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Homomorphism and Cartesian Product on Fuzzy Translation and Fuzzy Multiplication of PS-algebras

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Abstract. In this paper, we define Homomorphism and Cartesian product on fuzzy translation and fuzzy muliplication of PS-algebras and discussed some of its properties in detail by using the concepts of fuzzy PS-ideal and fuzzy PS-sub algebra.

Keywords: fuzzy-α-translation, fuzzy-α-multiplication of PS-algebra, fuzzy PS-ideal, fuzzy PS-sub algebra, homomorphism and Cartesian product.

AMS Mathematics Subject Classifications (2010): 06F35, 03G25

1. Introduction

The concept of fuzzy set was initiated by Zadeh in 1965 [4]. It has opened up keen insights and applications in a wide range of scientific fields. Since its inception, the theory of fuzzy subsets has developed in many directions and found applications in a wide variety of fields. The study of fuzzy subsets and its applications to various mathematical contexts has given rise to what is now commonly called fuzzy mathematics. Fuzzy algebra is an important branch of fuzzy mathematics. Fuzzy ideas have been applied to other algebraic structures such as groups, rings, modules, vector spaces and topologies. In this way, Iseki and Tanaka [1] introduced the concept of BCK-algebras in 1978. Iseki [2] introduced the concept of BCI-algebras in 1980. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Priya and Ramachandran [6,7] introduced the class of PS-algebras, which is a generalization of BCI / BCK/Q / KU / d algebras and studied various properties [5,8,9]. In this paper, we introduce the concept homomorphism and Cartesian product of fuzzy-α-translation, fuzzy-α-multiplication of PS-algebras and established some of its properties in detail.

2. Preliminaries

In this section, we introduced some fundamental definitions that will be used in the sequel.

Definition 2.1. [1] A BCK- algebra is an algebra $(X,*,0)$ of type(2,0) satisfying the following conditions:

i) $(x * y) * (x * z) \leq (z * y)$
ii) $x * (x * y) \leq y$
iii) $x \leq x$
iv) $x \leq y$ and $y \leq x \Rightarrow x=y$
v) $0 \leq x \Rightarrow x=0$, where $x \leq y$ is defined by $x * y = 0$, for all $x, y, z \in X$.

Definition 2.2. [2] A BCI-algebra is an algebra $(X,*,0)$ of type(2,0) satisfying the following conditions:

i) $(x * y) *(x * z) \leq (z*y)$
ii) $x * (x * y) \leq y$
iii) $x \leq x$
iv) $x \leq y$ and $y \leq x \Rightarrow x = y$
v) $x \leq 0 \Rightarrow x = 0$, where $x \leq y$ is defined by $x * y = 0$,for all $x, y, z \in X$.

Definition 2.3. [7] A nonempty set X with a constant 0 and a binary operation ' $*$ ' is called PS – algebra if it satisfies the following axioms.

1. $x * x = 0$
2. $x * 0 = 0$
3. $x * y = 0$ and $y * x = 0 \Rightarrow x = y$, $\forall$ $x, y \in X$.

Definition 2.4. [6] Let X be a PS-algebra. A fuzzy set μ in X is called a fuzzy PS-ideal of X if it satisfies the following conditions.
i) $\mu(0) \geq \mu(x)$
ii) $\mu(x) \geq \min\{\mu(y *x), \mu(y)\}$, for all $x, y \in X$.

Definition 2.5. [6] A fuzzy set μ in a PS-algebra X is called a fuzzy PS- sub algebra of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y\in X$.

Remark. Let X be a PS-algebra. For any fuzzy set μ of X, we define $T = 1– \sup\{\mu(x) / x \in X\}$, unless otherwise we specified.

Definition 2.6. ([3,5,12]) Let μ be a fuzzy subset of X and $\alpha \in [0,T]$. A mapping $\mu_\alpha^T: X \to[0, 1]$ is said to be a fuzzy-α-translation of μ if it satisfies $\mu_\alpha^T(x) = \mu(x) + \alpha$, $\forall$ $x\in X$.

Definition 2.7. ([3,5,12]) Let μ be a fuzzy subset of X and $\alpha \in [0,1]$. A mapping $\mu_\alpha^M : X \to[0, 1]$ is said to be a fuzzy-α-multiplication of μ if it satisfies $\mu_\alpha^M(x) = \alpha\, \mu(x)$, $\forall$ $x\in X$.

Example 2.8. [5] Let X = { 0, 1, 2 } be the set with the following table.

*	0	1	2
0	0	1	2
1	0	0	1
2	0	2	0

Then (X, $*$, 0) is a PS – algebra.

Define a fuzzy set μ of X by $\mu(x) = \begin{cases} 0.3 & if \;\; x \neq 1 \\ 0.2 & if \;\; x = 1 \end{cases}$.

Then μ is a fuzzy PS-sub algebra of X. Here T = 1 – sup $\{\mu(x) / x \in X\}$ = 1 – 0.3 = 0.7, Choose α= 0.4 $\in$ [0, T] and
β = 0.5 $\in$ [0, 1].

Then the mapping $\mu_{0.4}{}^{T} : X \to [0, 1]$ is defined by $\mu_{0.4}{}^{T} = \begin{cases} 0.3+0.4=0.7 & if \;\; x \neq 1 \\ 0.2+0.4=0.6 & if \;\; x = 1 \end{cases}$

which satisfies $\mu_{0.4}{}^{T}(x) = \mu(x) + 0.4, \forall x \in X$, is a fuzzy 0.4-translation and the mapping $\mu_{0.5}{}^{M} : X \to [0, 1]$ is defined by $\mu_{0.5}{}^{M}(x) = \begin{cases} (0.5)(0.3) = 0.15 & if \;\; x \neq 1 \\ (0.5)(0.2) = 0.10 & if \;\; x = 1 \end{cases}$

which satisfies $\mu_{0.5}{}^{M}(x) = (0.5)\mu(x), \forall x \in X$, is a fuzzy 0.5-multiplication.

3. Homomorphism on fuzzy translation and fuzzy multiplication

In this section, we discuss homomorphism on fuzzy translation and fuzzy multiplication of PS-algebra and proved certain results on the basis of fuzzy PS-ideal and fuzzy PS- sub algebra.

Definition 3.1. [10][13] Let f: X $\to$ X be an endomorphism and $\mu_\alpha{}^T$ be a fuzzy -α-translation of μ in X. We define a new fuzzy set in X by $(\mu_\alpha{}^T)_f$ in X as
$(\mu_\alpha{}^T)_f(x) = (\mu_\alpha{}^T)(f(x)) = \mu[f(x)] + \alpha, \forall x \in X$.

Theorem 3.2. Let f be an endomorphism of PS- algebra X. If μ is a fuzzy PS-ideal of X, then so is $(\mu_\alpha{}^T)_f$.

Proof: Let μ be a fuzzy PS-ideal of X.

Now, $(\mu_\alpha{}^T)_f(0) = \mu_\alpha{}^T[f(0)]$
$= \mu[f(0)] + \alpha$
$\geq \mu[f(x)] + \alpha$
$= (\mu_\alpha{}^T)(f(x))$
$= (\mu_\alpha{}^T)_f(x)$
$\Rightarrow (\mu_\alpha{}^T)_f(0) \geq (\mu_\alpha{}^T)_f(x)$

Let x,y $\in$ X. Then $(\mu_\alpha{}^T)_f(x) = \mu_\alpha{}^T[f(x)]$
$= \mu[f(x)] + \alpha$
$\geq \min\{\mu(f(y) * f(x)), \mu(f(y))\} + \alpha$
$= \min\{\mu(f(y * x)), \mu(f(y))\} + \alpha$
$= \min\{\mu(f(y * x)) + \alpha, \mu(f(y)) + \alpha\}$
$= \min\{\mu_\alpha{}^T[f(y * x)], \mu_\alpha{}^T[f(y)]\}$
$= \min\{(\mu_\alpha{}^T)_f(y * x), (\mu_\alpha{}^T)_f(y)\}$
$\therefore (\mu_\alpha{}^T)_f(x) \geq \min\{(\mu_\alpha{}^T)_f(y * x), (\mu_\alpha{}^T)_f(y)\}$

Hence $(\mu_\alpha{}^T)_f$ is a fuzzy PS-ideal of X.

Theorem 3.3. Let f: X $\rightarrow$ Y be an epimorphism of PS- algebra. If $(\mu_\alpha^T)_f$ is a fuzzy PS-ideal of X, then μ is a fuzzy PS-ideal of Y.

Proof: Let $(\mu_\alpha^T)_f$ be a fuzzy PS-ideal of X and let y $\in$ Y. Then there exists x $\in$ X such that f(x) = y.

Now,
$$\begin{aligned}\mu(0) + \alpha &= \mu_\alpha^T(0)\\ &= \mu_\alpha^T[f(0)]\\ &= (\mu_\alpha^T)_f(0)\\ &\geq (\mu_\alpha^T)_f(x)\\ &= \mu_\alpha^T[f(x)]\\ &= \mu[f(x)] + \alpha\end{aligned}$$

and so $\mu(0) \geq \mu[f(x)] = \mu(y)$. $\therefore$ $\mu(0) \geq \mu(y)$

Let $y_1, y_2 \in Y$.
$$\begin{aligned}\mu(y_1) + \alpha &= \mu_\alpha^T(y_1)\\ &= \mu_\alpha^T(f(x_1))\\ &= (\mu_\alpha^T)_f(x_1)\\ &\geq \min\{(\mu_\alpha^T)_f(x_2 * x_1), (\mu_\alpha^T)_f(x_2)\}\\ &= \min\{\mu_\alpha^T[f(x_2 * x_1)], \mu_\alpha^T[f(x_2)]\}\\ &= \min\{\mu_\alpha^T[f(x_2) * f(x_1)], \mu_\alpha^T[f(x_2)]\}\\ &= \min\{\mu_\alpha^T[y_2 * y_1], \mu_\alpha^T[y_2]\}\\ &= \min\{\mu(y_2 * y_1) + \alpha, \mu(y_2) + \alpha\}\\ &= \min\{\mu(y_2 * y_1), \mu(y_2)\} + \alpha\end{aligned}$$

$\therefore$ $\mu(y_1) \geq \min\{\mu(y_2 * y_1), \mu(y_2)\}$ $\Rightarrow$ μ is a fuzzy PS-ideal of Y.

Theorem 3.4. Let f: X $\rightarrow$ Y be a homomorphism of PS- algebra. If μ is a fuzzy PS-ideal of Y then $(\mu_\alpha^T)_f$ is a fuzzy PS-ideal of X.

Proof: Let μ be a fuzzy PS-ideal of Y and let x,y $\in$ X.

Then
$$\begin{aligned}(\mu_\alpha^T)_f(0) &= \mu_\alpha^T[f(0)]\\ &= \mu(f(0)) + \alpha\\ &\geq \mu(f(x)) + \alpha\\ &= \mu_\alpha^T[f(x)]\\ &= (\mu_\alpha^T)_f(x)\end{aligned}$$

$\Rightarrow$ $(\mu_\alpha^T)_f(0) \geq (\mu_\alpha^T)_f(x)$.

Also
$$\begin{aligned}(\mu_\alpha^T)_f(x) &= \mu_\alpha^T[f(x)]\\ &= \mu(f(x)) + \alpha\\ &\geq \min\{\mu(f(y) * f(x)), \mu(f(y))\} + \alpha\\ &= \min\{\mu(f(y * x)), \mu(f(y))\} + \alpha\\ &= \min\{\mu(f(y * x)) + \alpha, \mu(f(y)) + \alpha\}\\ &= \min\{\mu_\alpha^T[f(y * x)], \mu_\alpha^T[f(y)]\}\\ &= \min\{(\mu_\alpha^T)_f(y * x), (\mu_\alpha^T)_f(y)\}\end{aligned}$$

$\therefore$ $(\mu_\alpha^T)_f(x) \geq \min\{(\mu_\alpha^T)_f(y * x), (\mu_\alpha^T)_f(y)\}$.

Hence $(\mu_\alpha^T)_f$ is a fuzzy PS-ideal of X.

Theorem 3.5. If μ is a fuzzy PS- sub algebra of X, then $(\mu_\alpha^T)_f$ is also a fuzzy PS-sub algebra of X.

Proof: Let μ be a fuzzy PS- sub algebra of X. Let x, y$\in$ X.

Now, $(\mu_\alpha^T)_f (x*y) = \mu_\alpha^T [f (x *y)]$
$= \mu [f (x *y)] + \alpha$
$= \mu (f (x) * f(y)) + \alpha$
$\geq \min \{\mu (f (x)), \mu(f(y)) \} + \alpha$
$= \min \{\mu (f (x)) + \alpha, \mu (f(y)) + \alpha \}$
$= \min \{\mu_\alpha^T [f (x)], \mu_\alpha^T [f(y)] \}$
$= \min \{(\mu_\alpha^T)_f (x), (\mu_\alpha^T)_f (y) \}$

$\Rightarrow (\mu_\alpha^T)_f (x*y) \geq \min \{(\mu_\alpha^T)_f (x), (\mu_\alpha^T)_f (y) \}$

Hence $(\mu_\alpha^T)_f$ is a fuzzy PS-sub algebra of X.

Theorem 3.6. Let f: X→Y be a homomorphism of a PS-algebra X into a PS-algebra Y and μ_α^T be a fuzzy - α - translation of μ, then the pre- image of μ_α^T denoted by $f^{-1} (\mu_\alpha^T)$ is defined as $\{f^{-1} (\mu_\alpha^T)\}(x) = \mu_\alpha^T (f(x)), \forall x \in X$. If μ is a fuzzy PS- sub algebra of Y, then $f^{-1} (\mu_\alpha^T)$ is a fuzzy PS- sub algebra of X.

Proof: Let μ be a fuzzy PS- sub algebra of Y. Let x, y $\in$ X.

Now, $\{f^{-1} (\mu_\alpha^T)\}(x*y) = \mu_\alpha^T [f (x *y)]$
$= \mu [f (x *y)] + \alpha$
$= \mu [f (x) * f(y)] + \alpha$
$\geq \min \{ \mu [f (x)], \mu [f(y)] \} + \alpha$
$= \min \{ \mu [f (x)] + \alpha, \mu [f(y)] + \alpha \}$
$= \min \{ \mu_\alpha^T [f (x)], \mu_\alpha^T [f(y)] \}$
$= \min \{\{f^{-1}(\mu_\alpha^T)\} (x), \{f^{-1}(\mu_\alpha^T)\}(y) \}$

$\Rightarrow f^{-1} (\mu_\alpha^T)\}(x*y) \geq \min \{\{f^{-1}(\mu_\alpha^T)\} (x), \{f^{-1}(\mu_\alpha^T)\}(y) \}$

$f^{-1} (\mu_\alpha^T)$ is a fuzzy PS-sub algebra of X.

Definition 3.7. Let f: X $\to$ X be an endomorphism and μ_α^M be a fuzzy-α-multiplication of μ in X. We define a new fuzzy set in X by $(\mu_\alpha^M)_f$ in X as $(\mu_\alpha^M)_f (x) = (\mu_\alpha^M) [f(x)] = \alpha\, \mu[f(x)], \forall x\in X$.

Theorem 3.8. Let f be an endomorphism of PS- algebra X. If μ is a fuzzy PS-ideal of X, then so is $(\mu_\alpha^M)_f$.

Proof: Let μ be a fuzzy PS-ideal of X.

Now, $(\mu_\alpha^M)_f (0) = \mu_\alpha^M [f(0)]$
$= \alpha\, \mu [f(0)]$
$\geq \alpha\, \mu [f(x)]$
$= (\mu_\alpha^M) (f(x))$
$= (\mu_\alpha^M)_f (x)$

$\Rightarrow (\mu_\alpha^M)_f (0) \geq (\mu_\alpha^M)_f (x)$

Let x,y $\in$ X. Then $(\mu_\alpha^M)_f (x) = \mu_\alpha^M [f(x)]$
$= \alpha\, \mu [f(x)]$
$\geq \alpha \min \{ \mu (f(y) * f(x)), \mu(f (y))\}$
$= \alpha \min \{ \mu (f(y * x)), \mu(f (y))\}$

$= \min \{\alpha\, \mu\, (\, f(\, y * x\,)\,), \alpha\, \mu(f\,(y))\,\}$
$= \min \{\, \mu_\alpha{}^M\, [f(\, y * x\,)], \mu_\alpha{}^M [f(y)]\}$
$= \min \{\, (\mu_\alpha{}^M\,)_f\, (\, y * x\,), (\mu_\alpha{}^M\,)_f\, (y)\,\}$

$\therefore\, (\mu_\alpha{}^M\,)_f\, (\, x\,) \,\geq\, \min \{\, (\mu_\alpha{}^M\,)_f\, (\, y * x\,), (\mu_\alpha{}^M\,)_f\, (y)\,\}$

Hence $(\mu_\alpha{}^M\,)_f$ is a fuzzy PS-ideal of X.

Theorem 3.9. Let f: X $\rightarrow$ Y be an epimorphism of PS- algebra. If $(\mu_\alpha{}^M\,)_f$ is a fuzzy PS-ideal of X, then μ is a fuzzy PS-ideal of Y.

Proof: Let $(\mu_\alpha{}^M\,)_f$ be a fuzzy PS-ideal of X and let $y \in$ Y. Then there exists $x \in$ X such that f(x) = y.

Now, $\alpha\, \mu(0) = \mu_\alpha{}^M (0)$
$= \mu_\alpha{}^M [\, f(0)]$
$= (\mu_\alpha{}^M)_f\, (0)$
$\geq (\mu_\alpha{}^M)_f\, (x)$
$= \mu_\alpha{}^M [f(x)]$
$= \alpha\, \mu[f(x)]$

$\Rightarrow \mu(0) \geq \mu[f(x)] = \mu\, (y)$.

$\therefore\, \mu\, (0) \geq \mu\, (y)$

Let $y_1, y_2 \in$ Y.

$\alpha\, \mu(y_1) = \mu_\alpha{}^M [y_1]$
$= \mu_\alpha{}^M [f\, (x_1)]$
$= (\mu_\alpha{}^M)_f\, (\, x_1)$
$\geq \min \{\, (\mu_\alpha{}^M)_f (\, x_2 * x_1\,), (\mu_\alpha{}^M)_f (\, x_2\,)\,\}$
$= \min \{\, \mu_\alpha{}^M\, [f\, (x_2 * x_1)], \mu_\alpha{}^M\, [f(x_2)]\,\}$
$= \min \{\, \mu_\alpha{}^M\, [f\, (x_2) * f(x_1)], \mu_\alpha{}^M\, [f(\, x_2)]\,\}$
$= \min \{\, \mu_\alpha{}^M [\, y_2 * y_1\,], \mu_\alpha{}^M\, [y_2]\,\}$
$= \min \{\, \alpha\, \mu\, (y_2 * y_1),\, \alpha\, \mu(y_2)\,\}$
$= \alpha \min \{\mu\, (y_2 * y_1), \mu(y_2)\}$

$\therefore\, \mu\, (y_1) \geq\, \min \{\, \mu\, (y_2 * y_1\,), \mu(y_2)\}$

$\Rightarrow\, \mu$ is a fuzzy PS-ideal of Y.

Theorem 3.10. Let f: X $\rightarrow$ Y be a homomorphism of PS- algebra. If μ is a fuzzy PS-ideal of Y then $(\mu_\alpha{}^M\,)_f$ is a fuzzy PS-ideal of X.

Proof: Let μ be a fuzzy PS-ideal of Y and let $x,y \in$ X.

Then $(\mu_\alpha{}^M\,)_f\, (0) = \mu_\alpha{}^M [f(0)]$
$= \alpha\, \mu\, (f(0))$
$\geq \alpha\, \mu\, (f(x))$
$= \mu_\alpha{}^M [\, f(x)]$
$= (\mu_\alpha{}^M)_f\, (x)$

$\Rightarrow\, (\mu_\alpha{}^M)_f\, (0) \geq (\mu_\alpha{}^M\,)_f\, (x)$.

Also, $(\mu_\alpha{}^M)_f\, (x) = \mu_\alpha{}^M\, [\, f\, (x\,)\,] = \alpha\, \mu\, (f(x))$
$\geq\, \alpha \min \{\, \mu\, (\, f(y)\, * \, f(x)\,), \mu\, (\, f\, (y)\,)\,\}$
$= \alpha \min \{\, \mu\, (\, f(\, y * x)\,), \mu\, (\, f\, (y)\,)\,\}$
$= \min \{\, \alpha\, \mu\, (\, f(\, y * x)\,), \alpha\, \mu\, (\, f\, (y)\,)\,\}$

$= \min \{ \mu_\alpha^M [f(y * x)], \mu_\alpha^M [f (y)]\}$
$= \min \{ (\mu_\alpha^M)_f \ (y * x), (\mu_\alpha^M)_f (y) \}$

$\therefore (\mu_\alpha^M)_f (x) \geq \min \{ (\mu_\alpha^M)_f (y * x), (\mu_\alpha^M)_f (y) \}$. Hence $(\mu_\alpha^M)_f$ is a fuzzy PS-ideal of X.

Theorem 3.11. If μ be a fuzzy PS- sub algebra of X, then $(\mu_\alpha^M)_f$ is also a fuzzy PS-sub algebra of X.

Proof: Let μ be a fuzzy PS- sub algebra of X. Let x, y ∈ X.

Now, $(\mu_\alpha^M)_f (x* y) = \mu_\alpha^M [f (x *y)]$
$= \alpha\, \mu [f (x *y)]$
$= \alpha\, \mu (f (x) * f(y))$
$\geq \alpha \min\{\mu (f (x)),\mu(f(y))\}$
$= \min\{\alpha\, \mu (f (x)),\alpha\, \mu (f(y))\}$
$= \min\{\mu_\alpha^M [f (x)], \mu_\alpha^M [f(y)]\}$
$= \min \{(\mu_\alpha^M)_f \ (x), (\mu_\alpha^M)_f (y)\}$

$\Rightarrow (\mu_\alpha^M)_f$ is a fuzzy PS-sub algebra of X.

Theorem 3.12. Let f: X→Y be a homomorphism of a PS-algebra X into a PS-algebra Y and μ_α^M be a fuzzy - α - multiplication of μ, then the pre- image of μ_α^M denoted by $f^{-1}(\mu_\alpha^M)$ is defined as $\{f^{-1} (\mu_\alpha^M)\}(x) = \mu_\alpha^M (f(x)), \forall x \in X$. If μ is a fuzzy PS- sub algebra of Y, then $f^{-1} (\mu_\alpha^M)$ is a fuzzy PS- sub algebra of X.

Proof: Let μ be a fuzzy PS- sub algebra of Y. Let x, y ∈ X.

Now, $\{f^{-1} (\mu_\alpha^M)\}(x* y) = \mu_\alpha^M [f (x *y)]$
$= \alpha\, \mu [f (x *y)]$
$= \alpha\, \mu [f (x) * f(y)]$
$\geq \alpha \min \{ \mu [f (x)], \mu [f(y)] \}$
$= \min \{\alpha\, \mu [f (x)], \alpha\, \mu [f(y)] \}$
$= \min \{ \mu_\alpha^M [f (x)], \mu_\alpha^M [f(y)] \}$
$= \min \{\{f^{-1}(\mu_\alpha^M)\} (x), \{f^{-1}(\mu_\alpha^M)\}(y) \}$

$\Rightarrow f^{-1} (\mu_\alpha^M)\}(x* y) \geq \min \{\{f^{-1}(\mu_\alpha^M)\} (x), \{f^{-1}(\mu_\alpha^M)\}(y) \}$

$f^{-1} (\mu_\alpha^M)$ is a fuzzy PS-sub algebra of X.

4. Cartesian product on fuzzy translation and fuzzy multiplication

In this section, we discuss the Cartesian product of fuzzy translation and fuzzy multiplication of PS-algebras and establish some of its properties in detail on the basis of fuzzy PS-ideal and fuzzy PS- sub algebra.

Definition 4.1. ([11,14]) Let μ_α^T and δ_α^T be the fuzzy sets in X. Then Cartesian product $\mu_\alpha^T x\delta_\alpha^T$:X x X→[0,1] is defined by $(\mu_\alpha^T \text{ x } \delta_\alpha^T) (x, y) = \min \{\mu_\alpha^T (x), \delta_\alpha^T (y)\}$, for all x, y ∈ X.

Theorem 4.2. If μ and δ are fuzzy PS-idels in a PS– algebra X, then μ_α^T x δ_α^T is a fuzzy PS-ideal in X x X.

Proof: Let $(x_1, x_2) \in X \times X$.

$$\begin{aligned}
(\mu_\alpha^T \times \delta_\alpha^T)(0,0) &= \min\{\mu_\alpha^T(0), \delta_\alpha^T(0)\} \\
&= \min\{\mu(0)+\alpha, \delta(0)+\alpha\} \\
&= \min\{\mu(0), \delta(0)\}+\alpha \\
&\geq \min\{\mu(x_1), \delta(x_2)\}+\alpha \\
&= \min\{\mu(x_1)+\alpha, \delta(x_2)+\alpha\} \\
&= \min\{\mu_\alpha^T(x_1), \delta_\alpha^T(x_2)\} \\
&= (\mu_\alpha^T \times \delta_\alpha^T)(x_1,x_2)
\end{aligned}$$

Let $(x_1, x_2), (y_1, y_2) \in X \times X$.

$$\begin{aligned}
(\mu_\alpha^T \times \delta_\alpha^T)(x_1, x_2) &= \min\{\mu_\alpha^T(x_1), \delta_\alpha^T(x_2)\} \\
&= \min\{\mu(x_1)+\alpha, \delta(x_2)+\alpha\} \\
&= \min\{\mu(x_1), \delta(x_2)\}+\alpha \\
&\geq \min\{\min\{\mu(y_1 * x_1), \mu(y_1)\}, \min\{\delta(y_2 * x_2), \delta(y_2)\}\}+\alpha \\
&= \min\{\min\{\mu(y_1 * x_1), \mu(y_1)\}+\alpha, \min\{\delta(y_2 * x_2), \delta(y_2)\}+\alpha\} \\
&= \min\{\min\{\mu(y_1 * x_1)+\alpha, \mu(y_1)+\alpha\}, \min\{\delta(y_2 * x_2)+\alpha, \delta(y_2)+\alpha\}\} \\
&= \min\{\min\{\mu_\alpha^T(y_1 * x_1), \mu_\alpha^T(y_1)\}, \min\{\delta_\alpha^T(y_2 * x_2), \delta_\alpha^T(y_2)\}\} \\
&= \min\{\min\{\mu_\alpha^T(y_1 * x_1), \delta_\alpha^T(y_2 * x_2)\}, \min\{\mu_\alpha^T(y_1), \delta_\alpha^T(y_2)\}\} \\
&= \min\{(\mu_\alpha^T \times \delta_\alpha^T)((y_1 * x_1),(y_2 * x_2)), (\mu_\alpha^T \times \delta_\alpha^T)(y_1, y_2)\} \\
&= \min\{(\mu_\alpha^T \times \delta_\alpha^T)((y_1, y_2) * (x_1, x_2)), (\mu_\alpha^T \times \delta_\alpha^T)(y_1, y_2)\}
\end{aligned}$$

$\therefore (\mu_\alpha^T \times \delta_\alpha^T)(x_1, x_2) \geq \min\{(\mu_\alpha^T \times \delta_\alpha^T)((y_1, y_2) * (x_1, x_2)), (\mu_\alpha^T \times \delta_\alpha^T)(y_1, y_2)\}$.

Hence $\mu_\alpha^T \times \delta_\alpha^T$ is a fuzzy PS- ideal in X x X.

Theorem 4.3. Let μ & δ be fuzzy sets in a PS-algebra X such that $\mu_\alpha^T \times \delta_\alpha^T$ is a fuzzy PS-ideal of X x X. Then

(i) Either $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$ for all $x \in X$.

(ii) If $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ for all $x \in X$, then either $\delta_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$

(iii) If $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$ for all $x \in X$, then either $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\mu_\alpha^T(0) \geq \delta_\alpha^T(x)$

Proof: Let $\mu_\alpha^T \times \delta_\alpha^T$ be a fuzzy PS-ideal of X x X .

(i) Suppose that $\mu_\alpha^T(0) < \mu_\alpha^T(x)$ and $\delta_\alpha^T(0) < \delta_\alpha^T(x)$ for some $x, y \in X$.

$$\begin{aligned}
\text{Then } (\mu_\alpha^T \times \delta_\alpha^T)(x,y) &= \min\{\mu_\alpha^T(x), \delta_\alpha^T(y)\} \\
&> \min\{\mu_\alpha^T(0), \delta_\alpha^T(0)\} \\
&= (\mu_\alpha^T \times \delta_\alpha^T)(0, 0), \text{ which is a contradiction.}
\end{aligned}$$

Therefore $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$ for all $x \in X$.

(ii) Assume that there exists $x, y \in X$ such that $\delta_\alpha^T(0) < \mu_\alpha^T(x)$ and $\delta_\alpha^T(0) < \delta_\alpha^T(x)$.

Then $(\mu_\alpha^T \times \delta_\alpha^T)(0,0) = \min\{\mu_\alpha^T(0), \delta_\alpha^T(0)\} = \delta_\alpha^T(0)$ and hence $(\mu_\alpha^T \times \delta_\alpha^T)(x, y) = \min\{\mu_\alpha^T(x),\delta_\alpha^T(y)\} > \delta_\alpha^T(0) = (\mu_\alpha^T \times \delta_\alpha^T)(0,0)$ Which is a contradiction**.**

Hence, if $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ for all $x \in X$, then either $\delta_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$. Similarly, we can prove that if $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$ for all $x \in X$, then either $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\mu_\alpha^T(0) \geq \delta_\alpha^T(x)$, which yields (iii).

Theorem 4.4. Let μ & δ be fuzzy sets in a PS-algebra X such that $\mu_\alpha^T \times \delta_\alpha^T$ is a fuzzy PS-ideal of X x X. Then either μ or δ is a fuzzy PS-ideal of X.

Proof: First we prove that δ is a fuzzy PS-ideal of X.

Since by 4.5.3 (i) either $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$ for all $x \in X$.
Assume that $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$ for all $x \in X$.
$\Rightarrow \delta(0) + \alpha \geq \delta(x) + \alpha$.
$\therefore \delta(0) \geq \delta(x)$.
It follows from 4.5.3 (iii) that either $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\mu_\alpha^T(0) \geq \delta_\alpha^T(x)$.
If $\mu_\alpha^T(0) \geq \delta_\alpha^T(x)$, for any $x \in X$, then $\delta_\alpha^T(x) = \min\{\mu_\alpha^T(0), \delta_\alpha^T(x)\} = (\mu_\alpha^T \times \delta_\alpha^T)(0, x)$

$$
\begin{aligned}
\delta(x) + \alpha = \delta_\alpha^T(x) &= (\mu_\alpha^T \times \delta_\alpha^T)(0, x) \\
&\geq \min\{(\mu_\alpha^T \times \delta_\alpha^T)((0,y) * (0,x)), (\mu_\alpha^T \times \delta_\alpha^T)(0, y)\} \\
&= \min\{(\mu_\alpha^T \times \delta_\alpha^T)((0*0),(y*x)), (\mu_\alpha^T \times \delta_\alpha^T)(0, y)\} \\
&= \min\{(\mu_\alpha^T \times \delta_\alpha^T)(0,(y*x)), (\mu_\alpha^T \times \delta_\alpha^T)(0, y)\} \\
&= \min\{\delta_\alpha^T(y*x), \delta_\alpha^T(y)\} \\
&= \min\{\delta(y * x) + \alpha, \delta(y) + \alpha\} \\
&= \min\{\delta(y * x), \delta(y)\} + \alpha
\end{aligned}
$$

$\Rightarrow \delta(x) \geq \min\{\delta(y * x), \delta(y)\}$
Hence δ is a fuzzy PS-ideal of X.
Next we will prove that μ is a fuzzy PS-ideal of X.
Let $\mu_\alpha^T(0) \geq \mu_\alpha^T(x)$.
$\Rightarrow \mu(0) \geq \mu(x)$
Since by theorem 4.5.3 (ii), either $\delta_\alpha^T(0) \geq \mu_\alpha^T(x)$ (or) $\delta_\alpha^T(0) \geq \delta_\alpha^T(x)$.
Assume that $\delta_\alpha^T(0) \geq \mu_\alpha^T(x)$, then $\mu_\alpha^T(x) = \min\{\mu_\alpha^T(x), \delta_\alpha^T(0)\} = (\mu_\alpha^T \times \delta_\alpha^T)(x,0)$.

$$
\begin{aligned}
\mu(x) + \alpha = \mu_\alpha^T(x) &= (\mu_\alpha^T \times \delta_\alpha^T)(x,0) \\
&\geq \min\{(\mu_\alpha^T \times \delta_\alpha^T)((y,0) * (x,0)), (\mu_\alpha^T \times \delta_\alpha^T)(y,0)\} \\
&= \min\{(\mu_\alpha^T \times \delta_\alpha^T)((y * x), (0*0)), (\mu_\alpha^T \times \delta_\alpha^T)(y,0)\} \\
&= \min\{(\mu_\alpha^T \times \delta_\alpha^T)((y * x), 0), (\mu_\alpha^T \times \delta_\alpha^T)(y,0)\} \\
&= \min\{\mu_\alpha^T(y * x), \mu_\alpha^T(y)\} \\
&= \min\{\mu(y * x) + \alpha, \mu(y) + \alpha\} \\
&= \min\{\mu(y * x), \mu(y)\} + \alpha
\end{aligned}
$$

$\Rightarrow \mu(x) \geq \min\{\mu(y * x), \mu(y)\}$
Hence μ is a fuzzy PS-ideal of X.

Theorem 4.5. If μ and δ are fuzzy PS-sub algebras of a PS-algebra X, then $\mu_\alpha^T \times \delta_\alpha^T$ is also a fuzzy PS-sub algebra of X x X.
Proof: For any $x_1, x_2, y_1, y_2 \in X$.

$$
\begin{aligned}
(\mu_\alpha^T \times \delta_\alpha^T)((x_1, y_1) * (x_2, y_2)) &= (\mu_\alpha^T \times \delta_\alpha^T)(x_1 * x_2, y_1 * y_2) \\
&= \min\{\mu_\alpha^T(x_1 * x_2), \delta_\alpha^T(y_1 * y_2)\} \\
&= \min\{\mu(x_1 * x_2) + \alpha, \delta(y_1 * y_2) + \alpha\} \\
&= \min\{\mu(x_1 * x_2), \delta(y_1 * y_2)\} + \alpha \\
&\geq \min\{\min\{\mu(x_1), \mu(x_2)\}, \min\{\delta(y_1), \delta(y_2)\}\} + \alpha \\
&= \min\{\min\{\mu(x_1), \mu(x_2)\} + \alpha, \min\{\delta(y_1), \delta(y_2)\} + \alpha\} \\
&= \min\{\min\{\mu(x_1) + \alpha, \mu(x_2) + \alpha\}, \min\{\delta(y_1) + \alpha, \delta(y_2) + \alpha\}\} \\
&= \min\{\min\{\mu_\alpha^T(x_1), \mu_\alpha^T(x_2)\}, \min\{\delta_\alpha^T(y_1), \delta_\alpha^T(y_2)\}\} \\
&= \min\{\min\{\mu_\alpha^T(x_1), \delta_\alpha^T(y_1)\}, \min\{\mu_\alpha^T(x_2), \delta_\alpha^T(y_2)\}\} \\
&= \min\{(\mu_\alpha^T \times \delta_\alpha^T)(x_1, y_1), (\mu_\alpha^T \times \delta_\alpha^T)(x_2, y_2)\}
\end{aligned}
$$

$\Rightarrow (\mu_\alpha^T \times \delta_\alpha^T)\ ((x_1, y_1) * (x_2, y_2)) \geq \min\{(\mu_\alpha^T \times \delta_\alpha^T)(x_1, y_1), (\mu_\alpha^T \times \delta_\alpha^T)(x_2, y_2)\}$

This completes the proof.

Theorem 4.6. If μ and δ are fuzzy PS-ideals in a PS-algebra X, then $\mu_\alpha^M \times \delta_\alpha^M$ is a fuzzy PS-ideal in X x X.

Proof: Let $(x_1, x_2) \in X \times X$.

$$\begin{aligned}
(\mu_\alpha^M \times \delta_\alpha^M)(0,0) &= \min\{\mu_\alpha^M(0), \delta_\alpha^M(0)\}\\
&= \min\{\alpha\,\mu(0), \alpha\,\delta(0)\}\\
&= \alpha \min\{\mu(0), \delta(0)\}\\
&\geq \alpha \min\{\mu(x_1), \delta(x_2)\}\\
&= \min\{\alpha\,\mu(x_1), \alpha\,\delta(x_2)\}\\
&= \min\{\mu_\alpha^M(x_1), \delta_\alpha^M(x_2)\}\\
&= (\mu_\alpha^M \times \delta_\alpha^M)(x_1, x_2)
\end{aligned}$$

Let $(x_1, x_2), (y_1, y_2) \in X \times X$.

$$\begin{aligned}
(\mu_\alpha^M \times \delta_\alpha^M)(x_1, x_2) &= \min\{\mu_\alpha^M(x_1), \delta_\alpha^M(x_2)\}\\
&= \min\{\alpha\,\mu(x_1), \alpha\,\delta(x_2)\}\\
&= \alpha \min\{\mu(x_1), \delta(x_2)\}\\
&\geq \alpha \min\{\min\{\mu(y_1 * x_1), \mu(y_1)\}, \min\{\delta(y_2 * x_2), \delta(y_2)\}\}\\
&= \min\{\alpha \min\{\mu(y_1 * x_1), \mu(y_1)\}, \alpha \min\{\delta(y_2 * x_2), \delta(y_2)\}\}\\
&= \min\{\min\{\alpha\,\mu(y_1 * x_1), \alpha\,\mu(y_1)\}, \min\{\alpha\,\delta(y_2 * x_2), \alpha\,\delta(y_2)\}\}\\
&= \min\{\min\{\mu_\alpha^M(y_1 * x_1), \mu_\alpha^M(y_1)\}, \min\{\delta_\alpha^M(y_2 * x_2), \delta_\alpha^M(y_2)\}\}\\
&= \min\{\min\{\mu_\alpha^M(y_1 * x_1), \delta_\alpha^M(y_2 * x_2)\}, \min\{\mu_\alpha^M(y_1), \delta_\alpha^M(y_2)\}\}\\
&= \min\{(\mu_\alpha^M \times \delta_\alpha^M)((y_1 * x_1), (y_2 * x_2)), (\mu_\alpha^M \times \delta_\alpha^M)(y_1, y_2)\}\\
&= \min\{(\mu_\alpha^M \times \delta_\alpha^M)((y_1, y_2) * (x_1, x_2)), (\mu_\alpha^M \times \delta_\alpha^M)(y_1, y_2)\}
\end{aligned}$$

$\therefore (\mu_\alpha^M \times \delta_\alpha^M)(x_1, x_2) \geq \min\{(\mu_\alpha^M \times \delta_\alpha^M)((y_1, y_2) * (x_1, x_2)), (\mu_\alpha^M \times \delta_\alpha^M)(y_1, y_2)\}$.

Hence $\mu_\alpha^M \times \delta_\alpha^M$ is a fuzzy PS- ideal in X x X.

Theorem 4.7. Let μ & δ be fuzzy sets in a PS-algebra X such that $\mu_\alpha^M \times \delta_\alpha^M$ is a fuzzy PS-ideal of X x X. Then

(i) Either $\mu_\alpha^M(0) \geq \mu_\alpha^M(x)$ (or) $\delta_\alpha^M(0) \geq \delta_\alpha^M(x)$ for all $x \in X$.

(ii) If $\mu_\alpha^M(0) \geq \mu_\alpha^M(x)$ for all $x \in X$, then either $\delta_\alpha^M(0) \geq \mu_\alpha^M(x)$ (or) $\delta_\alpha^M(0) \geq \delta_\alpha^M(x)$

(iii) If $\delta_\alpha^M(0) \geq \delta_\alpha^M(x)$ for all $x \in X$, then either $\mu_\alpha^M(0) \geq \mu_\alpha^M(x)$ (or) $\mu_\alpha^M(0) \geq \delta_\alpha^M(x)$

Proof: Straight forward.

Theorem 4.8. Let μ & δ be fuzzy sets in a PS-algebra X such that $\mu_\alpha^M \times \delta_\alpha^M$ is a fuzzy PS-ideal of X x X. Then either μ or δ is a fuzzy PS-ideal of X.

Proof: First we prove that δ is a fuzzy PS-ideal of X.

Since by 4.5.3 (i) either $\mu_\alpha^M(0) \geq \mu_\alpha^M(x)$ (or) $\delta_\alpha^M(0) \geq \delta_\alpha^M(x)$ for all $x \in X$.

Assume that $\delta_\alpha^M(0) \geq \delta_\alpha^M(x)$ for all $x \in X$.

$\Rightarrow \alpha\,\delta(0) \geq \alpha\,\delta(x)$. $\therefore \delta(0) \geq \delta(x)$

It follows from 4.5.3 (iii) that either $\mu_\alpha^M(0) \geq \mu_\alpha^M(x)$ (or) $\mu_\alpha^M(0) \geq \delta_\alpha^M(x)$.

If $\mu_\alpha^M(0) \geq \delta_\alpha^M(x)$, for any $x \in X$, then $\delta_\alpha^M(x) = \min\{\mu_\alpha^M(0), \delta_\alpha^M(x)\} = (\mu_\alpha^M \times \delta_\alpha^M)(0, x)$

$\alpha\,\delta(x) = \delta_\alpha^M(x) = (\mu_\alpha^M \times \delta_\alpha^M)(0, x)$

$\geq \min \{(\mu_\alpha^M \text{ x } \delta_\alpha^M) ((0,y) * (0,x)), (\mu_\alpha^M \text{ x } \delta_\alpha^M) (0, y)\}$
$= \min \{(\mu_\alpha^M \text{ x } \delta_\alpha^M) ((0*0),(y*x)), (\mu_\alpha^M \text{ x } \delta_\alpha^M) (0, y)\}$
$= \min \{(\mu_\alpha^M \text{ x } \delta_\alpha^M) (0,(y*x)), (\mu_\alpha^M \text{ x } \delta_\alpha^M) (0, y)\}$
$= \min \{ \delta_\alpha^M (y*x), \delta_\alpha^M (y)\}$
$= \min \{ \alpha \delta (y * x), \alpha \delta (y) \}$
$= \alpha \min \{ \delta (y * x), \delta (y) \}$

$\Rightarrow \delta(x) \geq \min \{ \delta (y * x), \delta (y) \}$
Hence δ is a fuzzy PS-ideal of X. Similarly we will prove that μ is a fuzzy PS-ideal of X.

Theorem 4.9. If μ and δ are fuzzy PS-sub algebras of a PS-algebra X, then μ_α^M x δ_α^M is also a fuzzy PS-sub algebra of X x X.
Proof: For any $x_1, x_2, y_1, y_2 \in X$.

$(\mu_\alpha^M \text{ x } \delta_\alpha^M) ((x_1, y_1) * (x_2, y_2)) = (\mu_\alpha^M \text{ x } \delta_\alpha^M) (x_1 * x_2 , y_1 * y_2)$
$= \min \{ \mu_\alpha^M (x_1 * x_2), \delta_\alpha^M (y_1 * y_2) \}$
$= \min \{ \alpha \mu (x_1 * x_2), \alpha \delta (y_1 * y_2) \}$
$= \alpha \min \{ \mu (x_1 * x_2), \delta (y_1 * y_2) \}$
$\geq \alpha \min \{ \min \{\mu (x_1), \mu (x_2)\},\min\{\delta(y_1), \delta(y_2)\}\}$
$= \min \{ \alpha \min \{\mu (x_1), \mu (x_2)\}, \alpha \min\{\delta(y_1),\delta(y_2)\} \}$
$= \min\{\min\{\alpha \mu(x_1), \alpha \mu (x_2)\}, \min\{ \alpha \delta(y_1), \alpha \delta(y_2)\}\}$
$= \min\{\min\{\mu_\alpha^M (x_1), \mu_\alpha^M (x_2)\},\min \{\delta_\alpha^M (y_1), \delta_\alpha^M (y_2)\}$
$= \min\{\min\{\mu_\alpha^M (x_1),\delta_\alpha^M (y_1)\},\min \{\mu_\alpha^M (x_2),\delta_\alpha^M(y_2)\}\}$
$= \min \{ (\mu_\alpha^M \text{ x } \delta_\alpha^M) (x_1, y_1), (\mu_\alpha^M \text{ x } \delta_\alpha^M) (x_2, y_2) \}$

$\Rightarrow (\mu_\alpha^M \text{ x } \delta_\alpha^M) ((x_1, y_1) * (x_2, y_2)) \geq \min \{ (\mu_\alpha^M \text{ x } \delta_\alpha^M) (x_1, y_1), (\mu_\alpha^M \text{ x } \delta_\alpha^M) (x_2, y_2) \}$
This completes the proof.

5. Conclusion

In this article we have been discussed homomorphism and Cartesian product on fuzzy translation and fuzzy muliplication of PS-algebras. It adds an another dimension to the defined PS--algebras. This concept can further be generalized to intuitionistic fuzzy set, interval valued fuzzy sets for new results in our future work.

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