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Some Graph Labelings on Middle Graph of Extended Duplicate Graph of a Path

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Abstract. A graph labeling is an assignment of integers to the vertices, or edges, or both subject to certain conditions. In this paper, we prove the existence of Z_3- magic, and Z_4- bi magic labelings for the middle graph of extended duplicate graph of a path by presenting algorithms.

Keywords: Graph labeling, Middle graph, Extended duplicate graph

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for broad range of applications such as coding theory, X-ray, crystallography, radar, astronomy, circuit design, communication networks and data base management and models for constraint programming over finite domain. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic, anti-magic labeling, bi-magic labeling, prime labeling, cordial labeling, total cordial labeling, k-graceful labeling and odd graceful labeling etc., have been studied in over 1100 papers .The concept of extended duplicate graph was introduced by Thirusangu, et al. in [9] and they proved EDG(P_m) is cordial.

The original concept of an A-magic graph is due to sedlack, who defined it to be a graph with real- valued edge labeling such that distinct edges have distinct non-negative labels which satisfies the condition that the sum of the labels of the edges incident to a particular vertex is the same for all vertices. The concept of bi-magic labeling is due to Babujee in 2004. Bala and Thirusangu proved that 4-regular graphs of girth *j* admits bi-magic labeling and the extended triplicate graph of a path admits E-Cordial and Z_3 – magic labeling in [2] and [3] respectively. *In this paper, we prove the existence of Z_3 – magic and Z_4 - bi magic labeling for the middle graph of extended duplicate graph of a path by presenting algorithms.*

2. Preliminaries

Let G = G(V,E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels(usually the set of integers). In this paper we deal with the labeling with domain either the set of all vertices or the set of all edges or the set of all vertices and edges. We call these labelings as the vertex labeling or the edge labeling or the total labeling respectively.

Definition 2.1. Let G(V,E) be a simple graph. A Duplicate graph of G is $DG=(V_1,E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \varphi$ and f: $V \to V'$ is bijective(for v □ V, we write f(v) = v' for convenience) and the edge set E_1 of DG is defined as follows: The edge ab is in E if and only if both ab' and a'b are edges in E_1. Clearly duplicate graph of a path is disconnected.

Definition 2.2. Let $DG=(V_1,E_1)$be a duplicate graph of a path G(V,E). Add an edge between any one vertex from V to any other vertex in V ', except the terminal vertices of V and V'. For convenience let us take v_2 □ V and v_2' □ V' and thus the edge (v_2 v_2') is formed. This graph is called the Extended Duplicate of the path P_m and it is denoted by $EDG(P_m)$ where 'm' represents the length of the path P_m .

Definition 2.3. A graph G(V,E) is said to admit Z_3 -magic labeling if there exists a function f from E on to the set {1,2} such that the induced map f* on V defined by $f^*(v_i)=\sum f(e) \pmod 3) = k$, a constant where $e=(v_iv_j)$ □ E.

Definition 2.4. A graph G(V,E) is said to admit Z_4 -bi -magic labeling if there exists a function f from E on to the set {1,2,3} such that the induced map f* on V defined by $f^*(v_i)=\sum f(e)(\text{mod}4)=k_1$ or k_2, a constant where $e=(v_iv_j) \epsilon E$.

3. Main result

In this paper, we prove the existence of Z_3 magic, and Z_4-bi-magic for the middle graph of extended duplicate graph of a path by presenting algorithm.

Definition 3.1 : The middle graph of G(V,E) ,is defined with the vertex V(G) U E(G) where two vertices are adjacent iff they are either adjacent edges of G or one is a vertex and the other is an edge incident with it and it is denoted by M(G)

Definition 3.2 : The structure of middle graph of extended duplicate graph is defined as follows : Let $EDG(P_m)$ be a graph with 2m+2 vertices $\{v_1,v_2\ldots\ldots v_{m+1},v_1',v_2'\ldots\ldots v'_{m+1}\}$ and 2m+1 edges where 'm' represents the length of the path P_m. The middle graph of $EDG(P_m)$ is obtained by introducing a new vertex w_i on each edge as follows;

$$v_iv_{i+1}' \longleftarrow w_i \ ; 1 \le i \le m$$
$$v_iv_{i-1}' \longleftarrow w_{i+m-1} \ ; 2 \le i \le m+1$$
$$v_2v_2' \longleftarrow w_{2m+1}$$

Thus $MEDG(P_m)$, the middle graph of $EDG(P_m)$ is a (V,E) graph where $V=\{v_i \text{ U } v_i' \text{ U } w_k \ / \ 1 \le i \le m+1 \text{ and } 1 \le k \le 2m+1\}$ and $E= \{v_iw_i \ ; w_iv_{i+1}' \text{ for } 1 \le i \le m \}$ U $\{v_iw_{i+m-1};w_{i+m-1}\, v_{i-1}' \text{ for } 2 \le i \le m+1\}$ U $\{v_2w_{2m+1} \ ; w_{2m+1}v_2'\}U\{w_iw_{i+m+1} \text{ for } 1 \le i<m \}$ U $\{w_iw_{i-m+1} \text{ for } m<i<2m\}$U $\{w_iw_{2m+1} \text{ for } 1\le i \le 2$ and i=m+1 and i=m+2 }.

Thus MEDG(P_m) has 4m+3 vertices and 6m+4 edges.

Algorithm 3.1

procedure Z_4 – bi-magic labeling of MEDG(P_m)

Input : MEDG (P_m)

v_1w_1, $v_1'w_{m+1}$,v_2w_{2m+1},$v_2'w_{2m+1}$,$v_{m+1}w_{2m}$,$v_{m+1}'w_m$ ⟵ 1

v_2w_{m+1},$v_2'w_1$ ⟵ 2

w_1w_{m+2},w_2w_{m+1} ⟵ 3

for i=2 to m do

{

v_iw_i,$v_i'w_{i+m}$ ⟵ 2

}

end for

for i=2 to m-1 do

{

w_iw_{i+m} ⟵ 2

}

end for

for i=m+2 to 2m-1 do

{

w_iw_{i-m+1} ⟵ 2

}

end for

for i=3 to m do

{

v_iw_{i+m-1},$v_i'w_{i-1}$ ⟵ 3

}

end for

for i=1,2,m+1,m+2

{

w_iw_{2m+1} ⟵ 3

}

end for

output: Labeled MEDG (P_m)

Algorithm 3.2:

procedure Z_3- magic labeling for MEDG(P_m)

Input : MEDG (P_m) where m ≡ 1(mod 2)

// assignment of labels to the edges

v_1w_1,v_2w_2,v_2w_{m+1},$v_2'w_1$,$v_1'w_{m+1}$,$v_2'w_{m+2}$,w_1w_{m+2},w_1w_{2m+1},w_2w_{m+1}, $v_{m+1}w_{2m}$,$v_{m+1}'w_m$,$w_{m+1}w_{2m+1}$ ⟵ 1

v_2w_{2m+1},$v_2'w_{2m+1}$,w_2w_{2m+1},$w_{m+2}w_{2m+1}$ ⟵ 2

For i= 2 to m-1 and i ≡ 0(mod 2) do

{

$w_i w_{i+m+1}$ ⟵ 1

}

end for

for i = m+1 to 2m-1, i ≡ 1(mod 2) do

{
$w_i w_{i-m+1} \leftarrow 1$
}
end for
for i = 3 to m do
{
$v_i w_i$, $v_i w_{i+m-1}$, $v_i' w_{i-1}$, $v_i' w_{i+m} \leftarrow 2$
}
end for
for i = 3 to m-1, $i \equiv 1 \pmod 2$ do
{
$w_i w_{i+m+1} \leftarrow 2$
}
end for
for i=m+2 to 2m-1 and $i \equiv 0 \pmod 2$ do
{
$w_i w_{i-m+1} \leftarrow 2$
}
end for
output : Labeled MEDG(P_m)

Theorem 3.1. The middle graph of EDG(P_m) is Z_4 bi-magic.

Proof: From the structure of MEDG(P_m), it is clear that MEDG(P_m) has 4m + 3 vertices and 6m+4 edges. Denote the vertex set and edge set of MEDG(P_m) as given in the definition 3.2.

To prove MEDG(P_m) is Z_4 bi-magic, we have to show that there exists a function f from E on to the set {1,2,3} such that the induced map $f^* : V \rightarrow \{0, 1, 2, 3\}$ defined by $f^*(v)=[\{ \sum f(uv)\} \pmod 4) / u \in N(v)] = k_1$ or k_2, constants.

Using algorithm 3.1, the edges are labeled. Vertex labels are obtained as follows:

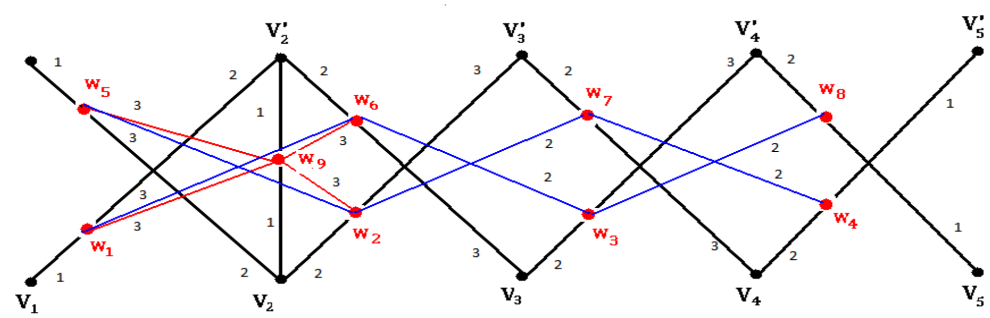
For $1 \leq i \leq m + 1$ and $1 \leq k \leq 2m + 1$

$$f^*(v_i) = \{\sum f(v_i w_j)\} \pmod 4 = 1$$
$$f^*(v_i') = \{\sum f(v_i' w_j)\} \pmod 4 = 1.$$

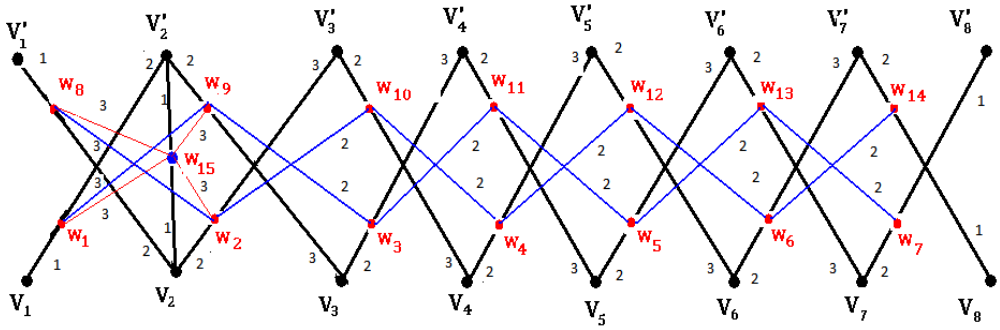
$$f^*(w_k) = \sum\{f(w_k v_j) + f(w_k v_j') + f(w_k w_j)\} \pmod 4$$
$$= \begin{cases} 1, & \text{if } 1 \leq k \leq 2m \\ 2, & \text{otherwise} \end{cases}$$

Thus all the weights of the vertices are either 1 or 2.
Therefore MEDG(P_m) admits Z_4 bi- magic labeling.

Example 3.1. Z_4 bi- magic labeling of MEDG(P_4) and MEDG(P_7) are given below.



MEDG(P_4)



MEDG(P_7)

Figure 1:

Theorem 3.2. The middle graph of EDG(P_m) , m $\equiv 1 \pmod 2$ is Z_3-magic.

Proof: From the construction of MEDG(P_m), we have MEDG(P_m) has 4m + 3 vertices and 6m+4 edges. Denote the vertex set and edge set of MEDG(P_m) as given in the definition 3.2.

To prove MEDG(P_m), m $\equiv 1 \pmod 2$ is Z_3-magic, we have to show that there exists a function f from E on to the set {1,2} such that the induced map $f^* : V \rightarrow \{0, 1, 2\}$ defined by $f^*(v)=[\{ \sum f(uv) \} (\text{mod } 3) / u \in N(v)] = k$, a constant.

Using algorithm 3.2, the edges are labeled. Vertex labels are obtained as follows. For $1 \leq i \leq m + 1$ and $1 \leq k \leq 2m + 1$

$$f^*(v_i) = \{\sum f(v_i w_k)\} (\text{mod } 3) = 1$$

$$f^*(v_i') = \{\sum f(v_i' w_k)\} (\text{mod } 3) = 1$$

$$f^*(w_k) = \sum \{f(w_k v_j) + f(w_k v_j') + f(w_k w_j)\} (\text{mod } 3) = 1$$

Since all the vertices receive the same label '1' , MEDG(P_m), m $\equiv 1 \pmod 2$ admits Z_3- magic labeling.

Example 3.2. Z_3 – magic labeling for MEDG(P_7) is given below.

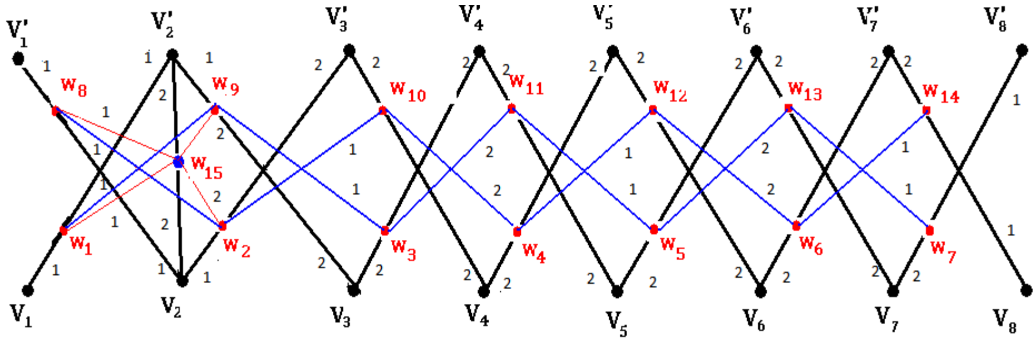


Figure 2:

4. Conclusion

In this paper, we have presented the algorithms and proved the existence of Z_3- magic and Z_4- bi magic labeling for the middle graph of extended duplicate graph of a path P_m.

REFERENCES

1. J. Akiyama, T. Hamada and I. Yoshimura, On characterization of middle graphs, *TRU Math.*, 11 (1975) 35-39.
2. E Bala and K.Thirusangu, On Bi-Magic labeling of 4-regular graphs, *Indian Journal of Science and Technology*, 4(4) (2011) 414-416.
3. E Bala and K. Thirusangu, E-Cordial and Z_3 – magic labeling in extended triplicate graph of a path, *General Mathematics Notes*, 5(2) (2011) 8-23.
4. J.A.Bondy and U.S.R.Murthy, *Graph Theory with Applications*, 1976.
5. M. Borowiecki, A characterization of middle graphs and a matroid associated with middle graphs of hyper graphs, *Fund. Math.*, CXVII (1983) 1-4.
6. J.A.Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatories*, 16 (2013) # DS6.
7. S.Paul, M.Pal and A.Pal,L(2,1)-labeling of circular-arc graph, *Annals of Pure and Applied Mathematics*, 5(2) (2014) 208-219
8. M.Skowronska, Maciej and M.Syslo, An algorithm to recognize a middle graph, *Discrete Applied Mathematics*, 7 (1984) 201-208.
9. P.P.Ulaganathan, K.Thirusangu and B.Selvam, Edge magic total labeling in extended duplicate graph of path, *International Journal of Applied Engineering Research*, 6(10) (2011) 1211-1217.
10. S.K.Vaidya and N.B.Vyas, Anti-magic labeling of some path and cycle releated graphs, *Annals of Pure and Applied Mathematics*, 3(2) (2013) 119-128.