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Novel Approach to Fuzzy Shortest Path Problem

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Abstract. The shortest path problem is an important classical network optimization problem arising from many applications including robotics, networking, and transportation. In most situations, however, some issues of a network-theoretic problem may be uncertain. In conventional shortest path problems, there always an assumption that one who takes the decision is certain about the parameters (distance, time etc.) between different possible vertices in the network G={V,E}. But while considering the real time cases, the possibility of existence of uncertainty about the parameters between different nodes is always high. In those situations, the representation of parameters are given by fuzzy numbers and here we consider the generalized trapezoidal fuzzy numbers, can be dealt with the uncertainty using fuzzy set theory. In order to provide solution for the uncertain shortest path problem, we proposed Hybrid Ant based optimization with ranking of generalized trapezoidal fuzzy numbers, which is proposed recently, as a fitness function. The proposed model is implemented using MATLAB with the test network of 30 nodes and the results reports that the algorithm converges in a more reasonable time in comparison with conventional approaches.

Keywords: Optimization algorithm, generalized trapezoidal fuzzy number, ranking function, shortest path problem

AMS mathematics Subject Classification (2010): 94D05

1. Introduction

The Shortest Path (SP) problem has accustomed abundant absorption in the literature. Many applications such as communication, robotics, scheduling, transportation and routing, in which, Shortest Path (SP) is applied importantly. While considering a network, the arc length may represent time or cost. Therefore, in real life applications, it can be advised to be a fuzzy set. Fuzzy set theory, proposed by Zadeh [19], is frequently used to accord with uncertainties in a problem.

The fuzzy shortest path problem was first analyzed by Dubois and Parade [6]. They utilized the conventional shortest path algorithms, to treat the fuzzy shortest path problem. Klein [12] proposed a dynamic programming recursion-based fuzzy algorithm. Lin and Chen [13] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Another algorithm for this problem was presented by Okada and Gen [16,17], using a generalization of Djikstra's algorithm. In this algorithm, the weights of the arcs were considered to be interval numbers and defined by a partial order of interval numbers.

We consider a directed network G = {V, E} where V represents the finite collection of vertices (nodes) and E represents the finite collection of directed edges. It is assumed that there is only one directed edge between any two vertices. We specify a source vertex and a destination vertex. Each edge length is represented by a generalized trapezoidal fuzzy number, and the length of a path is defined to be the fuzzy sum of edge lengths along the path. We are concerned with finding a path from the source vertex to destination vertex while optimizing the fuzzy length of the path using the properties of generalized fuzzy numbers. Blue et al. [1] give taxonomy of network fuzziness that distinguishes five basic types combining fuzzy or crisp vertex sets with fuzzy or crisp edge sets and fuzzy weights and fuzzy connectivity.

In order to make evolve the design of fuzzy systems, several met heuristic learning algorithms are projected. One major improvement class uses evolutionary algorithms (EAs) [18]. These algorithms are heuristic and random. They involve populations of individuals with a particular behavior like a biological development, like crossover or mutation. Another well-known SI is the ant-colony optimization (ACO) [15]. The ACO technique is impressed by real-ant-colony observations. It is a multiagent approach that was originally projected to resolve troublesome discrete combinatorial-optimization issues, like the traveling salesman problem (TSP) [4,5]. In some studies, completely different ACO models were applied to FS design problems, [9,15]. In these studies, the antecedent-part parameters of an FS were manually appointed ahead. The consequent-part parameters were optimized in discrete space using ACO. Since solely the consequent-part parameters are optimized and also the optimization space is restricted to be discrete, the designed FSs are unsuitable for issues wherever high accuracy could be a major concern.

This paper is organized as follows. In section 2, some basic definitions are reviewed and discussed. Section 3 explains the properties of generalized trapezoidal fuzzy numbers. Section 4 describes the ranking method of generalized trapezoidal fuzzy numbers. Section 5 briefs the network terminology. Section 6 explains the proposed hybrid ant based optimization approach. In section 7, numerical example is given. Section 8 deals with the implementation and results.

2. Basic definitions

The basic definitions of some of the required concepts are reviewed in this section.

2.1. Fuzzy set

Let X be an universal set of real numbers R, then a fuzzy set is defined as

$$A= \{[x, \mu_{\tilde{A}}(x)], x \in X\}$$

This is characterized by a membership function: X → [0 1], where, $\mu_{\tilde{A}}(x)$ denotes the degree of membership of the element x to the set A.

2.2. Characteristics of generalized fuzzy number

A fuzzy set $\tilde{A}$ which is defined on the universal set of real numbers R, is known to be a generalized fuzzy number if its membership function has the following characteristics

a) $\mu_{\tilde{A}} : R \to [0,1]$ is continuous.
b) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
c) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
d) $\mu_{\tilde{A}}(x) = w$, forall $x \in [b, c]$, where $0 < w \leq 1$.

2.3. Membership function of generalized trapezoidal fuzzy number

A generalized fuzzy number $\tilde{A}$ = (a, b, c, d; w) is said to be a generalized trapezoidal fuzzy number, if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{w(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

Let $\tilde{A} = (a, b, c, d; w)$ be a generalized trapezoidal fuzzy number then

a) $R(\tilde{A}) = \frac{w(a+b+c+d)}{4}$, b) mode $\tilde{A} = \frac{w(b+c)}{2}$, c) divergence $(\tilde{A}) = w(d-a)$,
d) Left spread $(\tilde{A}) = w(b-a)$, e) Right spread $(\tilde{A}) = w(d-c)$

2.4. Fitness function

Let $\tilde{A}$= (a_1, b_1, c_1, d_1; w_1) and $\tilde{B}$= (a_2, b_2, c_2, d_2; w_2) be two triangular fuzzy numbers then the addition is defined by $\tilde{A} \oplus \tilde{B}$ = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2; w_1+w_2).

3. Properties of generalized trapezoidal fuzzy number

The properties of generalized fuzzy numbers are described from some important results that pay ways to deal with the ranking of generalized trapezoidal fuzzy numbers.

Property 3.1. Let A = (a_1 , b_1 , c_1 , d_1 ; w_1) and B = (a_2 , b_2 , c_2 , d_2 ; w_2) be two generalized trapezoidal fuzzy numbers such that
(i) R(A) =R (B), (ii) mode (A) = mode (B) and (iii) divergence (A) =divergence (B) then
(a) Left spread (A) > Left spread (B) iff $w_1b_1 > w_2 b_2$
(b) Left spread (A) < Left spread (B) iff $w_1b_1 < w_2 b_2$
(c) Left spread (A) = Left spread (B) iff $w_1b_1 = w_2 b_2$

Property 3.2 Let A =(a_1 , b_1 , c_1 , d_1 ; w_1) and B = (a_2 , b_2 , c_2 , d_2 ; w_2) be two generalized trapezoidal fuzzy numbers such that
(i) R (A) =R (B), (ii) mode (A) = mode (B) and (iii) divergence (A) = divergence (B) then
(a) Left spread (A) > Left spread (B) iff Right spread (A) > Right spread (B)
(b) Left spread (A) < Left spread (B) iff Right spread (A) < Right spread (B)
(c) Left spread (A) = Left spread (B) iff Right spread (A) = Right spread (B).

4. Ranking of generalized trapezoidal fuzzy numbers

Let A = (a_1 , b_1 , c_1 , d_1 ; w_1) and B = (a_2 , b_2 , c_2 , d_2 ; w_2) be two generalized trapezoidal fuzzy numbers then use the following steps to compare A and B

Step 1: Find R (A) and R (B)

Case (i) If R (A) > R (B) then A >B

Case (ii) If R (A) < R (B) then A < B

Case (iii) If R(A) = R(B) then go to step 2.

Step 2: Find mode (A) and mode (B)

Case (i) If mode (A) > mode (B) then A > B

Case (ii) If mode (A) < mode (B) then A < B

Case (iii) If mode (A) = mode (B) then go to step 3.

Step 3: Find divergence (A) and divergence (B)

Case (i) If divergence (A) > divergence (B) then A > B

Case (ii) If divergence (A) < divergence (B) then A < B

Case (iii) If divergence (A) = divergence (B) then go to step 4

Step 4: Find Left spread (A) and Left spread (B)

Case (i) If Left spread (A) > Left spread (B)

i.e., $w_1b_1 > w_2 b_2$ then A > B (from property 3.1)

Case (ii) If Left spread (A) < Left spread (B)

i.e., $w_1b_1 < w_2 b_2$ then A < B (from property 3.1)

Case (iii) If Left spread (A) = Left spread (B)

i.e., $w_1b_1 = w_2 b_2$ then go to step 5 (from property 3.1)

Step 5: Find w_1 and w_2

Case (i) If $w_1 > w_2$ then A > B

Case (ii) If $w_1 < w_2$ then A< B

Case (iii) If $w_1 = w_2$ then A ~ B

5. Network terminology

Consider the directed network G(V, E) consisting of a finite set of nodes V = {1,2……n} and a set of m directed edges E $\subseteq V \times V$. Each edge is denoted by an ordered pair (i,j) where i, j$\in$ v and i$\neq$ j. In this network, we specify two nodes namely source node and the destination node. $\widetilde{d_{ij}}$ denotes the generalized trapezoidal fuzzy number associated with the edge (i,j). The fuzzy distance along the path P is denoted by $\tilde{d}$(P) = $\sum_{i,j\in P} \widetilde{d_{ij}}$.

6. Hybrid ant based optimization

Hybrid Ant based optimization is evolved in swarm intelligence from the ant colony algorithms family. The swarm intelligence involves met heuristic optimizations in which every ant has its own way of searching food from the colony to the food source. Over the iterations, the ant with the optimized path is selected and the process carried out till the optimized path becomes constant or simply equilibrium state. In the proposed method, a hybrid approach is designed with the generalized trapezoidal fuzzy numbers and Ant Colony Algorithm (ACO) to find the Shortest Path (SP) problem.

Assumptions

As described in network terminology, n represents the number of vertices in the network and it is assumed that the total possible directed edges in the network will be around$2^{n/2}$. Hence $2^{n/2}$edges are assumed as optimized number and $2^{n/2}$ ants are considered.

The property of each ant is assumed to be unique and every ant chooses only the valid vertex for its next visit. Valid vertex is the vertex in which there exists a path between both vertices.

Algorithm

Step 1: Generate the network with vertices and edges with generalized trapezoidal fuzzy numbers

Step 2: $2^{n/2}$ ants are placed in the source vertex.

Step 3: Apply uniqueness for every ant, in selecting the next possible vertex.

Step 4: Continue the traversal till the ant reaches the destination vertex.

Step 5: Calculate the fitness of each path in which, the ant is selected using definition 2.4

Step 6: Compare the path address that possess ranking of generalized trapezoidal fuzzy numbers by definition (Section 4)

Step 7: Choose the optimized solution

Step 8: Repeat the steps 2 to 7 by the given number of iterations.

Step 9: Report the ant with minimal length as the solution when the results of rank, mode, divergence, left spread and right spread becomes constant through the proceeding iterations.

7. Numerical example

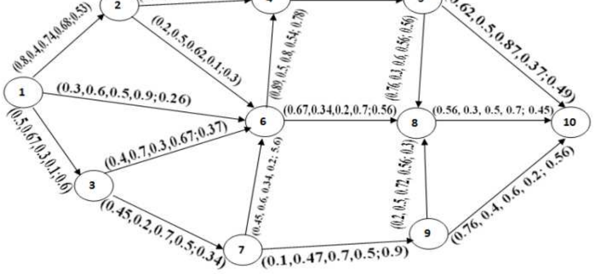

Figure 7.1:

We Consider the network G= {V, E} of 10 vertices. According to the assumption, we take $2^5\,(2^{10/2})$ ants to serve for the finding of shortest path. Every edge is represented by the generalized trapezoidal fuzzy number. The fitness, ranking and properties of generalized trapezoidal fuzzy number calculated.

7.1. Selection of ants

Ants that has to be used in the algorithm is based on the number of vertices and possible number of edges that is given by $2^{n/2}$ (ie) $2^5 = 32$.

7.2. Random traversal of ants

Ants are assume to have unique characteristics and the select of path is based on natural selection. The fuzzy distance between the paths is explained in the fitness function of generalized trapezoidal fuzzy number. It is also assumed that the paths traveled by the ants are valid.

Present Path	Selection	Fitness
1	6	(0.3,0.6,0.5,0.9;0.26)
1-6	8	(0.97,0.94,0.7,1.6;0.82)
1-6-8	10	(1.53,1.24,1.3,0.86;1.27)

Table 7.1: Example calculation for ant (say a_{10})

Ants	Path	Fitness
a_{10}	1-6-8-10	(1.53,1.24,1.3,0.86;1.27)
a_{15}	1-3-7-9-10	(1.81,1.74,2.3,1.3;2.4)
a_{20}	1-2-4-5-10	(2.89,1.69,2.56,1.35;2.23)
a_{25}	1-6-4-5-10	(2.61,1.94,2.28,2.01;2.29)
a_{30}	1-2-4-5-10	(2.89,1.69,2.56,1.35;2.23)

Table 7.2: The sample ants with their fitness from source to destination

7.3. Ranking of paths selected by ants

The ranking of generalized trapezoidal fuzzy numbers are used for the comparison of paths.

Example 7.3.1. Let (A) = (0.2, 0.4, 0.6, 0.8; 0.35) and (B) = (0.1, 0.2, 0.3, 0.4; 0.7)

Step 1: (A) = 0.175 and (B) = 0.175. Since (A) = (B), so go to step 2

Step 2: Mode (A) = 0.175 and mode (B) = 0.175. Since mode (A) = mode (B), so go to step 3

Step 3: Divergence (A) = 0.21 and divergence (B) = 0.21. Since divergence (A) = divergence(B), so go to step 4.

Step 4: Left spread (A) = 0.7 and Left spread (B) = 0.7. Since Left spread (A) = Left spread (B), so go to step 5

Step 5: w1 = 0.35, w2 = 0.7 since w1 < w2 => A < B For every iteration, the shortest path is updated. Whenever the shortest path obtained becomes constant for remaining iterations, the process is terminated and the final result is obtained.

8. Implementation and results

The implementation is carried out in Matlab 8.1 (R_{2013a}) 32 bit student version. The assumptions explained before are implemented and the selection of valid path is controlled using adjacency matrix.

The network G = {V,E} of 30 nodes with the edges of generalized trapezoidal fuzzy number is initialized. The algorithm is implemented as per the given description and demonstrated through numerical example. 3278 ants are separately generated for all iterations and the shortest path obtained is measured.

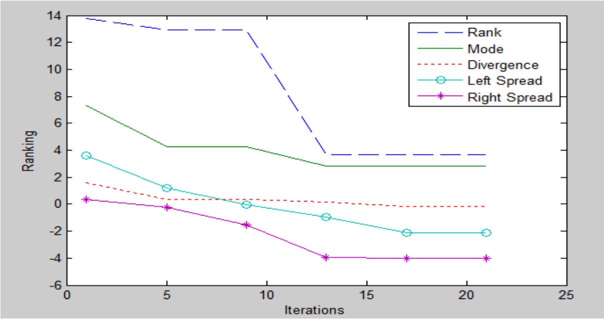

Figure 8.1: Comparison on ranking components along iterations

The random function with time constraint is used to implement the ants with unique behavior in all iterations. The outcome of shortest path is comprised and the rank, mode, divergence, left spread and right spread of shortest path in all iterations is compared using ranking method.

From the figure 8.1**,** it is clear that the ranking method depends on the rank,mode, divergence, left spread and right spread. The path at which all the components attain equilibrium is considered to be the shortest path. Here, iterations around 17-21 posses constant values for all components of ranking and the path obtained is considered to be shortest path.

9. Conclusion and future implication

The Shortest Path (SP) problem in many applications is uncertain in parameters (Distance, Range, etc.). Hence, there occurs the necessity of fuzzy numbers for uncertain parameters. We propose a hybrid ant based optimization algorithm along with the generalized trapezoidal fuzzy numbers and ranking method. The result clears that the proposed algorithm comprises the shortest path with minimal iterations using swarm intelligence.

The future enhancement of the research includes, reduction of assumptions carried out in proposed algorithm and enhance the algorithm for all pair shortest path problem with the minimal selection of ants and iterations involved.

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