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Intuitionistic Fuzzy Pushdown Automata and Intuitionistic Fuzzy Context-Free Languages

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Abstract. In this paper intuitionistic fuzzy pushdown automata, acceptance by empty stack and acceptance by final states and their equivalence is proved. It follows that intuitionistic fuzzy pushdown automata with empty stack and IFPDAs are equivalent. We propose the notions of intuitionistic fuzzy context-free grammars (IFCFGs), intuitionistic fuzzy languages generated by IFCFGs. Additionally, we introduce the concepts of intuitionistic Chomsky normal form grammar and Greibach normal form grammar.

Keywords: Intuitionistic fuzzy set; instantaneous description; IFCFG; IFCNF; derivation

AMS Mathematics Subject Classification (2010): 18B20

1. Introduction

Intuitionistic fuzzy set (IFS) introduced by Atanassov [1], which emerges from the simultaneous consideration of the degrees of membership and nonmembership with a degree of hesitancy has been found to be very useful in dealing with problems involving vagueness and uncertainty. IFS theory has been found to support a wealth of important applications in many fields such as fuzzy multiple attribute decision making, fuzzy pattern recognition, medical diagnosis, fuzzy control and fuzzy optimization [2]. Since formal languages are not powerful enough in processing human languages, Lee and Zadeh [3] introduced the notion of fuzzy languages and gave some characterizations. Fuzzy grammars, automata and languages have contributed to the development of lexical analysis and in simulating fuzzy discrete event dynamical systems and hybrid systems [4].

To enhance the processing ability of fuzzy automata, the membership grades were extended to several general algebraic structures. Primarily, Qiu has established automata theory based on complete residuated lattice-valued logic [5]. Li and Pedrycz [6] have studied automata theory with membership values in lattice-ordered monoids. Jin and Li [7] have established a fundamental framework of fuzzy grammars based on lattices. Fuzzy pushdown automata theory based on complete residuated lattice-valued logic has been established in recent years by Xing et al. [8]. This paper deals with the notions of intuitionistic fuzzy context-free grammars and intuitionistic fuzzy pushdown automata and some results concerning them. Intuitionistic fuzzy context-free languages are expected to reduce the gap between formal languages and the imprecision associated with natural languages.

The remaining part of the paper is arranged as follows. Section 2 describes some basic concepts of IFSs. Section 3 gives the definitions of intuitionistic fuzzy pushdown automata and languages. In section 4, we establish that every intuitionistic Fuzzy PDA that accepts intuitionistic Fuzzy Context Free Language with empty stack has an equivalent intuitionistic Fuzzy PDA that accepts the same language with final state and vice-versa. It follows that intuitionistic Fuzzy PDA with final states and empty stack are equivalent. Section 5 is devoted to the study of intuitionistic fuzzy context-free grammars (IFCFGs) and intuitionistic fuzzy context-free languages (IFCFLs). The notions of intuitionistic fuzzy Chomsky normal form (IFCNF) and intuitionistic fuzzy Greibach normal form (IFGNF) have been proposed. Conclusions and directions for future work are presented in Section 6.

2. Basic concepts

Definition 1. Let X be the universe of discourse, an Intuitionistic fuzzy set (IFS) A in X is defined as an object of the form

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\} \tag{1}$$

here the functions $\mu_A: X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership and degree of non-membership respectively. For every element $x \in X$

$$0 \le \mu_A(x) + \nu_A(x) \le 1. \tag{2}$$

For the sake of simplicity, we use the notation $A = (\mu_A, \nu_A)$ instead of $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$.

Note that if $\mu_A(x) + \nu_A(x) = 1$ for every $x \in X$, then IFS A reduces to a fuzzy set in X.

Definition 2. Let $\{A_i \mid i \in I\}$ be a family of IFSs in X. Then the infimum and supremum operations of IFSs are defined as follows:

$$\cap A_i = \{< x, \wedge \mu_{Ai}(x), \vee \nu_{Ai}(x) > \mid x \in X, i \in I\}$$
$$\cup A_i = \{< x, \vee \mu_{Ai}(x), \wedge \nu_{Ai}(x) > \mid x \in X, i \in I\} \tag{3}$$

here $\wedge$ and $\vee$ denote the infimum and supremum of real numbers respectively.

Definition 3. Two IFSs $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ are said to be *equal* if $\mu_A = \mu_B$ and $\nu_A = \nu_B$

Definition 4. Let X and Y be any two sets then an *Intuitionistic Fuzzy Relation* (IFR) from X to Y is an Intuitionistic fuzzy subset of $X \times Y$. The expression R is given by

$$R = \{< (x, y), \mu_R(x, y), \nu_R(x, y) > \mid x \in X, y \in Y\},$$

here the mappings $\mu_R: X \times Y \to [0, 1]$ and $\nu_R : X \times Y \to [0, 1]$ satisfy

$$0 \le \mu_R(x, y) + \nu_R(x, y) \le 1, \text{ for all } (x, y) \in X \times Y \tag{4}$$

Definition 5. *Intuitionistic Fuzzy Binary Relation* (IFBR) from X to X is an Intuitionistic fuzzy subset of $X \times X$. It is given by the relation

$$R = \{< (x, y), \mu_R(x, y), \nu_R(x, y) > \mid (x, y) \in X \times X\},$$

where the mappings $\mu_R : X \times X \to [0, 1]$ and $\nu_R: X \times X \to [0, 1]$ satisfy

$0 \leq \mu_R(x, y) + \nu_R(x, y) \leq 1$, for all $(x, y) \in X \times X$.

We will again use the notation $R = (\mu_R, \nu_R)$ instead of $R = \{< (x, y), \mu_R(x, y), \nu_R(x, y) > | (x, y) \in X \times X \}$. The *reflexive and transitive closure* of IFBR R over X is $R^* = \cup R^n$ for n = 0 to ∞. Here $R^{n+1} = R^n \circ R$ for $n \geq 0$ and $R^0 = (\mu_{id}, \nu_{id})$ defined as

$\mu_{id}(x, y) = 1$ if x = y, 0 otherwise and

$\nu_{id}(x, y) = 0$ if x = y, 1 otherwise.

for all $(x, y) \in X \times X$.

Definition 6. Let $R = (\mu_R, \nu_R)$ be an IFR from X to Y and $S = (\mu_S, \nu_S)$ be an IFR from Y to Z, the *composition* of *IFR*s R and S is an IFS $R \circ S = (\mu_{R\circ S}, \nu_{R\circ S})$ from X to Z given by

$$\mu_{R\circ S}(x, z) = \vee (\mu_R(x, y) \wedge \mu_S(y, z) \mid y \in Y)$$
$$\nu_{R\circ S}(x, z) = \wedge (\nu_R(x, y) \vee \nu_S(y, z) \mid y \in Y), \text{ for all } (x, z) \in X \times Z. \quad (5)$$

Definition 7. Let $A = (\mu_A, \nu_A)$ be an IFS from X to X. Then the *image set* of A denoted by Im(A) is defined as $\mathrm{Im}(A) = \mathrm{Im}(\mu_A) \cup \mathrm{Im}(\nu_A) = \{\mu_A(x) \mid x \in X\} \cup \{\nu_A(x) \mid x \in X\}$

Definition 8. For $\lambda_1, \lambda_2 \in [0, 1]$ where $\lambda_1 + \lambda_2 \leq 1$, *($\lambda_1$, λ_2)-cut of IFS A* is defined as $A_{(\lambda_1, \lambda_2)} = \{ x \in X \mid \mu_A(x) \geq \lambda_1 \text{ and } \nu_A(x) \leq \lambda_2\}$.

And *support set* of A is defined by supp $(A) = \{x \in X \mid \mu_A(x) > 0, \nu_A(x) < 1\}$. If supp ($A$) is finite, then A is called finite IFS.

3. Intuitionistic fuzzy pushdown automata (IFPDA)

Definition 9. An intuitionistic fuzzy pushdown automaton (IFPDA) is a *seven tuple*
$M = (Q, \Sigma, \Gamma, \delta, I, Z_0, F)$, where
Q is a finite nonempty set of states;
Σ is a finite nonempty set of input symbols;
Γ is a finite nonempty set of stack symbols
$\delta = (\mu_\delta, \nu_\delta)$ is a finite IF subset of $Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \times (Q \times \Gamma^*)$ defined by

$\mu_\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \times (Q \times \Gamma^*) \rightarrow [0, 1]$ and

$\nu_\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \times (Q \times \Gamma^*) \rightarrow [0, 1]$

$Z_0 \in \Gamma$ is the start stack symbol;
$I = (\mu_I, \nu_I)$ and $F = (\mu_F, \nu_F)$ are intuitionistic fuzzy subsets of Q, which are called the intuitionistic fuzzy subsets of initial and final states respectively.

Definition 10. The state (configuration) of IFPDA is an IF subset of $Q \times \Sigma^* \times \Gamma^*$ given by (q, w, u, μ, ν) which indicates that IFPDA is currently in state q with w as unread part of input string, u on the top of the stack with the degree of membership and nonmembership $\mu, \nu \in [0,1]$ respectively.

Definition 11. The *move* of an IFPDA M denoted by $\vdash_M = (\mu_{\vdash M}, \nu_{\vdash M})$ is an IFBR on $(Q \times \Sigma^* \times \Gamma^*)$ to $(Q \times \Sigma^* \times \Gamma^*)$ defined as $(q, aw, Zy) \vdash_M (p, w, xy) = (\mu_{\vdash M}, \nu_{\vdash M})$ where

$\mu_{\vdash M}((q, aw, Zy), (p, w, xy)) = \mu_\delta(q, a, Z, p, x)$ and

$\nu_{\vdash M}((q, aw, Zy), (p, w, xy)) = \nu_\delta(q, a, Z, p, x)$.

Here $p, q \in Q$, $a \in \Sigma \cup \{\varepsilon\}$, $w \in \Sigma^*$ and $x, y \in \Gamma^*$. (6)

$\vdash_M{}^*$ is the reflexive and transitive closure of $\vdash_M$. When no confusion arises, we denote $\vdash_M$ by $\vdash$ and $\vdash_M{}^*$ by $\vdash^*$, respectively. Note that $\vdash^* = (\mu_{\vdash}{}^*, \nu_{\vdash}{}^*)$ is an intuitionistic fuzzy subset defined as follows:

If $(q_1, w_1, \Upsilon_1, \mu_1, \nu_1) \vdash (q_2, w_2, \Upsilon_2, \mu_2, \nu_2) \vdash (q_3, w_3, \Upsilon_3, \mu_3, \nu_3) \vdash \ldots \vdash (q_k, w_k, \Upsilon_k, \mu_k, \nu_k)$ is the sequence of moves in IFPDA then $(q_1, w_1, \Upsilon_1, \mu_1, \nu_1) \vdash^* (q_k, w_k, \Upsilon_k, \mu_k, \nu_k)$. Here $q_i \in Q$, $w_i \in \Sigma^*$, $\Upsilon_i \in \Gamma^*$ and $\mu_i, \nu_i \in [0, 1]$ for i = 1 to k.

Definition 12. Let M = $(Q, \Sigma, \Gamma, \delta, I, Z_0, F)$ be an IFPDA. The IF *language accepted by* M can be defined in two ways:

i. Language accepted by M with *final states* denoted by L(M) = $(\mu_{L(M)}, \nu_{L(M)})$ where $\mu_{L(M)}$ and $\nu_{L(M)}$ are fuzzy subsets of Σ^* and are given by

$$\mu_{L(M)}(\omega) = \vee\{\mu_I(q_0) \wedge \mu_{\vdash^*}((q_0, \omega, z_0), (p, \varepsilon, r)) \wedge \mu_F(p) \mid q_0, p \in Q, r \in \Gamma^*\}$$

and

$$\nu_{L(M)}(\omega) = \wedge\{\nu_I(q_0) \vee \nu_{\vdash^*}((q_0, \omega, z_0), (p, \varepsilon, r)) \vee \nu_F(p) \mid q_0, p \in Q, r \in \Gamma^*\}$$

for all $\omega \in \Sigma^*$.

ii. Language accepted by M with *empty stack* denoted by N(M) = $(\mu_{L(N)}, \nu_{L(N)})$ where $\mu_{L(N)}$ and $\nu_{L(N)}$ are fuzzy subsets and are given by

$$\mu_{L(N)}(\omega) = \vee\{\mu_I(q_0) \wedge \mu_{\vdash^*}((q_0, \omega, z_0), (p, \varepsilon, \varepsilon)) \mid q_0, p \in Q\} \text{ and}$$

$$\nu_{L(N)}(\omega) = \wedge\{\nu_I(q_0) \vee \nu_{\vdash^*}((q_0, \omega, z_0), (p, \varepsilon, \varepsilon)) \mid q_0, p \in Q\} \text{ for all } \omega \in \Sigma^*.$$

4. Equivalence of IFPDA with final states and IFPDA with empty stack

Proposition 1. If f is an intuitionistic fuzzy language accepted with final states by an IFPDA M = $(Q, \Sigma, \Gamma, \delta, I, Z_0, F)$, then f is an IFS in Σ^*, and the image set of f is finite.

Proof. First we will prove that $f = (\mu_f, \nu_f)$ is an IFS in Σ^* by showing that $0 \le \mu_f(\omega) + \nu_f(\omega) \le 1$, for any $\omega = x_1 \cdots x_n$, $x_i \in \Sigma \cup \{\varepsilon\}$, $i = 1 \ldots n$. Clearly,

$$\begin{aligned}\mu_f(\omega) &= \vee\{\mu_I(q_0) \wedge \mu_{\vdash^*}((q_0, \omega, z_0), (p, \varepsilon, r)) \wedge \mu_F(p) \mid q_0, p \in F, r \in \Gamma^*\}\\ &= \vee\{\mu_I(q_0) \wedge \mu_{\vdash}((q_0, \omega, z_0), (q_1, x_2 \cdots x_n, z_1 r_1))\\ &\qquad \wedge \mu_{\vdash}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \cdots\\ &\qquad \wedge \mu_{\vdash}((q_{n-1}, x_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \wedge \mu_F(q_n) \mid\\ &\qquad q_0, q_1, \ldots, q_n) \in Q, \quad z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^*\},\end{aligned}$$

and

$$\begin{aligned}\nu_f(\omega) &= \wedge\{\nu_I(q_0) \vee \nu_{\vdash^*}((q_0, \omega, z_0), (p, \varepsilon, r)) \vee \nu_F(p) \mid q_0, p \in F, r \in \Gamma^*\}\\ &= \wedge\{\nu_I(q_0) \vee \nu_{\vdash}((q_0, \omega, z_0), (q_1, x_2 \cdots x_n, z_1 r_1))\\ &\qquad \vee \nu_{\vdash}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \cdots\\ &\qquad \vee \nu_{\vdash}((q_{n-1}, x_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \vee \nu_F(q_n) \mid\\ &\qquad q_0, q_1, \ldots, q_n \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^*\}.\end{aligned}$$

On the one hand, $0 \le \mu_f(\omega) + \nu_f(\omega)$; on the other hand, there exists a sequence $q_0, q_1, \ldots, q_n \in Q$, $z_1 \ldots z_{n-1} \in \Gamma$, $r_1, \ldots, r_n \in \Gamma^*$ such that

$$\begin{aligned}\mu_f(\omega) = \mu_I(q_0) &\wedge \mu_{\vdash}((q_0, \omega, z_0), (q_1, u_2 \cdots u_n, z_1 r_1))\\ &\wedge \mu_{\vdash}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \cdots\\ &\wedge \mu_{\vdash}((q_{n-1}, x_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \wedge \mu_F(q_n).\end{aligned}$$

Hence $\nu_f(\omega) \le \nu_I(q_0) \vee \nu_{\vdash}((q_0, \omega, z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) \vee (q_2, x_3 \cdots x_n, z_2 r_2)) \vee \cdots \vee \nu_{\vdash}((q_{n-1}, x_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \vee \nu_F(q_n)$

Therefore, $\mu_f(\omega) + \nu_f(\omega) \le (\mu_I(q_0) + \nu_I(q_0)) \vee (\mu_{\vdash}((q_0, \omega, z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) + \nu_{\vdash}((q_0, \omega, z_0), (q_1, x_2 \cdots x_n, z_1 r_1))) \vee (\mu_{\vdash}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots$

x_n , z_2r_2)) + $\nu_\vdash$ ((q_1 ,$x_2 \cdots x_n$, z_1r_1), (q_2 , $x_3 \cdots x_n$, z_2r_2))) ∨ · · · ∨ ($\mu_\vdash$ ((q_{n-1} , x_n ,$z_{n-1} r_{n-1}$), (q_n , ε, r_n)) + $\nu_\vdash$ ((q_{n-1} , x_n , $z_{n-1} r_{n-1}$), (q_n , ε, r_n))) ∨ (μ_F (q_n) + ν_F (q_n)) ≤ 1 ∨ 1 ∨ · · · ∨ 1 = 1.

To prove that Im(f) is finite, let X= Im(μ_I) ∪ Im(μ_δ) ∪ Im(μ_F) and Y= Im(ν_I)∪ Im(ν_δ)∪Im(ν_F). Since $\delta = (\mu_\delta, \nu_\delta)$ and $F = (\mu_F, \nu_F)$ are finite IFS, $\mu_f(\omega) \in X$ and $\nu_f(\omega) \in$ Y for any $\omega = x_1 \cdots x_n$.
Therefore, Im(f) = Im(μ_f) ∪ Im(ν_f) is finite.

Proposition 2. If f is a fuzzy language accepted with empty states by some IFPDA M = (Q, Σ, Γ, δ, I, Z_0, F), then f is an IFS in Σ^*, and the image set of f is finite.
Proof. Similar to the Proposition 1

Proposition 3. Let f be IFS in Σ*, then the following statements are equivalent:

i. f can be accepted by some IFPDA M = (Q, Σ, Γ, δ, I, Z_0, F),
ii. f can be accepted by some IFPDM M' = (Q', Σ, Γ', δ, q_0, X_0, F ') where q_0 □ Q'

Proof. (i) → (ii).
Construct an IFPDA M ' = (Q', Σ, Γ', δ', I', X_0, F') as follows:
$Q' = Q \cup \{q_0\}$, $\Gamma' = \Gamma \cup \{X_0\}$, where $q_0 \notin Q$, $X_0 \notin \Gamma$.
Define an IFS I' in Q' by
$\mu_I'(q) = 1$ if q = q_0, $\mu_I(q)$ if $q \neq q_0$ and $\nu_I'(q) = 0$ if q = q_0, $\nu_I(q)$ if $q \neq q_0$
Define an IFS F' in Q' by
$\mu_F'(q) = 0$ if q = q_0, $\mu_F(q)$ if $q \neq q_0$ and $\nu_F'(q) = 1$ if q = q_0, $\nu_F(q)$ if $q \neq q_0$

Define an IFS δ' in $Q' \times (\Sigma \cup \{\varepsilon\}) \times \Gamma' \times Q' \times \Gamma'^*$ by mappings μ_δ', ν_δ': $Q' \times (\Sigma \cup \{\varepsilon\}) \times \Gamma' \times Q' \times \Gamma'^* \to [0, 1]$ defined as
$\mu_\delta'(q_0, \varepsilon, X_0, p, Z_0) = \mu_I(p)$,
$\nu_\delta'(q_0, \varepsilon, X_0, p, Z_0) = \nu_I(p)$,
$\mu_\delta'(q, a, z, p, \gamma) = \mu_\delta(q, a, z, p, \gamma)$,
$\nu_\delta'(q, a, z, p, \gamma) = \nu_\delta(q, a, z, p, \gamma)$, where $q, p \in Q$, $a \in \Sigma \cup \{\varepsilon\}$, $z \in \Gamma$, $\gamma \in \Gamma^*$;
For $a \in \Sigma$, $p \in Q$, $\gamma \in \Gamma^*$ define $\mu_\delta'(q_0, a, z, p, \gamma) = 0$ and $\nu_\delta'(q_0, a, z, p, \gamma) = 1$.

Then for any $\omega = x_1 \cdots x_n \in \Sigma^*$, $x_i \in \Sigma \cup \{\varepsilon\}$, $i = 1\ldots n$, we have

$$\begin{aligned}
\mu_{L(M')}(\omega) = \bigvee\{&\mu_I'(q) \wedge \mu_{\vdash M}'((q, \omega, X_0), (p_0, \omega, Z_0)) \wedge \mu_{\vdash M'}((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) \\
&\wedge \mu_{\vdash M}'((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \wedge \cdots \\
&\wedge \mu_{\vdash M}'((q_{n-1}, x_n, z_{n-1} x_{n-1}), (q_n, \varepsilon, r_n)) \wedge \mu_F'(q_n) \mid \\
&q \in Q', p_0, q_1, \ldots, q_n \in Q', z_1, \ldots, z_{n-1} \in \Gamma', r_1, \ldots, r_n \in \Gamma'^* \} \\
= \bigvee \{1 \wedge &\mu_I(p_0) \wedge \mu_{\vdash M}((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) \\
&\wedge \mu_{\vdash M}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \wedge \cdots \\
&\wedge \mu_{\vdash M}((q_{n-1}, x_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \wedge \mu_F(q_n) \mid \\
&p_0, q_1, q_n \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \mu_{L(M)}&(\omega), \text{ and}
\end{aligned}$$

$$\begin{aligned}
\nu_{L(M')}(\omega) = \bigwedge\{&\nu_I'(q) \vee \nu_{\vdash M}'((q, \omega, X_0), (p_0, \omega, {}_0)) \vee \nu_{\vdash M}'((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) \\
&\vee \nu_{\vdash M}'((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \vee \cdots \\
&\vee \nu_{\vdash M}'((q_{n-1}, x_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \vee \nu_F'(q_n) \mid \\
&q \in Q', (p_0, q_1, \ldots, q_n) \in Q', z_1, \ldots, z_{n-1} \in \Gamma', r_1, \ldots, r_n \in \Gamma'^{*\}}
\end{aligned}$$

$= \bigwedge \{0 \vee \nu_I (p_0) \vee \nu_{\vdash M} ((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1))$
$\vee \nu_{\vdash M} ((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, u_3 \cdots u_n, z_2 r_2)) \vee \cdots$
$\vee \nu_{\vdash M} ((q_{n-1}, u_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \vee \nu_F (q_n) \mid$
$p_0, q_1, \ldots, q_n \in Q, \; z_1 \ldots z_{n-1} \in \Gamma, r_1 \ldots r_n \in \Gamma^*\} = \nu_{L(M)} (\omega)$.

Therefore L (M') = L (M).

From the above proposition we can see that any IFPDA can be assumed to be M = $(Q, \Sigma, \Gamma, \delta, q_0, X_0, F')$.

Proof. (ii) → (i).
Suppose the IFS A is accepted by the IFPDA M= $(Q', \Sigma, \Gamma', \delta', q_0, X_0, F')$. We construct an IFS I in Q' as follows
$\mu_I (q) = 1$ if $q=q_0$, 0 if $q \neq q_0$
$\nu_I (q) = 0$ if $q=q_0$, 1 if $q \neq q_0$
Then, it follows that M accepts f.

Proposition 4. Let f be IFS in a nonempty set Σ^*. Then the following statements are equivalent:
(i) f can be accepted by an IFPDA M = $(Q, \Sigma, \Gamma, \delta, I, Z_0, \phi)$ by empty state;
(ii) There exists an IFPDA M' = $(Q', \Sigma, \Gamma', \delta', q_0, X_0, \phi)$ recognizing f

Proof. Similar to Proposition 3

Proposition 5. Let f be IFS in a nonempty set Σ^{*}. Then the following statements are equivalent:
(i) f can be accepted by an IFPDA M = $(Q, \Sigma, \Gamma, \delta, I, Z_0, F)$ by final state;
(ii) There exists an IFPDA M' = $(Q', \Sigma, \Gamma', \delta', q_0, X_0, \phi)$ recognizing f with empty stack, where q_0 □ Q'

Proof. (i) → (ii).
Let M = $(Q, \Sigma, \Gamma, \delta, I, Z_0, F)$ be a IFPDA that accepts $f = (\mu_f, \nu_f)$ by final state.
Construct an IFPDA M ' = $(Q', \Sigma, \Gamma', \delta', I', X_0, F')$ as follows:
$Q' = Q \cup \{q_0', q_e\}$, $\Gamma' = \Gamma \cup \{X_0\}$, where $q_0 \notin Q$, $X_0 \notin \Gamma$.
Define an IFS I' in Q' by
$\mu_I' (q) = 1$ if $q = q_0'$, $\mu_I(q)$ if $q \neq q_0'$ and $\nu_I' (q) = 0$ if $q = q_0'$, $\nu_I (q)$ if $q \neq q_0'$
Define an IFS F' in Q' by
$\mu_F' (q) = 0$ if $q = q_0'$, $\mu_F (q)$ if $q \neq q_0$ and $\nu_F' (q) = 1$ if $q = q_0'$, $\nu_F (q)$ if $q \neq q_0$
Define an IFS δ' in $Q' \times (\Sigma \cup \{\varepsilon\}) \times \Gamma' \times Q' \times \Gamma'^*$ by mappings μ_{δ}', ν_{δ}': $Q' \times (\Sigma \cup \{\varepsilon\}) \times \Gamma' \times Q' \times \Gamma'^* \to [0, 1]$ as
$\mu_{\delta}' (q_0', \varepsilon, X_0, p, Z_0) = \mu_I (p)$,
$\nu_{\delta}' (q_0', \varepsilon, X_0, p, Z_0) = \nu_I (p)$,
$\mu_{\delta}' (q, \varepsilon, \text{any}, q_e, \varepsilon) = \mu_F (q)$, $\forall q \in F$,
$\nu_{\delta}' (q, \varepsilon, \text{any}, q_e, \varepsilon) = \nu_F (q)$ $\forall q \in F$,
$\mu_{\delta}' (q_e, \varepsilon, \text{any}, q_e, \varepsilon) = 1$
$\nu_{\delta}' (q_e, \varepsilon, \text{any}, q_e, \varepsilon) = 0$

$\mu_{\delta}'(q, \text{a}, z, p, \gamma) = \mu_{\delta}(q, \text{a}, z, p, \gamma)$,
$\nu_{\delta}'(q, \text{a}, z, p, \gamma) = \nu_{\delta}(q, \text{a}, z, p, \gamma)$, where $q, p \in Q$, $\text{a} \in \Sigma \cup \{\varepsilon\}$, $z \in \Gamma$, $\gamma \in \Gamma^*$;
Otherwise, $\mu_{\delta'}(q_0, \text{a}, z, p, \gamma) = 0$ and $\nu_{\delta'}(q_0, \text{a}, z, p, \gamma) = 1$.

Then for any $\omega = \text{x}_1 \cdots \text{x}_n \in \Sigma^*$, $\text{x}_i \in \Sigma \cup \{\varepsilon\}$, $i = 1...n$, we have

$$\begin{aligned}
\mu_{L(M')}(\omega) = \bigvee\{&\mu_I'(q) \wedge \mu_{\vdash M'}((q, \omega, X_0), (p_0, \omega, Z_0)) \wedge \mu_{\vdash M'}((p_0, \omega, Z_0), (q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1)) \\
&\wedge \mu_{\vdash M}'((q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1), (q_2, \text{x}_3 \cdots \text{x}_n, z_2 r_2)) \wedge \cdots \\
&\wedge \mu_{\vdash M}'((q_{n-1}, \text{x}_n, z_{n-1}\text{x}_{n-1}), (q_n, \varepsilon, r_n)) \\
&\wedge \mu_{\vdash M}'((q_n, \varepsilon, r_n)), (q_e, \varepsilon, \varepsilon)) \mid \\
&p_0, q_1, \ldots, q_n \in Q', z_1, \ldots, z_{n-1} \in \Gamma', r_1, \ldots, r_n \in \Gamma'^* \} \\
= \bigvee\{&1 \wedge \mu_I(p_0) \wedge \mu_{\vdash M}((p_0, \omega, Z_0), (q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1)) \\
&\wedge \mu_{\vdash M}((q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1), (q_2, \text{x}_3 \cdots \text{x}_n, z_2 r_2)) \wedge \cdots \\
&\wedge \mu_{\vdash M}((q_{n-1}, \text{x}_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \wedge 1 \wedge 1 \wedge 1 \ldots \wedge 1 \mid \\
&(p_0, q_1, \ldots, q_n) \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \bigvee\{&\mu_I(p_0) \wedge \mu_{\vdash M}((p_0, \omega, Z_0), (q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1)) \\
&\wedge \mu_{\vdash M}((q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1), (q_2, \text{x}_3 \cdots \text{x}_n, z_2 r_2)) \wedge \cdots \\
&\wedge \mu_{\vdash M}((q_{n-1}, \text{x}_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \mid \\
&(p_0, q_1, \ldots, q_n) \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \mu_{L(M)}&(\omega), \text{ and}
\end{aligned}$$

$$\begin{aligned}
\nu_{L(M')}(\omega) = \wedge\{&\nu_I'(q) \vee \nu_{\vdash M'}((q, \omega, X_0), (p_0, \omega, Z_0)) \wedge \nu_{\vdash M'}((p_0, \omega, Z_0), (q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1)) \\
&\vee \nu_{\vdash M}'((q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1), (q_2, \text{x}_3 \cdots \text{x}_n, z_2 r_2)) \wedge \cdots \\
&\vee \nu_{\vdash M}'((q_{n-1}, \text{x}_n, z_{n-1}\text{x}_{n-1}), (q_n, \varepsilon, r_n)) \\
&\vee \nu_{\vdash M}'(((q_n, \varepsilon, r_n)), (q_e, \varepsilon, \varepsilon)) \mid \\
&(p_0, q_1, \ldots, q_n) \in Q', z_1, \ldots, z_{n-1} \in \Gamma', r_1, \ldots, r_n \in \Gamma'^* \} \\
= \wedge\{&0 \vee \nu_I(p_0) \vee \mu_{\vdash M}((p_0, \omega, Z_0), (q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1)) \\
&\vee \nu_{\vdash M}((q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1), (q_2, \text{x}_3 \cdots \text{x}_n, z_2 r_2)) \vee \cdots \\
&\vee \nu_{\vdash M}((q_{n-1}, \text{x}_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \vee 0 \vee 0 \vee 0 \ldots \vee 0 \mid \\
&(p_0, q_1, \ldots, q_n) \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \wedge\{&\nu_I(p_0) \vee \nu_{\vdash M}((p_0, \omega, Z_0), (q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1)) \\
&\vee \nu_{\vdash M}((q_1, \text{x}_2 \cdots \text{x}_n, z_1 r_1), (q_2, \text{x}_3 \cdots \text{x}_n, z_2 r_2)) \vee \cdots \\
&\vee \nu_{\vdash M}((q_{n-1}, \text{x}_n, z_{n-1} r_{n-1}), (q_n, \varepsilon, r_n)) \mid \\
&p_0, q_1, \ldots, q_n \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \nu_{L(M)}&(\omega)
\end{aligned}$$

Therefore L (M') = L (M).

Proposition 6. Let f be IFS in a nonempty set Σ^*. Then the following statements are equivalent:
(i) f can be accepted by an IFPDA M = $(Q, \Sigma, \Gamma, \delta, I, Z_0, \phi)$ by empty stack;
(ii) There exists an IFPDA M' = $(Q', \Sigma, \Gamma', \delta', q_0, X_0, \text{F})$ recognizing f with final state
Proof. (i) $\rightarrow$ (ii).
Let M = $(Q, \Sigma, \Gamma, \delta, I, Z_0, \phi)$ be a IFPDA that accepts f = (μ_f, ν_f) by empty stack.
Construct an IFPDA M' = $(Q', \Sigma, \Gamma', \delta', I', X_0, F')$ as follows:
$Q' = Q \cup \{q_0', q_f\}$, $\Gamma' = \Gamma \cup \{X_0\}$, where $q_0 \notin Q$, $X_0 \notin \Gamma$.
Define an IFS I' in Q' by
$\mu_I'(q) = 1$ if $q = q_0'$, $\mu_I(q)$ if $q \neq q_0'$ and $\nu_I'(q) = 0$ if $q = q_0'$, $\nu_I'(q)$ if $q \neq q_0'$

Define an IFS F' in Q' by
$\mu_F'(q) = 0$ if $q = q_0'$, $\mu_F(q)$ if $q \neq q_0$ and $\nu_F'(q) = 1$ if $q = q_0'$, $\nu_F(q)$ if $q \neq q_0$

Define an IFS δ' in $Q' \times (\Sigma \cup \{\varepsilon\}) \times \Gamma' \times Q' \times \Gamma'^*$ by mappings μ_δ', ν_δ': $Q' \times (\Sigma \cup \{\varepsilon\}) \times \Gamma' \times Q' \times \Gamma'^* \to [0, 1]$ defined as
$\mu_{\delta}'(q_0', \varepsilon, X_0, p, Z_0) = \mu_I(p)$,
$\nu_\delta'(q_0', \varepsilon, X_0, p, Z_0) = \nu_I(p)$,
$\mu_{\delta}'(q, \varepsilon, X_0, q_f, \varepsilon) = 1$,
$\nu_\delta'(q, \varepsilon, \text{any}, q_f, \varepsilon) = 0$,
$\mu_\delta'(q, a, z, p, \gamma) = \mu_\delta(q, a, z, p, \gamma)$,
$\nu_\delta'(q, a, z, p, \gamma) = \nu_\delta(q, a, z, p, \gamma)$, where $q, p \in Q$, $a \in \Sigma \cup \{\varepsilon\}$, $z \in \Gamma$, $\gamma \in \Gamma^*$;
Otherwise, $\mu_\delta'(q_0, a, z, p, \gamma) = 0$ and $\nu_\delta'(q_0, a, z, p, \gamma) = 1$.

Then for any $\omega = x_1 \cdots x_n \in \Sigma^*$, $x_i \in \Sigma \cup \{\varepsilon\}$, $i = 1 \ldots n$, we have

$$\begin{aligned}
\mu_{L(M')}(\omega) = \bigvee\{&\mu_I'(q) \wedge \mu_{\vdash M'}((q, \omega, X_0), (p_0, \omega, Z_0)) \wedge \mu_{\vdash M'}((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) \\
&\wedge \mu_{\vdash M}'((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \wedge \cdots \\
&\wedge \mu_{\vdash M}'((q_{n-1}, x_n, z_{n-1} x_{n-1}), (q_n, \varepsilon, r_n)) \\
&\wedge \mu_{\vdash M}'(((q_n, \varepsilon, X_0)), (q_f, \varepsilon, Z_0)) \mid \\
&(p_0, q_1, \ldots, q_n) \in Q', z, \ldots, z_{n-1} \in \Gamma', r_1, \ldots, r_n \in \Gamma'^* \} \\
= \bigvee \{&1 \wedge \mu_I(p_0) \wedge \mu_{\vdash M}((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) \\
&\wedge \mu_{\vdash M}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \wedge \cdots \\
&\wedge \mu_{\vdash M}((q_{n-1}, x_n, z_{n-1} r_{n-1}), (q_f, \varepsilon, r_n)) \wedge 1 \mid \\
&(p_0, q_1, \ldots, q_n) \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \bigvee\{&\mu_I(p_0) \wedge \mu_{\vdash M}((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1))\, \mu_{\vdash M}'(((q_n, \varepsilon, X_0)), (q_f, \varepsilon, Z_0)) \\
&\wedge \mu_{\vdash M}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \wedge \cdots \\
&\wedge \mu_{\vdash M}'(((q_n, \varepsilon, X_0)), (q_f, \varepsilon, Z_0)) \mid \\
&(p_0, q_1, \ldots, q_n) \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \mu_{L(M)}&(\omega), \text{ and}
\end{aligned}$$

$$\begin{aligned}
\nu_{L(M')}(\omega) = \bigwedge\{&\nu_I'(q) \vee \nu_{\vdash M'}((q, \omega, X_0), (p_0, \omega, Z_0)) \wedge \nu_{\vdash M'}((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) \\
&\vee \nu_{\vdash M}'((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \wedge \cdots \\
&\vee \nu_{\vdash M}'((q_{n-1}, x_n, z_{n-1} x_{n-1}), (q_n, \varepsilon, r_n)) \\
&\vee \nu_{\vdash M}'(((q_n, \varepsilon, r_n)), (q_f, \varepsilon, \varepsilon)) \mid \\
&(p_0, q_1, \ldots, q_n) \in Q', z_1, \ldots, z_{n-1} \in \Gamma', r_1, \ldots, r_n \in \Gamma'^* \} \\
= \bigwedge \{&0 \vee \nu_I(p_0) \vee \mu_{\vdash M}((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z r_1)) \\
&\vee \nu_{\vdash M}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \vee \cdots \\
&\vee \mu_{\vdash M}((q_{n-1}, x_n, z_{n-1} r_{n-1}), (q_f, \varepsilon, r_n)) \vee 0 \mid \\
&(p_0, q_1, \ldots, q_n) \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \bigwedge \{&\nu_I(p_0) \vee \nu_{\vdash M}((p_0, \omega, Z_0), (q_1, x_2 \cdots x_n, z_1 r_1)) \\
&\vee \nu_{\vdash M}((q_1, x_2 \cdots x_n, z_1 r_1), (q_2, x_3 \cdots x_n, z_2 r_2)) \vee \cdots \\
&\vee \mu_{\vdash M}'(((q_n, \varepsilon, X_0)), (q_f, \varepsilon, Z_0)) \mid \\
&(p_0, q_1, \ldots, q_n) \in Q, z_1, \ldots, z_{n-1} \in \Gamma, r_1, \ldots, r_n \in \Gamma^* \} \\
= \nu_{L(M)}&(\omega)
\end{aligned}$$

Therefore L (M') = L (M).

Theorem 1. If L is an intuitionistic fuzzy language (IFL) accepted by an IFPDA M with final states, there exists an IFPDA M′ that accepts L with empty stack

Proof: By Proposition 5 and Proposition 6, it follows that the two IFLs accepted by empty stack and final state are equivalent.

5. Intuitionistic fuzzy context-free grammars and languages

Definition 12. An *Intuitionistic Fuzzy Grammar* (IFG) is a system $G = (N, T, \mathrm{I}, \mathrm{P})$, where

i. N is a finite nonempty set of variables;
ii. T is a finite nonempty set of terminals, $T \cap N = \phi$
iii. I intuitionistic fuzzy set of start symbols(variables)
iv. P is a finite set of productions over $T \cup N$, $P = \{ x \to \beta \mid x \in (N \cup T)^* N (N \cup T)^*$, $y \in (N \cup T)^* \}$ is an IFS over $(N \cup T)^* \times (N \cup T)^*$ defined as $P = (\mu_P, \nu_P)$ where
$\mu_P(x, y) = \mu_P(x \to y)$ and
$\nu_P(x, y) = \nu_P(x \to y)$

are membership degree and nonmembership degree that x will be replaced by y, respectively.

For $\alpha, \beta \in (N \cup T)^*$, if $x \to y \in P$, then $\alpha y \beta$ is said to be *directly derivable* from $\alpha x \beta$, denoted by $\alpha x \beta \Rightarrow \alpha y \beta$, and define $\mu_P(\alpha x \beta \Rightarrow \alpha y \beta) = \mu_P(x \to y)$, $\nu_P(\alpha x \beta \Rightarrow \alpha y \beta) = \nu_P(x \to y)$. If $\alpha_1 \ldots \alpha_m$ are strings in $(N \cup T)^*$ and $\alpha_1 \to \alpha_2, \ldots, \alpha_{m-1} \to \alpha_m \in P$, then α_1 is said to derive α_m in G, or, equivalently, α_m is derivable from α_1 in G. This is expressed by $\alpha_1 \Rightarrow^* \alpha_m$ or simply $\alpha_1 \Rightarrow^* \alpha_m$. The expression $\alpha_1 \to \alpha_2 \to \cdots \to \alpha_m$ is referred to as a *derivation chain* from α_1 to α_m.

Proposition 7. The language generated by IFG is an intuitionistic fuzzy language (IFL)

Proof: An intuitionistic fuzzy grammar G generates an intuitionistic fuzzy language $\mathrm{L}(G) = (\mu_G, \nu_G)$ in the following manner. For any string $\omega_n \in T^*$, $n \geq 1$, $\mu_G(\omega_n) = \bigvee\{\mu_I(\omega_0) \wedge \mu_P(\omega_0 \Rightarrow \omega_1) \wedge \cdots \wedge \mu_P(\omega_{n-1} \Rightarrow \omega_n) \mid \omega_0 \in N, \omega_1, \ldots, \omega_{n-1} \in (N \cup T)^*\}$, and $\nu_G(\omega_n) = \bigwedge\{\nu_I(\omega_0) \vee \nu_P(\omega_0 \Rightarrow \omega_1) \vee \cdots \vee \nu_P(\omega_{n-1} \Rightarrow \omega_n) \mid \omega_0 \in N, \omega_1 \ldots \omega_{n-1} \in (N \cup T)^*\}$. $\mu_G(\omega_n)$ and $\nu_G(\omega_n)$ express the membership and non membership degrees of ω_n in the language generated by grammar G, respectively. Obviously, $\mathrm{L}(G) = (\mu_G, \nu_G)$ is well defined. And also, for any string $\omega_n \in T^*$, $n \geq 1$, there is a derivation from ω_0 to ω_n, that is, $\omega_0 \Rightarrow \omega_1 \Rightarrow \cdots \Rightarrow \omega_{n-1} \Rightarrow \omega_n$. Therefore,
$\mu_G(\omega_n) + \nu_G(\omega) \leq \mu_G(\omega_n) + \nu_I(\omega_0) \vee \nu_\rho(\omega_0 \Rightarrow \omega_1) \vee \cdots \vee \nu_\rho(\omega_{n-1} \Rightarrow \omega_n) = (\mu_G(\omega_n) + \nu_I(\omega_0)) \vee (\mu_G(\omega_n) + \nu_P(\omega_0 \Rightarrow \omega_1)) \vee \cdots \vee (\mu_G(\omega) + \nu_P(\omega_{n-1} \Rightarrow \omega_n)) \leq (\mu_I(\omega_0) + \nu_I(\omega_0)) \vee (\mu_P(\omega_0 \Rightarrow \omega_1) + \nu_P(\omega_0 \Rightarrow \omega_1)) \vee \cdots \vee (\mu_P(\omega_{n-1} \Rightarrow \omega_n) + \nu_P(\omega_{n-1} \Rightarrow \omega_n)) \leq 1$.

Definition 13. For any intuitionistic fuzzy grammars G_1 and G_2, if $\mathrm{L}(G_1) = \mathrm{L}(G_2)$ in the sense of equality of intuitionistic fuzzy sets, then the grammars G_1 and G_2 are said to be *equivalent*

Proposition 8. Let A be IFS over T^*. Then the following statements are equivalent:
(i) A is generated by a certain IFG $G = (N, T, P, I)$
(ii) A is generated by an IFG $G' = (N', T', P', S)$

Proof: (i) $\rightarrow$ (ii).
Let A be generated by an intuitionistic fuzzy grammar $G = (N, T, P, I)$. Then we construct an IFG $G' = (N', T', P', S)$ as follows:
$N' = N \cup \{S\}$, $S \notin N$; $T' = T$, $P' = P \cup P_1$, where
$P_1 = \{S \rightarrow q \mid q\ \omega\ \text{supp}\ (I), \mu_P (S \rightarrow q) = \mu_I (q), \nu_P (S \rightarrow q) = \nu_I (q)\}$.
Next we show that L (G') = L (G).
In fact, $G' = (N', T', P', I')$ where I' is an IFS over N' defined as $\mu_I'(S) = 1$, $\nu_I'(S) = 0$; $\mu_I'(q) = 0$ and $\nu_I'(q) = 1\ \forall q \in N$. For any $\omega_n \in T^*$, $n \geq 1$,

$$\begin{aligned}
\mu_G'(\omega_n) &= \bigvee\{\mu_I'(\omega_0)\wedge\mu_P(\omega_0\Rightarrow\omega_1)\wedge\cdots\wedge\mu_P(\omega_{n-1}\Rightarrow\omega_n) \mid \omega_0 \in N', \omega_1, \ldots, \omega_{n-1} \in (N'\cup T')^*\}\\
&= \bigvee\{\mu_P (S \Rightarrow \omega_1) \wedge \mu_P (\omega_1 \Rightarrow \omega_2) \wedge \cdots \wedge \mu_P (\omega_{n-1}\Rightarrow\omega_n) \mid \omega_1, \ldots, \omega_{n-1} \in (N'\cup T')^*\}\\
&= \bigvee\{\mu_P(S\Rightarrow q) \wedge \mu_P(q\Rightarrow\omega_2) \wedge \cdots \wedge \mu_P(\omega_{n-1}\Rightarrow\omega_n) \mid q \in N, \omega_2, \ldots, \omega_{n-1} \in (N \cup T)^*\}\\
&= \bigvee\{\mu_I (q) \wedge \mu_P (q \Rightarrow \omega_2) \wedge \cdots \wedge \mu_P (\omega_{n-1} \Rightarrow \omega_n) \mid q \in N, \omega_2, \ldots, \omega_{n-1} \in (N\cup T)^*\}\\
&= \mu_G(\omega_n) \text{ and}
\end{aligned}$$

$$\begin{aligned}
\nu_G'(\omega_n) &= \bigwedge\{\nu_I'(\omega_0)\vee\nu_P(\omega_0\Rightarrow\omega_1)\vee\cdots\vee\nu_P(\omega_{n-1}\Rightarrow\omega_n) \mid \omega_0 \in N', \omega_1, \ldots, \omega_{n-1} \in (N'\cup T')^*\}\\
&= \bigwedge\{\nu_P (S \Rightarrow \omega_1) \vee \nu_P (\omega_1\Rightarrow\omega_2) \vee \cdots \vee \nu_P (\omega_{n-1}\Rightarrow\omega_n) \mid \omega_1, \ldots, \omega_{n-1} \in (N' \cup T')^*\}\\
&= \bigwedge\{\nu_P (S\Rightarrow q)\vee\nu_P(q \Rightarrow \omega_2) \vee \cdots \vee \nu_P (\omega_{n-1}\Rightarrow\omega_n) \mid q \in N, \omega_2, \ldots, \omega_{n-1} \in (N \cup T)^*\}\\
&= \bigwedge\{\nu_I (q) \vee \nu_P (q\ \omega_2) \vee \cdots \vee \nu_P (\omega_{n-1} \Rightarrow \omega_n) \mid q \in N, \omega_2, \ldots, \omega_{n-1} \in (N \cup T)^*\}\\
&= \nu_G(\omega_n).
\end{aligned}$$

Hence L(G') = L(G).
(i) $\rightarrow$ (ii). The proof is obvious

Definition 14.

(1) An IFG $G = (N, T, P, I)$ is said to be *intuitionistic context free grammar* (IFCFG) if it has only productions of the form $A \rightarrow \omega \in P$ with $A \in N$ and $\omega \in (N \cup T)^*$. And the language L(G), generated by the IFCFG G, is said to be an *intuitionistic fuzzy context-free language* (IFCFL).

(2) An IFCFG $G = (N, T, P, S)$ is called an *intuitionistic fuzzy Chomsky normal form* (IFCNF) if it has only productions of the form $A \rightarrow BC$/a $\in P$ or $S \rightarrow \varepsilon$, where A, B, C $\in N$, $B \neq S$, $C \neq S$ and $a \in T$.

(3) An IFCFG $G = (N, T, P, S)$ is called an *intuitionistic fuzzy Greibach normal form* (IFGNF) if all the productions are of the form $A \rightarrow ax \in P$ or $S \rightarrow \varepsilon$, where $A \in N$, $a \in T$, and $x \in (N - \{S\})^*$

6. **Conclusions**

We have defined intuitionistic fuzzy pushdown automata and the two different ways accepting languages by empty stack and final states. We also established that the languages accepted by IFPDA (with final state) are equivalent to those accepted by IFPDA (with empty stack). Secondly, we have introduced the notions of IFCFGs, IFCNFs, and IFGNFs. Our future work will be on related concepts, such as the equivalence of IFCFG and IFPDA, Converting IFCFG to IFPDA and vice-versa, algebraic properties of IFCFLs.

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