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The Middle Edge Dominating Graph of Prime Cycles

A.Elumalai[1] **and** ***M.Karthikeyan***[2]

[1] Department of Mathematics, Valliammai Engineering College, Chennai-603203, India
E-mail: aelu2000@yahoo.com
[2] Department of Mathematics, Agni College of Technology, Chennai-600130, India
E-mail: kmkmkarthikeyan@gmail.com

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Abstract. The middle edge dominating graph $M_{ed}(G)$ of a graph G=(V,E) is a graph with the vertex set $E \cup S$ where S is the set of all minimal edge dominating set G and with two vertices u, v ϵ $E \cup S$ adjacent if u ϵ E and V=F is a minimal edge dominating set of G containing u or u,v are not disjoint minimal edge dominating sets in S .In this paper we discuss about the middle edge dominating graph of Prime cycles

Keywords: Graph, Cycle, Edge dominating graph, Middle edge dominating graph

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

All graphs considered here are finite, undirected without loops, isolated vertices or multiple edges. Any undefined term in this paper may be found in Kulli [4]. Let G = (V, E) be a graph with |V|=p≥2 and |E|=q. A set $D \subseteq V$ is a dominating set if every vertex in V-D is adjacent to some vertex in D. The domination number γ(G) is the minimum cardinality of a dominating set of G.A set $F \subseteq E$ of edges in E-F is adjacent to at least one edge in F. An edge dominating set F of G is a minimal edge dominating set if for every e in F, F-e is not an edge dominating set of G. The edge domination number γ'(G) of G is the minimum cardinality of an edge dominating set of G.

Let A be a finite set. Let F= $\{A_1, A_2, \ldots, A_n\}$ be a partition of A. Then the intersection graph Ω(G) of F is the graph whose vertices are the subsets in F and in which two vertices A_i and A_j are adjacent if and only if $A_i \cap A_j = \Phi$

The minimal edge dominating graph $MD_e(G)$ of a graph G is the intersection graph defined on the family of all minimal edge dominating sets of G. This concept was introduced in [2].Some other dominating graphs are studied, for example, in [1] [3] [5] [6].

The edge dominating graph $D_e(G)$ of a graph G is the graph with the vertex set $E \cup S$ where S is the set of all minimal edge dominating sets of G and with two vertices u, v in $E \cup S$ adjacent if uϵE and v=f is a minimal edge dominating set of G containing u. This concept was introduced by kulli [5].

The middle edge dominating graph $M_{ed}(G)$ of a graph G=(V,E) is the graph with the vertex set $E \cup S$ where S is the set of all minimal edge dominating sets of G and with two vertices u,v $E \cup S$ adjacent if u∈E and V=F is a minimal edge dominating set of G containing u or u,v are not disjoint minimal edge dominating sets in G .

In Figure 1, a graph G and its middle edge dominating graph $M_{ed}(G)$ are shown

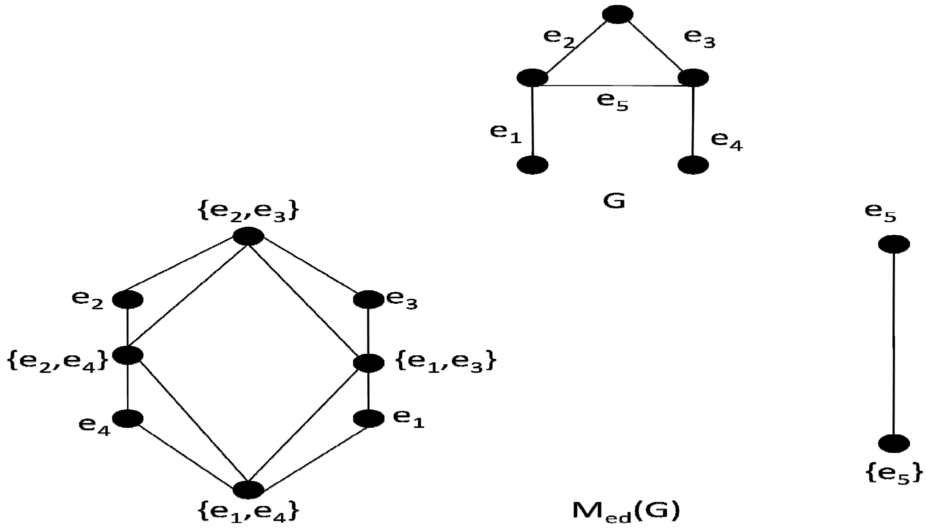

Figure1

We note that the middle edge dominating graph $M_{ed}(G)$ is defined only if G has not isolated vertices.

The degree of an edge uv is defined to be degu+degv-2.An edge called an isolated edge if deguv=0.Let Δ_1 (G) denote the maximum degree among the edges of G

In this paper, we discuss about the middle edge dominating graph of prime cycles.

2. Results of Middle Edge Dominating Graph

Theorem 2.1. Let G be a graph without isolated vertices and with at least two edges. The edge dominating graph $D_e(G)$ is connected if and only if $\Delta_1(G)<q-1$.

Remark 1. For any graph G without isolated vertices, $D_e(G)$ is a subgraph of $M_{ed}(G)$.

Remark 2. For any graph G without isolated vertices, $D_e(G)$ and $MD_e(G)$ are edge disjoint subgraphs of $M_{ed}(G)$.

Theorem 2.2. $M_{ed}(G)= k_{1,p}$ if and only if $G=pk_2, p\geq 1$.

Proof: Suppose $M_{ed}(G)= k_{1,p}$ .Assume $G\neq pk_2$. Then there exist at least two minimal edge dominating sets. Thus $|V(M_{ed}(G))|\geq p+2$ Which is a contradiction. Thus $G=pk_2$.

Conversely suppose $G=pk_2$. Then there exists exactly one minimal edge dominating set containing all the edges of G .From the definition of $M_{ed}(G)$, the result follows.

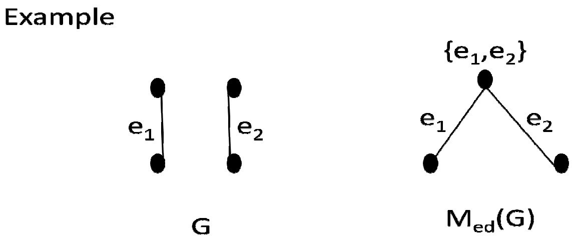

Figure 2:

Theorem 2.3. The middle edge dominating graph $M_{ed}(G)$ of G is complete bipartite if and only if $G=pk_2, p\geq1$.

Theorem 2. 4. $M_{ed}(G)= pk_2$ if and only if $G= k_{1,p}$ $p\geq1$or k_3

Proof: Suppose $M_{ed}(G)= pk_2$, $p\geq1$. Assume $G\neq k_{1,p}$ or k_3.Then there exists at least one minimal edge dominating set S containing two or more edges of G. By definition, S will form a subgraph P_3 in $M_{ed}(G)$, which is a contradiction.

Conversely suppose $G= k_{1,p}$ or k_3. Then each edge e_i of G forms a minimal edge dominating set $\{e_i\}$.Thus e and $\{e_i\}$ are adjacent vertices in $M_{ed}(G)$. Since each minimal edge dominating set $\{e_i\}$ contains only one edge no two vertices of G are adjacent in $M_{ed}(G)$ and no two corresponding vertices of minimal edge dominating sets are adjacent in $M_{ed}(G)$.Thus $M_{ed}(G)= pk_2$ or k_3.

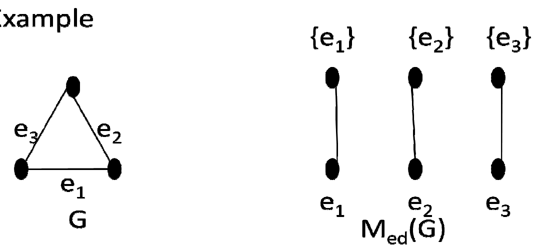

Figure 3:

3. Main results

In this section we study about the middle edge dominating graph of prime cycles and one edge union of prime cycles is shown below.

Definition 3.1. A one edge union C_n^k of K copies of cycles is the graph obtained by taking e as a common edge such that any two cycles C_n^i and C_n^j (i,j) are edge disjoint and do not have any vertex in common except v_i and v_j.

Theorem 3.1. $M_{ed}(G)= pK_{1,(n+1)}$ for n=1,2,3... if and only if $G=C_p$ for all p=5,7,11,...

Proof: Suppose $M_{ed}(G)= pK_{1,(n+1)}$ for n=1,2,3... . Assume $G\neq C_p$ p=5,7,11,...Then there exists at least one minimal edge dominating set S containing two or more edges of G. By definition, S will form a subgraph P_n in $M_{ed}(G)$, which is a contradiction.

Conversely suppose $G= C_p$ p=5,7,11,.... Then each edge e_i of G forms a minimal edge dominating set $\{e_i\}$.Thus e and $\{e_i\}$ are adjacent vertices in $M_{ed}(G)$. Since each minimal edge dominating set $\{e_i\}$ contains only one edge no two vertices of G are adjacent in $M_{ed}(G)$ and no two corresponding vertices of minimal edge dominating sets are adjacent in $M_{ed}(G)$.Thus $M_{ed}(G)= pK_{1,(n+1)}$.

Example

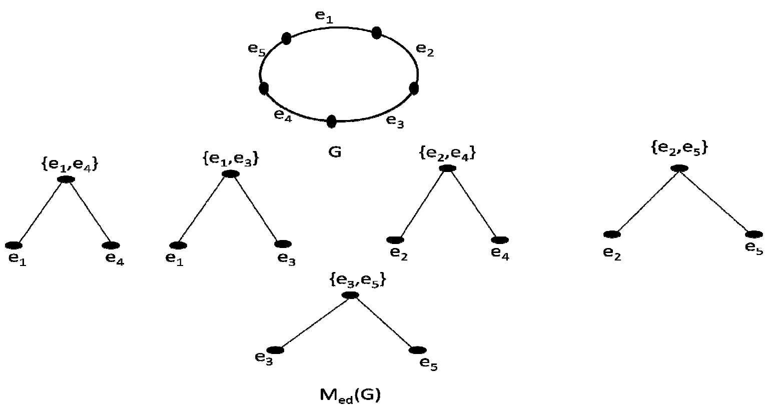

Figure 4:

Theorem 3.2. $M_{ed}(G)= K_{1,(n+1)}$ for n=1,2,3,...if and only if $G=C_p$ where p=5,7,11,... with one edge is common

Proof: Suppose $M_{ed}(G)= K_{1,(n+1)}$ for n=1,2,3... . Assume $G\neq C_p$ p=5,7,11,...with one edge is common. Then there exists at least one minimal edge dominating set S containing two or more edges of G. By definition, S will form a subgraph P_n in $M_{ed}(G)$, which is a contradiction.

Conversely suppose $G= C_p$ p=5,7,11,...with one edge is common and take that edge as a one of the dominating edge and remaining edge e_i of G forms a minimal edge dominating set $\{e_i\}$.Thus e and $\{e_i\}$ are adjacent vertices in $M_{ed}(G)$. Since each minimal edge dominating set $\{e_i\}$ contains only one edge no two vertices of G are adjacent in $M_{ed}(G)$ and no two corresponding Vertices of minimal edge dominating sets are adjacent in $M_{ed}(G)$.Thus $M_{ed}(G)= K_{1,(n+1)}$ for n=1,2,3...

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