strong chromatic number of fuzzy graphs, *Annals of Pure and Applied Mathematics*, 7(2) (2014) 52-60.

2. R.Balakrishnan and K.Ranganathan, *A Textbook of Graph Theory*, Springer-Verlag, New York, 2000.
3. J.A.Bondy and U.S.R.Murthy, *Graph Theory with Applications,* The Macmillan Press Ltd, 1976.
4. Eslahchi and B.N.Onagh, Vertex strength of fuzzy graphs, *International Journal of Mathematics and Mathematical Sciences,* 1–9 (2006) 53-72.
5. F.Harary, *Graph Theory*, Addison-Wesley Publishing Company, 1969.
6. J.N.Mordeson and P.S.Nair, Cycles and cocycles of fuzzy graphs, *International Journal of Information Sciences,* 90 (1996) 39-49.
7. J.N.Mordeson and P.S.Nair, *Fuzzy Graphs and Fuzzy hyper graphs*, PhysicaVerlag, Heidelberg, 1998.
8. J.N.Mordeson and C.S.Peng, Operations on fuzzy graphs, *International Journal of Information Sciences,* 79 (1994) 159- 170.
9. S.Lavanya and R Sattanathan, Fuzzy Total coloring of fuzzy graphs, *International Journal of Information Technology and Knowledge Management,* 2(2009) 37-39.
10. S.Lavanya and R.Sattanathan, Fuzzy vertex coloring of fuzzy graphs, *Inter. Review of Fuzzy Mathematics*, 6 (3) (2011) 53-61.
11. M.Tom and M.S.Sunitha, Sum distance in fuzzy graphs, *Annals of Pure and Applied Mathematics,* 7(2) (2014) 73-89.
12. T.Munoz, J.Ortuno, J.Ramirez and J.Yanez, Coloring fuzzy graphs, *The International Journal of Management Sciences,* 33(2005) 211-221.
13. A.Nagoorgani and M.B.Ahamed, Order and size of fuzzy graphs, *Bulletin of Pure and Applied Sciences*, 22 (2003) 145-148.
14. A.Nagoorgani and V.T.Chandrasekaran, *A First look at Fuzzy graph Theory,* Allied Publishers Pvt. Ltd, 2010.
15. A.Nagoorgani and K.Radha, The degree of a vertex in some fuzzy graphs, *International Journal of Algorithms, Computing and Mathematics*, 2 (2009) 107-116.
16. A.Roseneld, *Fuzzy graphs, in:* L.A.Zadeh, K.S. Fu,, K .Tanaka, M.Shimura (Eds), *Fuzzy sets and Their Applications to Cognitive and Decision Processes,* Academic Press, New York, (1975) 77-95.
17. M.S.Sunitha and Sunil Mathew, Fuzzy graph theory: a survey, *Annals of Pure and Applied Mathematics*, 4(1) (2013) 92-110.
18. L.A.Zadeh, *Fuzzy sets, Information and Control*, 8(1965) 338-353.
19. H.J.Zimmermann, *Fuzzy Set Theory and its Applications*, Kluwer-Nijhoff, Boston, 1985.
20. S.Samanta and M.Pal, Fuzzy tolerance graphs**,** *International Journal of Latest Trends in Mathematics*, 1(2) (2011) 57-6.
21. S.Samanta and M.Pal, Bipolar fuzzy hypergraphs, *International Journal of Fuzzy Logic Systems,* 2(1) (2012) 17-28.
22. S.Samanta and M.Pal, Irregular bipolar fuzzy graphs**,** *International Journal of Applications of Fuzzy Sets,* 2 (2012) 91-102.
23. S.Samanta and M.Pal, fuzzy *k*-competition graphs and *p*-competition fuzzy graphs, *Fuzzy Inf. Eng., Elsevier,* 5(2) (2013) 191-204
24. S.Samanta and M.Pal, Fuzzy threshold graphs, *CiiT International Journal of Fuzzy Systems,* 3(9)(2011) 360-364.

25. H.Rashmanlou and M.Pal, Isometry on Interval-valued fuzzy graphs, *Intern. J. Fuzzy Mathematical Archiv*e, 3 (2013) 28-35.
26. H.Rashmanlou and M.Pal, Balanced interval-valued fuzzy graphs, *Journal of Physical Sciences*, 17 (2013) 43-57.
27. H.Rashmanlou and M.Pal, Antipodal interval-valued fuzzy graphs, *International Journal of Applications of Fuzzy Sets and Artificial Intelligence,* 3 (2013) 107-130.
28. H.Rashmanlou and M.Pal, Some properties of highly irregular interval valued fuzzy graphs, *World Applied Sciences Journal,* 27(12) (2013) 1756-1773.