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Labelings on Jahangir Graph and Extended Duplicate Graph of Jahangir Graph

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Abstract. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. In this paper, we prove the existence of product cordial labeling for the Jahangir graph and also we show the existence of even and odd mean labeling for the extended duplicate graph of Jahangir graph.

Keywords: Graph labeling, Jahangir graph, extended duplicate of Jahangir graph

AMS Mathematics Subject Classification (2010): 05C78

1. Introduction

The concept of Jahangir graph was introduced by Tomescu and Javaid which has been studied from its metric dimension [1]. Jahangir graph $J_{2,8}$ appears on Jahangir's tomb in his mausoleum. It is situated in 5 km northwest of Lahore, Pakisthan, across the River Ravi.

Ulaganadhan el at. [2] have introduced an Extended Duplicate graph of G(V,E). Let G(V,E) be a simple graph. A duplicate graph of G is $DG(V_1,E_1)$where the vertex set $V_1=V\cup V'$ and $V\cap V'=\emptyset$ and f : $V\rightarrow V'$ is bijective (for $v\in V$, we write f(v) =v' for conveience) and the edge set $E_1$of DG is defined as follows; the edge ab is in E if and only if ab' and a'b are edges in E_1. Clearly the duplicate graph of the path graph is disconnected. To make it as connected by add an edge between any one vertex from V to any other vertex in V'. We call this new graph as the Extended duplicate graph of the path P_m.

Let G=G(V,E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one to one mapping that carries a set of graph elements onto a set of numbers called labels. In this paper we deal with the labeling with domain either the set of all vertices or set of all edges or the set of all vertices and edges. We call those labels as the vertex labeling or the edge labeling or the total labeling respectively. Somasundaram and Ponraj have introduced the notion of mean labelings of graph [3]. A graph G with p vertices and q edges is called a mean graph, if there is an injective function f from the vertices of G to {0,1,2,…q} such that each edge uv is labeled with (f(u)+f(v))/2 if f(u)+f(v) is even and (f(u)+f(v)+1) if f(u)+f(v) is odd, then the resulting edge labels are distinct. The notion of even and odd mean labeling was introduced by Nirmala. A function f is called an even mean labeling of a (p,q) graph , if f is an injection from the vertices of G to the set {2,4,6…..2q} such that when each edge uv is assigned the label (f(u)+f(v))/2, then the resulting edge labels are distinct. A graph which admits a even mean labeling is called an even mean graph. A function f is called an odd mean labeling of a (p,q) graph, if f is an injection from the vertices of G to the set{1,3,5…..2q-1} such that when each edge uv is assigned the label (f(u)+f(v))/2, then the resulting edge labels are distinct. A graph which admits an odd mean labeling is called an odd mean graph.

Sundaram et al. have introduced the notion of product cordial labeling [4]. A Product Cordial labeling of a graph G with vertex set V is a function from V(G) to {0,1} such that if each edge uv is assigned the label f(u)f(v), the number of vertices labeled **'**0**'** and the number of vertices labeled **'**1**'** differ by atmost 1 and number of edges labeled '1' and the number of edges labeled **'**0' differ by atmost 1. A graph that admits product cordial labeling is called product cordial graph.

2. Preliminaries

Definition 2.1. Jahangir graph $J_{n,m}$ for m≥3 and n=2 is a graph on nm+1 vertices. That is a graph consisting of a cycle C_{2m} with one additional vertex which is adjacent to m vertices of C_{2m} at distance n to each other on C_{2m}.

Algorithm 2.2.

Procedure (Structure of Extended Duplicate graph of Jahangir Graph $J_{2,m}$)
$V \leftarrow \{v_1, v_2, \ldots, v_{2m+1}, v_1', v_2', \ldots, v_{2m+1}'\}$
$E \leftarrow \{e_0, e_1, e_2, \ldots, e_{3m}, e_1', e_2', \ldots, e_{3m}'\}$
    $v_1 v_1' \leftarrow e_0$
for i=2 to m+1
    $v_1' v_i \leftarrow e_{i-1}$
    $v_1 v_i' \leftarrow e_{i-1}'$
end for
for i=2 to m
    $v_i v_{m+i}' \leftarrow e_{2i+5}$
    $v_i v_{m+i+1}' \leftarrow e_{2i+4}$
    $v_i' v_{m+i} \leftarrow e_{2i+5}'$
    $v_i' v_{m+i+1} \leftarrow e_{2i+4}'$
end for
    $v_{m+1} v_{2m+1}' \leftarrow e_{3m-1}$
    $v_{m+1} v_{m+2}' \leftarrow e_{3m}$
    $v_{m+1}' v_{2m+1} \leftarrow e_{3m-1}$
    $v_{m+1}' v_{2m+1} \leftarrow e_{3m}'$
end procedure

3. Main result

In this section, we present an algorithm to get product cordial labeling for Jahangir graph and also odd and even mean labeling for Extended duplicate graph of Jahangir graph and prove the existence of product cordial for Jahangir and the existence of odd and even mean for Extended duplicate of Jahangir Graph.

Algorithm3.1.
Procedure (Product cordial labeling for Jahangir Graph $J_{2,m}$)

$V \leftarrow \{v_1, v_2, \ldots\ldots, v_{2m+1}\}$

$E \leftarrow \{e_1, e_2, \ldots\ldots, e_{3m}\}$

If m≡1(mod 2)

$$f(v_i) = \begin{cases} 1, & 1 \le i \le \frac{m+3}{2} \\ 0, & \frac{m+5}{2} \le i \le m+1 \end{cases}$$

$$f(v_i) = \begin{cases} 1, & m+3 \le i \le \frac{3m+3}{2} \\ 0, & \frac{3m+5}{2} \le i \le 2m+1 \end{cases}$$

$f(v_{m+2}) = 0$

end if

end procedure

Algorithm 3.2.
Procedure (odd mean labeling for extended duplicate graph of Jahangir Graph $J_{2,m}$ with p vertices and q edges)

$V \leftarrow \{v_1, v_2, \ldots\ldots, v_{2m+1}, v_1', v_2', \ldots\ldots, v_{2m+1}'\}$

$E \leftarrow \{e_0, e_1, e_2, \ldots\ldots, e_{3m}, e_1', e_2', \ldots\ldots, e_{3m}'\}$

$f(v_1) = 1$

$f(v_1') = 2q-1$

fori=2 to m+1

$f(v_i) = 2q-2i+1$

$f(v_i') = 2i-1$

end for

fori=m+2 to 2m

$$f(v_i) = \begin{cases} 2(q+2m-3i)-1 & m \equiv 0 (mod\ 4) \\ 2(q+3m-4i)+3 & m \equiv 1 (mod\ 4) \\ 2(q+2m-3i)+1 & m \equiv 2 (mod\ 4) \\ 2(q+2m-3i)-3 & m \equiv 3 (mod\ 4) \end{cases}$$

$$f(v_{2m+1})=\begin{cases}2q-2m-5 & m\equiv 0(mod\ 4)\\ 2q-2m-3 & m\equiv 1(mod\ 4)\\ 2q-2m-3 & m\equiv 2(mod\ 4)\\ 2q-2m-9 & m\equiv 3(mod\ 4)\end{cases}$$

end for

for i=m+2 to 2m+1

$$f(v_i')=\begin{cases}2(q+3m-4i)+9 & m\equiv 0(mod\ 4)\\ 2(q+m-2i)-3 & m\equiv 1(mod\ 4)\\ 2(q+2m-3i)+5 & m\equiv 2(mod\ 4)\\ 2(q+2m-3i)+1 & m\equiv 3(mod\ 4)\end{cases}$$

end for

end procedure

Algorithm 3.3.
Procedure (even mean labeling for extended duplicate graph of Jahangirgraph $J_{2,m}$ with p vertices and q edges)

$V\leftarrow\{v_1,v_2,\ldots\ldots,v_{2m+1},v_1',v_2',\ldots\ldots,v_{2m+1}'\}$
$E\leftarrow\{e_0,e_1,e_2,\ldots\ldots,e_{3m},e_1',e_2',\ldots\ldots,e_{3m}'\}$

For i=1 to m+1

$f(v_i)= 2q-2i+2$
$f(v_i')=2i$

end for

for i=m+2 to 2m

$$f(v_i)=\begin{cases}2(q+2m-3i)m\equiv 0(mod\ 4)\\ 2(q+3m-4i)+4 & m\equiv 1(mod\ 4)\\ 2(q+2m-3i)+2 & m\equiv 2(mod\ 4)\\ 2(q+2m-3i)-4 & m\equiv 3(mod\ 4)\end{cases}$$

$$f(v_{2m+1})=\begin{cases}2(q-m-2) & m\equiv 0(mod\ 4)\\ 2(q-m-1) & m\equiv 1(mod\ 4)\\ 2(q-m-1) & m\equiv 2(mod\ 4)\\ 2(q-m-4) & m\equiv 3(mod\ 4)\end{cases}$$

end for

for i=m+2 to 2m+1

$$f(v_i\square)=\begin{cases}2(q+3m-4i)+10 & m\equiv 0(mod\ 4)\\ 2(q+m-2i)-2 & m\equiv 1(mod\ 4)\\ 2(q+2m-3i)+6 & m\equiv 2(mod\ 4)\\ 2(q+2m-3i)+2 & m\equiv 3(mod\ 4)\end{cases}$$

end for
end procedure

Theorem 3.4. Jahangir graph $J_{2,m}$ admits product cordial labeling.
Proof: We know that Jahangir graph $J_{2,m}$ has 2m+1 vertices and 3m edges.
Consider the arbitrary vertex $v_i \in V$. To label the vertices, using algorithm 3.1, define a map $f : V \rightarrow \{0,1\}$. From this definition of the labeling functions, we have the total number of vertices labeled '0' is m and the number of vertices '1' is m+1.

Hence the number of vertices labeled '0' and the number of vertices labeled '1' differ by atmost 1.

In order to get the labels for the edges, define the induced map $f^* : E \rightarrow \{0,1\}$ such that $f^*(v_iv_j) = \{f(v_i) \times f(v_j) \mid v_iv_j \in E\}$. Now for all $v_iv_j \in E$.

(i) $f^*(v_1v_i) = \begin{cases} 1, & 2 \le i \le \frac{m+3}{2} \\ 0, & \frac{m+5}{2} \le i \le m+1 \end{cases}$

(ii) $f^*(v_iv_{m+i}) = \begin{cases} 1, & 3 \le i \le \frac{m+3}{2} \\ 0, & \frac{m+5}{2} \le i \le m+1 \end{cases}$

(iii) $f^*(v_iv_{m+i+1}) = \begin{cases} 1, & 2 \le i \le \frac{m+1}{2} \\ 0, & \frac{m+3}{2} \le i \le m \end{cases}$

(iv) $f^*(v_2v_{m+2}) = f^*(v_{m+1}v_{m+2}) = 0$

Under this map, the number of edges labeled '0' is $\frac{3m+1}{2}$ and the number of edges labeled '1' is $\frac{3m-1}{2}$.

Thus the number of edges labeled '0' and the number of edges labeled '1' differ by atmost 1. Hence Jahangir graph admits product cordial labeling for all odd m.

Theorem 3.5. Extended duplicate graph of Jahangir graph $J_{2,m}$ admits odd mean labeling.
Proof: Clearly Extended Duplicate graph of Jahangir Graph $EDG(J_{2,m})$ has 4m+2 vertices and 6m+1 edges for all m.

To label the vertices, using algorithm3.2, define a map $f : V \rightarrow \{1,3,5\ldots..2q\text{-}1\}$ such that for any $v_iv_j \in E$, $f^*(v_iv_j) = (f(v_i)+f(v_j))/2$. Now the edge labels are as follows:

$f^*(v_1'v_1) = q$

(i) for i= 2 to m+1

$f^*(v_1'v_i) = 2q\text{-}i.$

$f^*(v_1v_i') = i$

(ii) for i=2 to m

$$f^*(v_i v_{m+i}') = \begin{cases} 2q-(5i+m-5) & m \equiv 0 (mod\ 4) \\ 2q-(3i+m+1) & m \equiv 1 (mod\ 4) \\ 2q-(4i+m-3) & m \equiv 2 (mod\ 4) \\ 2q-(4i+m-1) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_i v_{m+i+1}') = \begin{cases} 2q-(5i+m-3) & m \equiv 0 (mod\ 4) \\ 2q-(3i+m+3) & m \equiv 1 (mod\ 4) \\ 2q-(4i+m) & m \equiv 2 (mod\ 4) \\ 2q-(4i+m+2) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_{m+1} v_{2m+1}') = \begin{cases} 2q-6m & m \equiv 0 (mod\ 4) \\ 2q-(4m+4) & m \equiv 1 (mod\ 4) \\ 2q-(5m+1) & m \equiv 2 (mod\ 4) \\ 2q-(4m+6) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_{m+1} v_{m+2}') = \begin{cases} 2q-(2m+4) & m \equiv 0 (mod\ 2) \\ 2q-(2m+6) & m \equiv 1 (mod\ 2) \end{cases}$$

(iii) for i= 2 to m-1

$$f^*(v_i' v_{m+i}) = \begin{cases} q-(2i+m+1) & m \equiv 0 (mod\ 4) \\ q-(3i+m-1) & m \equiv 1 (mod\ 4) \\ q-(2i+m) & m \equiv 2 (mod\ 4) \\ q-(2i+m+2) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_i' v_{m+i+1}) = \begin{cases} q-(2i+m+4) & m \equiv 0 (mod\ 4) \\ q-(3i+m+3) & m \equiv 1 (mod\ 4) \\ q-(2i+m+3) & m \equiv 2 (mod\ 4) \\ q-(2i+m+5) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_m' v_{2m}) = \begin{cases} q-(3m+1) & m \equiv 0 (mod\ 4) \\ q-(4m-1) & m \equiv 1 (mod\ 4) \\ q-(3m) & m \equiv 2 (mod\ 4) \\ q-(3m+2) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_m' v_{2m+1}) = \begin{cases} q-3 & m \equiv 0 (mod\ 4) \\ q-1 & m \equiv 1 (mod\ 4) \\ q-1 & m \equiv 2 (mod\ 4) \\ q-5 & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_{m+1}' v_{2m+1}) = \begin{cases} q-2 & m \equiv 0 (mod\ 4) \\ q-1 & m \equiv 1 (mod\ 4) \\ q-1 & m \equiv 2 (mod\ 4) \\ q-4 & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_{m+1}'v_{m+2}) = \begin{cases} q-6 & m \equiv 0 (mod\ 4) \\ q-6 & m \equiv 1 (mod\ 4) \\ q-5 & m \equiv 2 (mod\ 4) \\ q-7 & m \equiv 3 (mod\ 4) \end{cases}$$

Thus the edges are labeled distinct from 1,2,3….2q-1 for all m.
Hence the Extended Duplicate Graph EDG($J_{2,m}$) admits odd mean labeling for all m.

Theorem 3.6. Extended Duplicate graph of Jahangir Graph $J_{2,m}$ admits Even mean labeling.

Proof: We know that Extended Duplicate graph of Jahangir Graph EDG($J_{2,m}$) has 4m+2 vertices and 6m+1 edges for all m.

To label the vertices for all m, using algorithm3.3, define a map f : V → {2,4,6…..2q}.

In order to get the labels for the edges for all m, define the induced map f^* : E → {1,2,3…..2q} such that for any $v_iv_j \in$ E,$f^*(v_iv_j) = (f(v_i)+f(v_j))/2$. We get the edge labels are as follows:

$f^*(v_1'v_1) = q+1$

(i) for i= 2 to m+1

$f^*(v_1'v_i) = 2q-i+1$

$f^*(v_1v_i') = i+1$

(ii) for i=2 to m

$$f^*(v_iv_{m+i}') = \begin{cases} 2q-(5i-m+2) & m \equiv 0 (mod\ 4) \\ 2q-(3i+m) & m \equiv 1 (mod\ 4) \\ 2q-(4i-m+4) & m \equiv 2 (mod\ 4) \\ 2q-(3i+m) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_iv_{m+i+1}') = \begin{cases} 2q-(5i+m-2) & m \equiv 0 (mod\ 4) \\ 2q-(3i+m+2) & m \equiv 1 (mod\ 4) \\ 2q-(4i+m-1) & m \equiv 2 (mod\ 4) \\ 2q-(4i+m+1) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_{m+1}v_{2m+1}') = \begin{cases} 2q-(5m+2) & m \equiv 0 (mod\ 4) \\ 2q-(4m+3) & m \equiv 1 (mod\ 4) \\ 2q-(4m+5) & m \equiv 2 (mod\ 4) \\ 2q-(5m+1) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_{m+1}v_{m+2}') = \begin{cases} 2q-(2m+3) & m \equiv 0 (mod\ 2) \\ 2q-(2m+5) & m \equiv 1 (mod\ 2) \end{cases}$$

(iii) for i= 2 to m-1

$$f^*(v_i'v_{m+i}) = \begin{cases} q-(2i+m) & m \equiv 0 (mod\ 4) \\ q-(3i+m-2) & m \equiv 1 (mod\ 4) \\ q-(2i+m-1) & m \equiv 2 (mod\ 4) \\ q-(2i+m+1) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_i'v_{m+i+1}) = \begin{cases} q-(2i+m+3) & m \equiv 0 (mod\ 4) \\ q-(3i+m+2) & m \equiv 1 (mod\ 4) \\ q-(2i+m+2) & m \equiv 2 (mod\ 4) \\ q-(3i+m+4) & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_m'v_{2m}) = \begin{cases} q-(2m+4) & m \equiv 0 (mod\ 4) \\ q-(4m-2) & m \equiv 1 (mod\ 4) \\ q-(3m-3) & m \equiv 2 (mod\ 4) \\ q-(3m+1) & m \equiv 3 (mod\ 4) \end{cases} \qquad f^*(v_m'v_{2m+1}) = \begin{cases} q-2 & m \equiv 0 (mod\ 4) \\ q-1 & m \equiv 1 (mod\ 4) \\ q-1 & m \equiv 2 (mod\ 4) \\ q-4 & m \equiv 3 (mod\ 4) \end{cases}$$

$$f^*(v_{m+1}'v_{2m+1}) = \begin{cases} q-1 & m \equiv 0 (mod\ 4) \\ q & m \equiv 1 (mod\ 4) \\ q & m \equiv 2 (mod\ 4) \\ q-3 & m \equiv 3 (mod\ 4) \end{cases} \qquad f^*(v_{m+1}'v_{m+2}) = \begin{cases} q-5 & m \equiv 0 (mod\ 4) \\ q-5 & m \equiv 1 (mod\ 4) \\ q-4 & m \equiv 2 (mod\ 4) \\ q-6 & m \equiv 3 (mod\ 4) \end{cases}$$

Thus the edges are labeled distinct from 1,2,3….2q for all m.

Hence the Extended Duplicate Graph EDG($J_{2,m}$) admits Even mean labeling for all m.

Examples 3.7. Product cordial labeling of jahangir graph edg($j_{2,5}$) and odd mean and even mean labeling of extended duplicate graph EDG($J_{2,5}$) are given in Figure 1,2 and 3respectively.

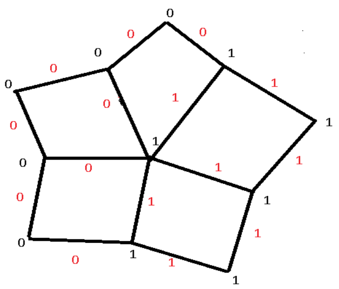

Figure 1: Product cordialof EDG($J_{2,5}$)

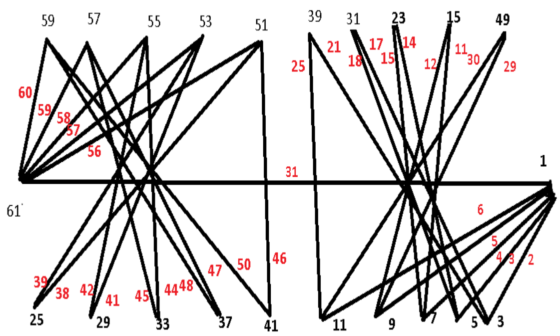

Figure 2: Odd mean of EDG($J_{2,5}$)

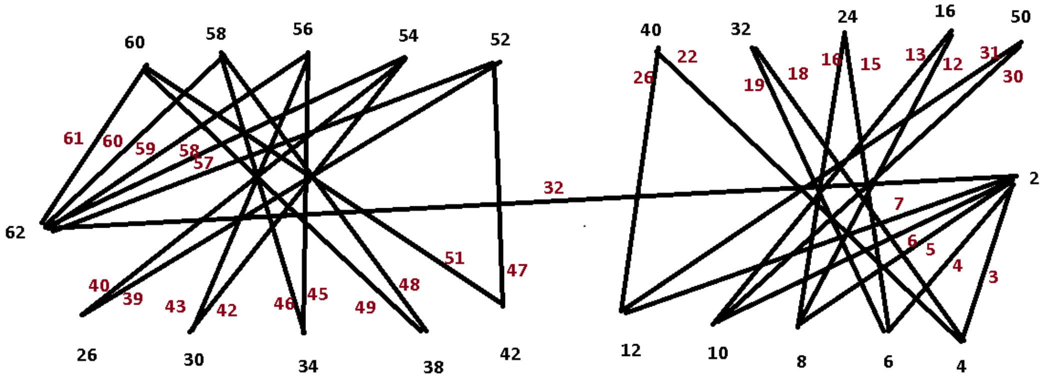

Figure 3: Even mean of EDG($J_{2,5}$)

4. Application

Labeled graphs have been applied, in determining ambiguities in X-Ray Crystallographic analysis, to Design Communication Network Addressing Systems, in determining Optimal Circuit Layouts and Radio-Astronomy, etc.

5. Conclusion

In this paper, we have proved that for all odd m,the Jahangir Graph $J_{2,m}$, admits Product Cordial Labeling. We have shown that, the Extended DuplicateGraph of Jahangir Graph $J_{2,m}$, admits odd mean and even mean labeling.

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