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Weakly *g Closed Sets in Topological Spaces

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Abstract. The aim of this paper is to introduce a new class of sets called w*g- closed sets and investigate some of the basic properties of this class of sets which is the weaker form of *g closed sets.

Keywords: w *g closed sets, w *g open sets

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1. Introduction

Levine introduced generalized closed sets [10] and Semiopen sets [11]. Abd El-Monsef, El-Deeb and Mahmoud[1] introduced β sets and Njastad introduced α sets and Mashour, Abd El-Monsef and El-Deeb introduced Pre open sets. Andregvic [2] called β sets as Semipre open sets. Veerakumar introduced g* closed sets [18]. AlsoVeera kumar introduced the notion of *g closed sets and studied its properties. The aim of this paper is to introduce w *g closed sets and investigate some fundamental properties and the relations with related generalized closed sets .

2. **Preliminaries**

Definitions 2.1. A subset A of a space (X, τ) is called

1. A **semi-open** set if $A \subset cl$ (int A) and a semi-closed set if int (cl A) $\subset$A and SCL(X,τ) denotes the class of all semi-closed subsets of (X,τ).
2. A **preopen** set if $A \subset$int cl (A) and a preclosed set if cl (int (A)) $\subset$ A, PC(X, τ) denotes the class of all preclosed subsets of (X, τ).
3. An **α-open** set if $A \subset$int(cl(int(A))) and an α-closed set if cl(int cl(A)) $\subset$ A, αC (X, τ) denotes the class of all α-closed subsets of(X, τ).
4. A **semi-preopen** set (=β-open) if $A \subset$cl (int (cl (A))) and a semi-preclosed set (=β-closed) if int (cl (int A)) $\subset$A. The semi-closure (respectively pre-closure, α-closure, semi-preclosure) of a preclosed) sets that contain A and is denoted by sclA (rep. pclA, α cl A,sp cl A).
5. A **Regular open** if int ((cl (A)) =A and Regular closed if cl (int (A)) =A.

Definition 2.2. A subset A of a space (X, τ) is called

1. A generalized closed (briefly g-closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$U and U is open in (X,τ); the complement of a g-closed set is called a g-open set.
2. A α **-generalized** closed set (briefly αg- closed) if αcl (A)$\subseteq$U whenever A $\subseteq$ U and U is open in (X, τ); the complement of an αg-closed se is called a αg-open set.
3. A generalizedα - closed set (briefly gα - closed) if αcl(A)$\subseteq$U whenever A $\subseteq$ U and U is α - open in (X, τ). ; the complement of a gα -closed set is called a gα -open set.
4. A rg- closed set if cl(A)$\subseteq$U whenever A $\subseteq$ U and U is Regular open in (X, τ); the complement of a rg -closed set is called a open set.
5. A generalized semi- pre closed set (briefly gsp- closed) if spcl(A)$\subseteq$U whenever A $\subseteq$ U and U is open in (X, τ). ; the complement of a gsp-closed set is called a gsp-open set.
6. A ĝ closed set if cl(A)$\subseteq$U whenever A $\subseteq$ U and U is Semi open in (X, τ); the complement of a ĝ -closed set is called a ĝ –open.
7. A g# closed if cl (int(A)) $\subseteq$U whenever A$\subseteq$ U and U is αg open in (X, τ); the complement of a g# -closed set is called a g# –open.
8. A g*-closed set if cl(A)$\subseteq$U whenever A $\subseteq$ U and U is g open in (X,τ); the complement of a g*-closed set is called a g*-open.
9. A *g closed if cl(A) $\subseteq$U whenever A$\subseteq$U and U is ĝ -open the complement of a *g-closed set is called a *g-open.

3. w *g- closed sets and their properties

Definition 3.1. A subset A of a space (X,τ) is called a weakly *g-closed set(briefly w*g closed) if cl(int(A))$\subseteq$U whenever A$\subseteq$U and U is ĝ open.

Example 3.2. Let X = {a, b, c} and τ = {φ, X, {a}, {b}, {a,b}}.
Closed sets are φ, X, {b,c},{a,c},{c}.
Semi open sets are {φ, X, {a}, {b}, {a,b},{a,c},{b,c}}.
ĝ open sets are {φ, X, {a}, {b}, {a,b}}
w*g closed sets are {φ, X,{b,c},{a,c},{c}}.

Theorem 3.3. If a subset A of a topological space (X, τ) is closed then it is w *g closed.
Proof: Let A be closed then A = cl (A).
Let A$\subseteq$U, where U is ĝ open
Now cl (int (A)) $\subseteq$cl (A) = A$\subseteq$U
=> A is w *g closed.
The converse of the above theorem need not be true.

Example 3.4. Let X= {a,b,c} τ= { φ,X,{a},{a,b}}.
Closed sets {φ, X, {b,c},{c}}.
Semi open sets = {φ,X,{a},{a,b},{a,c}}.
ĝ –open sets are {φ, X, {a,b},{a},{b}}
Here {a} is w*g is closed but not closed.

Theorem 3. 5. If a subset A of a topological space (X, τ) is Regular closed then it is w*g closed.
Proof: Since A is regular closed cl (int (A)) =A.
Let $A \subseteq U$ and U is $\hat{g}$ –open.Now Cl (int (A)) =A $\subseteq$ U
Thus A is w *g closed.
The converse of the above theorem need not be true.

Example 3.6. Let X= {a,b,c} τ = {φ, X, {a}, {b}, {a,b}} closed sets {φ, X,{b,c},{a,c},{c}}.
Regular closed sets are {φ, X, {b,c},{a,c}}
Here {c} is w*g closed but not regular closed.

Theorem 3.7. If a subset A of a topological space (X, τ) is pre-closed then it is w*g closed.
Proof: Let $A \subseteq U$ where U is $\hat{g}$ open.
Since A is pre-closed, cl (int (A)) $\subseteq$ A,
=>cl (int (A)) $\subseteq$ U.
Thus A is w *g closed.
The converse of the above theorem need not be true.

Example 3.8. Let X= {a,b,c} τ = {φ, X, {a}, {a,c}} closed sets { φ, X,{b,c},{b}}.
Semiopensets are {φ, X, {a},{a,b},{a,c}}
$\hat{g}$ –open sets are {φ, X, {a,c},{a,}}
Here {a,b} is w*g closed but not pre-closed.

Theorem 3.9. If a subset A of a topological space (X, τ) is α - closed then it is w*g closed.
Proof : Let $A \subseteq U$ and U be $\hat{g}$ open.
Since A is α closed cl (int (cl (A)) $\subseteq$ A,
Thus cl (int (A)) $\subseteq$ U
=> A is w *g closed.
The converse of the above theorem need not be true.

Example 3.10. Let X= {a,b,c} τ = {φ, X, {a}, {a,c}}.
Closed sets { φ, X,{b,c},{b}}.
α Open sets {φ, X, {a}, {a,b},{a,c}}.
α Closed sets {φ, X, {b,c},{b},{c}}.
Semi opensets are {φ, X, {a}, {a,b},{a,c}}
$\hat{g}$ –open sets are {φ, X, {a,c},{a,}}
Here {a,b}is w *g closed but not α - closed.

Theorem 3. 11. If a subset A of a topological space (X, τ) is *g closed then it is w*g closed.
Proof: Let $A \subseteq U$ is $\hat{g}$ open.

Since A is *g closed cl (A) $\subseteq$U, whenever A$\subseteq$U and U is ĝ open.

Now cl (int (A)) $\subseteq$cl (A) $\subseteq$U where U is ĝ open

=> A is w *g closed.

The converse of the above theorem need not be true.

Example 3.12. Let X= {a,b,c} τ= { φ,,{b},{a,b}}.

Closed set {φ, X, {a,c},{c}}

Semi pre open sets = {φ, X, {b,c},{b},{a,b }}

Semi pre closed sets = {φ, X, {a}, {a,c},{c }}

Here {a} is w *g closed but not *g closed.

Theorem 3.13. If a subset A of a topological space (X, τ) is g* closed then it is w*g closed.

Proof: Since A is g* closed**,** cl (A) $\subseteq$U, A$\subseteq$U where U is g-open.

Let A$\subseteq$U, where U is ĝ open **.** Now cl (int (cl(A)) $\subseteq$ cl (A) $\subseteq$ U**,** where U is g -open.

Since ĝ open => g-open, cl (int (A)) $\subseteq$ U where U is ĝ open.

Thus A is w *g closed.

The converse of the above theorem need not be true.

Example 3.14. Let X= {a,b,c} τ = {φ, X, {a}, {a,b}}.

In this space {b} is not g∗ closed. But {b} is w *g closed.

Theorem 3.15. If a subset A of a topological space (X, τ) is w*g closed then it is gsp closed.

Proof:. Let A be w *g closed.

Let A$\subseteq$U, where U is open .Since A is w*g closed we have

cl (int (A)) $\subseteq$ U.Now open => ĝ open

Int (cl (int (A))) $\subseteq$ U. A U int (cl (int (A))) $\subseteq$ A $\subseteq$ U.That is Spcl (A) $\subseteq$ U

=> A is gsp closed.

The converse of the above theorem need not be true.

Example 3.16. Let X = {a,b,c}**,** τ = {φ, X, {a}, {b}, {a,b}} closed sets are { φ, X,{b,c},{a ,c},{c}}

Semi pre open sets are {φ, X, {a}, {b}, {,a,c},{b,c}}

Semi pre closed sets = {φ, X, {b,c} , {a ,c},{b},{a}} ĝ open sets are ={ φ, X,{a,b},{a} , {b}} {a} is gsp closed but not w *g closed

Theorem 3.17. If a subset A of a topological space (X, τ) is g# closed then it is w*g closed.

Proof: Let A$\subseteq$U where U is ĝ open.

Since A is g# closed cl (int (A)) $\subseteq$U whenever A$\subseteq$ U and U is αg open.

Since ĝ open=> αg open, we have cl(int(A)) $\subseteq$U whenever A$\subseteq$ U and U is ĝ open

=>A is w *g closed

The converse of the above theorem need not be true.

Example 3.18. Let X = {a, b, c}, τ = {φ, X, {a}, {b,c}} closed sets are { φ, X,{b,c},{a}} semi open sets are {φ, X, {a}, {b,c}} $\hat{g}$ open sets are { φ, X,{a,b},{a},{b},{a,c},{b,c},{c}}.
Here {b} is w *g closed but not g# closed

4. Properties of w*g closed sets

Theorem 4.1. A set A is w *g closed iff cl(int(A) –A contain no non empty $\hat{g}$ closed sets

Proof: Let F be $\hat{g}$ closed sets such that F $\subseteq$ cl (int (A)) – A (1)

Then F $\subset$ A^c and

F^c is $\hat{g}$ open & A $\subseteq$ F^c

By the definition of w *g closed set cl (int(A)) $\subseteq$ F^c

F $\subseteq$ $(cl (int (A))^c$ (2)

∴ From (1)and (2) F = φ.

Conversely Let A $\subseteq$ F where U is $\hat{g}$ open.

If cl (int (A)) $\not\subset$ U then cl (int(A)) $\cap U^c$ is non empty $\hat{g}$ closed set of cl (int(A)) – A, which is a contradiction.

Theorem 4.2. A w *g closed is regular closed iff cl (int (A)) – A is $\hat{g}$ closed.

Proof: Since A is regular closed A = cl (int (A))

∴ cl (int(A)) – A = φ is regular closed and hence $\hat{g}$ closed

Conversely, suppose cl (int (A)) – A is $\hat{g}$ closed by theorem4.1

cl (int(A)) – A = φ

=> A is regular closed

Theorem 4.3. If a subset A of a topological space X is open and w *g closed then A is closed.

Proof: Let A be open. Since every open set is $\hat{g}$ open set, A is $\hat{g}$ open. since A is w *g closed, cl(int(A)) $\subseteq$ U where U is $\hat{g}$ open.

Taking A = U we've cl(int(A)) $\subseteq$ A

=>cl (A) $\subseteq$ A (since Int(A) = A)

But A $\subseteq$ cl(A).

Hence cl(A)=A and hence A is closed.

Theorem 4.4. If A is both open and w *g closed in X then it is both regular open and regular closed in X.

Proof: If A is open and w *g closed, then by theorem (4.3) A is closed

i.e. cl(A) = A.

Also as A is open, int(A) = A

=>A = int (cl (A))

=>A is regular open

Also A = cl(int(A))
=>A is regular closed.

Theorem 4.5. If A is both open and w *g closed in X then it is rg-closed.
Proof: If A is both open and w *g closed in X then by theorem (4.4) A is closed
i.e. cl(A) = A.
Also By theorem (4.4), A is regular open and regular closed.
Let $A \subseteq U$, where U is regular open.

Therefore, cl(A) $\subseteq$ U= A **,** whenever A $\subseteq$ U is regular open
=>A is **r**g-closed.

Theorem 4.6. If A is both semi-open and w *g closed in X then it is *g closed.
Proof: Let A be both w *g closed and semi-open.
Let A $\subseteq$ U where U is $\hat{g}$ open.
Then by the definition of w *g closed, cl(int(A)) $\subseteq$ U since A is semi-open, cl(A) $\subseteq$ cl(int(A)) **,** ([2] Theorem 1.1)
=>cl (A) $\subseteq$ cl(int (A)) $\subseteq$ U
=>cl (A) $\subseteq$ U where U is $\hat{g}$ -open
Hence A is *g closed.

Theorem 4.7. Let A be w *g closed and suppose that F is closed then A∩F is w *g closed.

Proof: Let A be a w *g closed and F be closed.
To Prove : A∩F is w *g closed
Let A∩F $\subseteq$ U, U is $\hat{g}$ –open since F is closed, A∩F is closed in A A∩F $\subseteq$ A
=>cl (int (A∩F)) $\subseteq$ cl ((A∩F)) = A∩F $\subseteq$ U,
=>cl (int (A∩F)) $\subseteq$ U
=>A∩F is w *g closed.

Theorem 4.8. If a subset A of (X,τ) is both closed and α g-closed then A is w *g-closed
Proof: Let A be α g-closed **.**
Let A $\subseteq$ U where U is $\hat{g}$ –open since every $\hat{g}$ –open set is α g-open, U is α g-open
α cl(A) $\subseteq$ Uwhich implies $\bigcup$ cl(int(cl(A)) $\subseteq$ U
Since A is closed cl (A) =A, cl (int (A)) $\subseteq$ U
=>A is w*g closed.

Theorem 4.9. Let (X,τ) be a topological space and A $\subseteq$ Y $\subseteq$ X. If A is w *g closed in X then A is w *g closed relative to Y.
Proof: Let A $\subseteq$ Y∩U where U is $\hat{g}$ –open in (X,τ)**,** since A is w *g closed,
=>cl (int (A)) $\subseteq$ U where A $\subseteq$ U and U is $\hat{g}$ –open

$\Rightarrow Y \cap cl(int(A)) \subseteq Y \cap U$

i.e. $cl_y(int(A)) \subseteq Y \cap U$

$\Rightarrow$A is w *g closed in Y.

Theorem 4.10. If a subset A of a topological space (X,τ) is nowhere dense, then it is w *g closed

Proof: If A is nowhere dense, then $int(A) = \varphi$

Let $A \subseteq U$ where U is $\hat{g}$ –open

$\Rightarrow cl(int(A)) = cl(\varphi) = \varphi \subseteq U$ and hence A is w*g closed.

5. Weakly *g open sets

Definition 5.1. A subset A of a topological space (X,τ) is called weakly *g open(w *g open) set if its complement is w *g closed in (X,τ).

Theorem 5.2. If subset A of a topological space (X,τ) is open then it is w *g open

Proof: Let A be an open set in (X,τ).

Then A^c is closed in (X,τ) since every closed set is w *g-closed, A^c is w *g-closed

Hence A is w *g open.

Theorem 5.3. A subset A of a topological space (X,τ) is regular open then it is w *g open set

Proof: Let A be a regular open then A^c is regular closed

i.e. $A^c = cl(int(A^c))$.

Let $A^c \subseteq U$, where U is $\hat{g}$ –open

$\Rightarrow cl(int(A^c)) = A^c \subseteq U$, where U is $\hat{g}$ –open

$\Rightarrow cl(int(A^c)) \subseteq U$, where U is $\hat{g}$ –open

$\Rightarrow A^c$ is w *g closed.

Hence A is w *g open.

Proposition 5.4.

1. Every α open set in(X,τ) is w*g open in (X,τ) but not conversely.
2. Every pre open set in(X,τ) is w*g open in (X,τ) but not conversely.
3. Every *g open set in(X,τ) is w*g open in (X,τ) but not conversely.
4. Every g# open set in(X,τ) is w*g open in (X,τ) but not conversely.
5. Every w*g open set in(X,τ) is gsp open in (X,τ) but not conversely.

Theorem 5.5. A subset A of a topological space (X,τ) is w *g open if and only if $G \subseteq int(cl(A))$ whenever $G \subseteq A$ and G is $\hat{g}$ –closed.

Proof: Assume that A is w *g open then A^c is w *g closed.

Let G be $\hat{g}$ –closed set in (X,τ) contained in A.

Then G^c is $\hat{g}$ –open containing A^c

i.e. $A^c \subseteq G^c$

$\Rightarrow cl(int(A^c)) \subseteq G^c$ (since A^c is w *g closed)

$\Rightarrow G \subseteq \text{int}(\text{cl}(A))$

Conversely, assume that $G \subseteq \text{int}(\text{cl}(A))$ whenever $G \subseteq A$ (1)

Now since G^c is $\hat{g}$ –open containing A^c

i.e. $A^c \subseteq G^c$

$\text{cl}(\text{int}(A^c) \subseteq G^c$

$\Rightarrow A^c$ is w*g closed

$\Rightarrow A$ is w*g open.

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