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# Time Minimizing Fuzzy Transportation Problem

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***Abstract*.** This paper shows a procedure for solving the time Minimizing fuzzy transportation problem by assuming that a decision-maker is uncertain about the precise value of the fuzzy transportation time, fuzzy source and fuzzy destination parameters. These parameters have been expressed as generalized non-normal p-norm trapezoidal fuzzy numbers. A numerical example illustrating the method is included.

***Keywords:*** Time Minimizing – fuzzy transportation problem – generalized non-normal p-norm trapezoidal fuzzy numbers – signed distance

***AMS Mathematics Subject Classification*:** 90C08, 90C90

## 1. Introduction

The Time Minimizing fuzzy transportation problem is more important than the fuzzy cost factor. Fuzzy Transportation model plays a vital role to ensure the efficient movement and in-time availability of raw materials and finished goods from fuzzy sources and fuzzy destinations. For instance, the time of despatch of the military personnel, equipment's, food, medicine etc. are to be sent from their basis to fronts. The objective of the fuzzy transportation is to determine the shipping schedule that minimizes the total shipping fuzzy cost while satisfying the fuzzy demand and fuzzy supply limit was given by Kantiswarup et al. [9]. If we are able to minimize the fuzzy transportation time, fuzzy transportation cost comes down naturally. In literature the time minimizing fuzzy transportation has already been studied by Arora and Puri[1], Burkand et al. [3], Garfrinkle and Rao [6], Hammer [7], Iserman [8], Szwarc [13], Mathur and Puri [11] and Bhatia et al. [2] and others. Theoretical approach of Multi-Objective programming and goal programming are shown in [14].

In this paper the fuzzy transportation time, fuzzy supply and fuzzy demand is expressed as generalized non-normal p-norm trapezoidal fuzzy numbers which is solved by the ranking function signed distance. A numerical example illustrating the method is also given.

## 2. Preliminaries

### 2.1. Basic Definitions

In this section some basic definitions are reviewed as follows:

**Definition 2.1.[10]** The characteristic function $\mu_A$of a crisp set. A⊆X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range. i.e. $\mu_{\tilde{A}}$ : X → [0, 1]. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A}$ = $\{(x, \mu_{\tilde{A}}(x)): x\epsilon X\}$ defined by $\mu_{\tilde{A}}(x)$ for each X is called a fuzzy set.

**Definition 2.2. [10]** A fuzzy number $\tilde{A}$ = (a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x\epsilon(-\infty, a] \cup [d, +\infty) \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{x-d}{c-d}, & c \leq x \leq d \end{cases}$$

**Definition 2.3.** A fuzzy set $\tilde{A}$ defined on the Universal set of real numbers ℝ is said to be generalized trapezoidal fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}(x)$: ℝ → [0, w] is continuous
2. $\mu_{\tilde{A}}(x)$ = 0 for all $x\epsilon(-\infty, a] \cup [d, +\infty)$
3. $\mu_{\tilde{A}}(x)$is strictly increasing on [a, b] and strictly decreasing on [c, d]
4. $\mu_{\tilde{A}}(x)$ = w for all $x\epsilon[b, c]$ where 0 < w≤ 1

**Definition 2.4. [4]** A generalized trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d, w) is said to be a non-normal trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x\epsilon(-\infty, a] \cup [d, +\infty) \\ w\left(\frac{x-a}{b-a}\right), & a \leq x \leq b \\ w\ , & b \leq x \leq c, 0 < w < 1 \\ w\left(\frac{x-d}{c-d}\right), & c \leq x \leq d \end{cases}$$

The generalized trapezoidal fuzzy number is said to be a normal trapezoidal fuzzy number if w = 1.

**Definition 2.5. [5]** A non-normal fuzzy number $\tilde{A}$ = (a, b, c, d, w)$_p$ is said to be a non-normal $p$-norm trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x\epsilon(-\infty, a] \cup [d, +\infty) \\ w\left[1 - \left(\frac{x-b}{a-b}\right)^p\right]^{\frac{1}{p}} & a \leq x \leq b \\ w\ , & b \leq x \leq c, \\ w\left[1 - \left(\frac{x-c}{d-c}\right)^p\right]^{\frac{1}{p}} & c \leq x \leq d \end{cases}$$

where $p$ is a positive integer.

The left and right inverse functions of $\mu_{\tilde{A}}(x)$ are,

$$L_{\tilde{A}}^{-1}(\mathrm{y}) = \mathrm{b} + (\mathrm{a}-\mathrm{b})\left[1-\left(\frac{y}{w}\right)^{p}\right]^{\frac{1}{p}}, 0 \le y \le w$$

$$R_{\tilde{A}}^{-1}(\mathrm{y}) = \mathrm{c} + (\mathrm{d}-\mathrm{c})\left[1-\left(\frac{y}{w}\right)^{p}\right]^{\frac{1}{p}}, 0 \le y \le w$$

**2.2. Arithmetic operations [4]**

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1 : w_1)_p$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2 : w_2)_p$ be two non-normal $p$-norm trapezoidal fuzzy numbers defined on the universal set of real numbers $\mathbb{R}$. Then the arithmetic operations between $\tilde{A}_1$ and $\tilde{A}_2$ are

i. $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2 : w)_p$
ii. $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2 : w)_p$
iii. $\tilde{A}_1 \otimes \tilde{A}_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2 : w)_p$
iv. $\tilde{A}_1 \oslash \tilde{A}_2 = (a_1 / d_2, b_1 / c_2, c_1 / b_2, d_1 / a_2 : w)_p$ where $w = min(w_1, w_2)$
v. $\lambda\tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1 : w_1)_p, \lambda \ge 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1 : w_1)_p, \lambda < 0 \end{cases}$

While considering more than two non-normal $p$-norm trapezoidal fuzzy numbers let $w = min\,(w_1, w_2)$.

**Definition 7. [12]** A ranking function is a function $\mathfrak{R}$:F($\mathbb{R}$) → $\mathbb{R}$, where F($\mathbb{R}$) is a set of fuzzy numbers defined on set of real numbers which maps each fuzzy number into the real line.

Let $\tilde{A} = (\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} : \mathrm{w})_{\mathrm{p}}$ where $w = 1$ and $p = 1$ be the p-normal trapezoidal fuzzy number. The signed distance from $0_1$(y-axis) is given as

$$\mathrm{d}(\tilde{A}, 0_1) = \frac{w}{2}\left[(\mathrm{a} - \mathrm{b} - \mathrm{c} + \mathrm{d} : \mathrm{w})_{\mathrm{p}} \frac{\Gamma\left(\frac{1}{\mathrm{p}}+1\right)\Gamma\left(\frac{1}{\mathrm{p}}\right)}{\mathrm{p}\Gamma\left(\frac{2}{\mathrm{p}}+1\right)} + (\mathrm{b}+\mathrm{c})\right];$$

If $w > 1$ and $p \ge 1$ then non-normal $p$-norm trapezoidal fuzzy number.

For $\tilde{A} = (a_1, b_1, c_1, d_1 : w_1)_p$ and $\tilde{B} = (a_2, b_2, c_2, d_2 : w_2)_p$ two non-normal $p$-norm trapezoidal fuzzy numbers with different heights.

i. d $(\tilde{A}, \tilde{B}) > 0$ iff d$(\tilde{A}, 0_1) >$ d $(\tilde{B}, 0_1)$ iff $\tilde{B} \prec \tilde{A}$
ii. d $(\tilde{A}, \tilde{B}) < 0$ iff d$(\tilde{A}, 0_1) <$ d $(\tilde{B}, 0_1)$ iff $\tilde{A} \prec \tilde{B}$
iii d $(\tilde{A}, \tilde{B}) = 0$ iff d$(\tilde{A}, 0_1) =$ d $(\tilde{B}, 0_1)$ iff $\tilde{B} \approx \tilde{A}$.

**3. Mathematical formulation**

In a time minimizing fuzzy transportation problem, the time of transporting goods from m origins to n destinations is minimized, satisfying certain conditions in respect of fuzzy availabilities at fuzzy sources and required at the fuzzy destinations.

Thus a time minimizing fuzzy transportation is $\tilde{t}_{ij}, \tilde{x}_{ij}, \tilde{a}_i, \tilde{b}_j$ are represented as a generalized non-normal p-norm trapezoidal fuzzy numbers is

$Minimize \tilde{Z} =$ [Max$_{(i,j)}$ $\tilde{t}_{ij}/\tilde{x}_{ij} > 0$ ]
subject to the constraints,

$$\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, i = 1,2,3 \ldots m$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, j = 1,2,3 \ldots n$$

$$\tilde{x}_{ij} \geq 0, \forall i, j.$$

$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

where $m$: Total number of sources,
$n$:Total number of destinations.
$\tilde{a}_i$: The fuzzy supply of the product at $i^{\text{th}}$ source.
$\tilde{b}_j$: The fuzzy demand of the product at $j^{\text{th}}$destination.
$\tilde{t}_{ij}$: The fuzzy transportation time for transporting goods.
$\sum_{i=1}^{m} \tilde{a}_i$: Total fuzzy supply of the product.
$\sum_{j=1}^{n} \tilde{b}_j$: Total fuzzy demand of the product.

A fuzzy feasible solution $\tilde{X} = [\tilde{x}_{ij}]$ for which[ Max$_{(i,j)}$ $\tilde{t}_{ij}/\tilde{x}_{ij} > 0$] is minimal is called an fuzzy optimal solution.

## 4. Proposed method: algorithm for solving time minimizing fuzzy transportation problem

**Step 1:** Find the total fuzzy supply$\sum_{i=1}^{m} \tilde{a}_i$ and the total fuzzy demand $\sum_{j=1}^{n} \tilde{b}_j$.
Examine that the problem is balanced or not $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ or $\sum_{i=1}^{m} \tilde{a}_i \neq \sum_{j=1}^{n} \tilde{b}_j$.

**Step 2:** Determine an initial fuzzy basic feasible solution which can be found by the methods applicable in the case of the common cost minimizing fuzzy transportation preferably generalized fuzzy Vogel's approximation method.

**Step 3:(i)** The time units shown in the non-empty cells are the times in which their materials despatch are complete. The time units are given in generalized non-normal p-norm trapezoidal fuzzy numbers.

Let $\tilde{T} =$ [ Max$_{(i,j)}$ $\tilde{t}_{ij}/\tilde{x}_{ij} > 0$] and if $\tilde{t}_{ij} < \tilde{T}$then within time $\tilde{T}$despatches of all cells will be complete.

If $\tilde{t}_{ij} \geq \tilde{T}$ then we omit these empty cells.

(ii) Now, the aim will be to determine another type of distribution of materials for if $\tilde{t}_{ij} < \tilde{T}$ and to find an adjacent better basic feasible solution.

(iii) Construct a loop for the basic cells corresponding to $\tilde{T}$ in such a way that when the values at the corner cells are shifted around the value at the cell towards (not necessarily zero) zero and no variable becomes zero. If no such closed path can be formed the solution under test is fuzzy optimal.

**Step 4:** Defuzzify by signed distance to get the minimum transportation time.

**Step 5:** Repetition of step 3 till no better adjacent basic feasible solution can be found.

**5. Numerical example**

The proposed method is illustrated by the following example:

Let there be three godowns$S_1, S_2, S_3$have the stocks $(1,4,6,9:2)_1(2,5,9,14:2)_1(1,2,3,4:2)_1$units of materials which are to be sent to the three destinations $D_1, D_2, D_3$ demanding $(1,4,6,9:2)_1, (1,2,5,8:2)_1, (2,5,7,10:2)_1$ respectively with the following fuzzy transportation time $\tilde{t}_{ij}, i = 1,2,3. j = 1,2,3$.

| Godowns | Destination | | | Supply |
|---|---|---|---|---|
| | **$D_1$** | **$D_2$** | **$D_3$** | |
| $S_1$ | $(1,4,6,9:2)_1$ | $(0.2,0.8,1,2:2)_1$ | $(3,4,5,8:4)_1$ | $(1,4,6,9:2)_1$ |
| $S_2$ | $(0,1,2,3:2)_1$ | $(2,3,4,5:2)_1$ | $(3,4,5,6:2)_1$ | $(2,5,9,14:2)_1$ |
| $S_3$ | $(2,5,7,10:2)_1$ | $(3,5,8,12:2)_1$ | $(2,3,4,7:4)_1$ | $(1,2,3,4:2)_1$ |
| **Demand** | $(1,4,6,9:2)_1$ | $(1,2,5,8:2)_1$ | $(2,5,7,10:2)_1$ | $(4,11,18,27:2)_1$ |

**Table 1:** Balanced Fuzzy transportation time

**Step 1:** $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ is a balanced fuzzy transportationtime which has shown in Table 1.

**Step 2:** Generalized fuzzy Vogel's Approximation method gives the initial fuzzy basic feasible Solution shown in Table 2.

| Godowns | Destination | | | Supply |
|---|---|---|---|---|
| | $D_1$ | $D_2$ | $D_3$ | |
| $S_1$ | $(1,4,6,9:2)_1$ | $(\mathbf{1,2,5,8:2})_{\mathbf{1}}$ ------------------- $(0.2,0.8,1,2:2)_1$ | $(\mathbf{0,2,1,1:2})_{\mathbf{1}}$ ----------------- $(3,4,5,8:4)_1$ | $(1,4,6,9:2)_1$ |
| $S_2$ | $(\mathbf{1,4,6,9:2})_{\mathbf{1}}$ ---------------- $(0,1,2,3:2)_1$ | $(2,3,4,5:2)_1$ | $(\mathbf{1,1,3,5:2})_{\mathbf{1}}$ ---------------- $(3,4,5,6:2)_1$ | $(2,5,9,14:2)_1$ |
| $S_3$ | $(2,5,7,10:2)_1$ | $(3,5,8,12:2)_1$ | $(\mathbf{1,2,3,4:2})_{\mathbf{1}}$ ----------------- $(2,3,4,7:4)_1$ | $(1,2,3,4:2)_1$ |
| **Demand** | $(1,4,6,9:2)_1$ | $(1,2,5,8:2)_1$ | $(2,5,7,10:2)_1$ | $(4,11,18,27:2)_1$ |

**Table 2:** An initial fuzzy basic feasible solution $\tilde{X}_1$ is given below

Bold numerals denote the allocations in the basic cells of the solution.
Therefore, in order to complete the shipment it takes time

$$\tilde{T}_1 = Max\{\tilde{t}_{12}, \tilde{t}_{13}, \tilde{t}_{21}, \tilde{t}_{23}, \tilde{t}_{33}\}$$

$\tilde{T}_1 = Max\{(0.2,0.8,1,2:2)_1, (3,4,5,8:4)_1, (0,1,2,3:2)_1, (3,4,5,6:2)_1, (2,3,4,7:4)_1\}$
$= (3,4,5,8:4)_1$ units of time.

Thus, ($S_1$, $D_3$) enters the basis.

**Step 3:**

| Godowns | Destination | | | Supply |
|---|---|---|---|---|
| | $D_1$ | $D_2$ | $D_3$ | |
| $S_1$ | $(\mathbf{0,2,1,1:2})_{\mathbf{1}}$ ----------------- $(1,4,6,9:2)_1$ | $(\mathbf{1,2,5,8:2})_{\mathbf{1}}$ ------------------- $(0.2,0.8,1,2:2)_1$ | $(3,4,5,8:4)_1$ | $(1,4,6,9:2)_1$ |
| $S_2$ | $(\mathbf{1,2,5,8:2})_{\mathbf{1}}$ ---------------- $(0,1,2,3:2)_1$ | $(2,3,4,5:2)_1$ | $(\mathbf{1,3,4,6:2})_{\mathbf{1}}$ ---------------- $(3,4,5,6:2)_1$ | $(2,5,9,14:2)_1$ |
| $S_3$ | $(2,5,7,10:2)_1$ | $(3,5,8,12:2)_1$ | $(\mathbf{1,2,3,4:2})_{\mathbf{1}}$ ----------------- $(2,3,4,7:4)_1$ | $(1,2,3,4:2)_1$ |
| **Demand** | $(1,4,6,9:2)_1$ | $(1,2,5,8:2)_1$ | $(2,5,7,10:2)_1$ | $(4,11,18,27:2)_1$ |

**Table 3:** The new solution $\tilde{X}_2$ is given below

Using Step 3(iii) of the proposed method, we get the solution $\tilde{T}_2 = (2,3,4,7:4)_1 > \tilde{T}_1$ which has shown in Table 3. Therefore, ($S_1$, $D_3$) is omitted.

Thus, ($S_3$, $D_3$) enters the basis.

**Step 4:**

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| Godowns | Destination | | | Supply |
|---|---|---|---|---|
| | **$D_1$** | **$D_2$** | **$D_3$** | |
| **$S_1$** | $(\mathbf{0},\mathbf{2},\mathbf{1},\mathbf{1}:\mathbf{2})_{\mathbf{1}}$<br>-----------------<br>$(1,4,6,9:2)_1$ | $(\mathbf{1},\mathbf{2},\mathbf{5},\mathbf{8}:\mathbf{2})_{\mathbf{1}}$<br>-------------------<br>$(0.2,0.8,1,2:2)_1$ | $(3,4,5,8:4)_1$ | $(1,4,6,9:2)_1$ |
| **$S_2$** | $(\mathbf{0},\mathbf{0},\mathbf{2},\mathbf{4}:\mathbf{2})_{\mathbf{1}}$<br>----------------<br>$(0,1,2,3:2)_1$ | $(2,3,4,5:2)_1$ | $(\mathbf{2},\mathbf{5},\mathbf{7},\mathbf{10}:\mathbf{2})_{\mathbf{1}}$<br>----------------<br>$(3,4,5,6:2)_1$ | $(2,5,9,14:2)_1$ |
| **$S_3$** | $(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}:\mathbf{2})_{\mathbf{1}}$<br>-----------------<br>$(2,5,7,10:2)_1$ | $(3,5,8,12:2)_1$ | $(2,3,4,7:4)_1$ | $(1,2,3,4:2)_1$ |
| **Demand** | $(1,4,6,9:2)_1$ | $(1,2,5,8:2)_1$ | $(2,5,7,10:2)_1$ | $(4,11,18,27:2)_1$ |

**Table 4:** The new solution $\tilde{X}_3$ is given below

Again using Step 3(iii) of the proposed method, we get the solution $\tilde{T}_3 = (2,5,7,10:2)_1 > \tilde{T}_2$which has shown in Table 4. Therefore,( $S_3$, $D_3$) is omitted.
Thus ( $S_3$, $D_1$) enters the basis.

Now we cannot form any loop originating from the cell ($S_3$,$D_1$). Thus the process terminates.Therefore, in order to complete the shipment it takes time

$$\tilde{T}_3 = Max\{\tilde{t}_{11}, \tilde{t}_{12}, \tilde{t}_{21}, \tilde{t}_{23}, \tilde{t}_{33}\}$$

$$\tilde{T}_3 = Max\{(1,4,6,9:2)_1, (0.2,0.8,1,2:2)_1, (0,1,2,3:2)_1, (3,4,5,6:2)_1, (2,5,7,10:2)_1\}$$
$$= (2,5,7,10:2)_1$$

Hence, the fuzzy optimal time is $\tilde{T} = (2,5,7,10:2)_1$ .
Rank of time minimizing fuzzy transportation = 12
The fuzzy optimal solution is

$$\tilde{x}_{11} = (0,2,1,1:2)_1, \tilde{x}_{12} = (1,2,5,8:2)_1, \tilde{x}_{21} = (0,0,2,4:2)_{\mathbf{1}}, x_{23} = (2,5,7,10:2)_1, \tilde{x}_{33} = (1,2,3,4:2)_1$$

## 5. Conclusion

The method presented and discussed above gives us the fuzzy optimal solution for time minimizing fuzzy transportation problem using generalized fuzzy non-normal$p$-norm trapezoidal fuzzy numbers. Sometimes we can get a fuzzy optimal solution directly.

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