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Shortest Path Problem on Intuitionistic Fuzzy Network

***Saibal Majumder*[1] and *Anita Pal*[2]**

[2]Department of Mathematics, National Institute of Technology, Durgapur
Durgapur-713209, West Bengal, India
[1]Email: saibaltufts@gmail.com, [2]Email: anita.buie@gmail.com

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Abstract. Finding shortest paths in graphs has been the area of interest for many researchers. Shortest paths are one of the fundamental and most widely used concepts in networks. In this paper the authors present an algorithm to find an Intuitionistic Fuzzy Shortest Path (IFSP) in a directed graph in which the cost of every edges are represented by a trapezoidal intuitionistic fuzzy numbers(TrIFNs) which is the most generalized form of Trapezoidal Fuzzy Numbers(TrFNs) consisting of degree of acceptance and degree of rejection. The proposed algorithm uses Induced Intutionistic Trapezoidal Fuzzy Order Weighted Geometric (I-ITFOWG) operator for finding Intuitionistic Fuzzy Shortest Path (IFSP). A numerical example is given to explain the proposed algorithm.

Keywords: Intuitionistic Fuzzy Set, Trapezoidal Intuitionistic Fuzzy Number, Centroid point.

AMS Mathematics Subject Classification (2010): 05C72

1. Introduction

Shortest Path problems are among the fundamental problems studied in computational geometry and other areas including graph algorithms, Geographical Information Systems (GIS), network optimization etc. The classical shortest path problems having certain edge length have been studied intensively by many researchers but in real world application we have to deal with many uncertain information. The shortest path problem under the uncertain environment was first analyzed by Dubois and Prade [3]. In this paper we will be dealing with intuitionistic trapezoidal fuzzy numbers which are expressed as the edge costs for the underlined directed graph. We proposed an algorithm which will first find out all possible paths between the source and sink vertices of the directed graph, then I-ITFOWG operator is applied to the set of TrIFNs comprising in each of those paths to find an aggregated ITrFN. We then calculate the centroid of each of these TrIFN. The TrIFN with largest centroid value is considered to be the aggregated ITrFN with maximum acceptance and minimum rejection degree. And the path corresponding to that aggregated TrIFN is recognized to be the shortest path of the network.

The paper is organized as follows:

Section 2 discussses briefly the preliminary concepts of TrIFN, Ordered Weighted Averaging (OWA) operator, Geometric Mean (GM) operator, Ordered Weighted Geometric (OWG) operator Induced Intuitionistic Fuzzy Ordered Weighted Geometric (I-ITFOWG) operator. Section 3 explain about ranking of trapezoidal intuitionistic fuzzy numbers based on its centroid point. Section 4 detailed our proposed algorithm to find the shortest path of a network whose edge costs are trapezoidal intuitionistic fuzzy numbers. Section 5 gives a numerical explanation of our proposed algorithm and finally we conclude our paper in section 6.

2. Preliminaries

Atanassov [1] first introduce the concept of intuitionistic fuzzy set as follows:
Let X be the universe of discourse then an intuitionistic fuzzy set I on U is defined as $\mathrm{I} = \{(\mathrm{x}, \mu_{\mathrm{I}}(\mathrm{x}), \upsilon_{\mathrm{I}}(\mathrm{x}))|\mathrm{x} \in \mathrm{X}\}$ where functions $\mu_{\mathrm{I}}(\mathrm{x}): \mathrm{X} \rightarrow [0,1]$ and $\upsilon_{\mathrm{I}}(\mathrm{x}): \mathrm{X} \rightarrow [0,1]$ are the degree of membership and non membership of the element $\mathrm{x} \in \mathrm{X}$ and $\forall \mathrm{x} \in \mathrm{X}, 0 \leq \mu_{\mathrm{I}}(\mathrm{x}) + \upsilon_{\mathrm{I}}(\mathrm{x}) \leq 1$.

2.1. Definition Intuitionistic Fuzzy Set

Atanassov **[1]** first introduce the concept of intuitionistic fuzzy set (IFS) as follows:
Let X be the universe of discourse then an intuitionistic fuzzy set I on U is defined as $I = \{(x, \mu_I(x), \upsilon_I(x))|x \in X\}$ where functions $\mu_I(x): X \rightarrow [0,1]$ and $\upsilon_I(x): X \rightarrow [0,1]$ are the degree of membership and non membership of the element $x \in X$ and $\forall x \in X, 0 \leq \mu_I(x) + \upsilon_I(x) \leq 1$.

2.2. Definition Trapezoidal Intuitionistic Fuzzy Number

A Trapezoidal Intuitionistic Fuzzy Number **(TrIFN)** $\tilde{\mathrm{s}} = \langle(\mathrm{s}_1, \mathrm{s}_2, \mathrm{s}_3, \mathrm{s}_4); \mathrm{w}_{\tilde{\mathrm{s}}}, \mathrm{u}_{\tilde{\mathrm{s}}}\rangle$ is a special case of intuitionistic fuzzy number defined on the set of real number $\mathfrak{R}$, whose membership and non-membership functions are defined as:

$$\mu_{\tilde{s}}(x) = \begin{cases} \frac{(x-s_1)}{(s_2-s_1)} w_{\tilde{s}} ; s_1 \leq x \leq s_2 \\ w_{\tilde{s}} ; s_2 \leq x \leq s_3 \\ \frac{(s_4-x)}{(s_4-s_3)} w_{\tilde{s}} ; s_3 \leq x \leq s_4 \\ 0 ; x < s_1 \; and \; x >; s_4 \end{cases} \quad \text{and } \upsilon_{\tilde{s}}(x) = \begin{cases} \frac{(s_2-x)+u_{\tilde{s}}(x-s_1)}{(s_2-s_1)} ; s_1 \leq x \leq s_2 \\ u_{\tilde{s}} ; s_2 \leq x \leq s_3 \\ \frac{(x-s_3)+u_{\tilde{s}}(s_4-x)}{(s_4-s_3)} ; s_3 \leq x \leq s_4 \\ 1 ; x < s_1 \; and \; x >; s_4 \end{cases}$$

The graphical representation of which is shown in Fig. 1. The values $\mathrm{w}_{\tilde{\mathrm{s}}}$ and $\mathrm{u}_{\tilde{\mathrm{s}}}$ represents the maximum membership degree and minimum non-membership degree respectively such that$0 \leq \mathrm{w}_{\tilde{\mathrm{s}}} \leq 1$, $0 \leq \mathrm{u}_{\tilde{\mathrm{s}}} \leq 1$ and $0 \leq \mathrm{w}_{\tilde{\mathrm{s}}} + \mathrm{u}_{\tilde{\mathrm{s}}} \leq 1$ holds. The parameters $\mathrm{w}_{\tilde{\mathrm{s}}}$ and $\mathrm{u}_{\tilde{\mathrm{s}}}$ defines the confidence level and the non-confidence level of the TrIFN $\tilde{\mathrm{s}} = \langle(\mathrm{s}_1, \mathrm{s}_2, \mathrm{s}_3, \mathrm{s}_4); \mathrm{w}_{\tilde{\mathrm{s}}}, \mathrm{u}_{\tilde{\mathrm{s}}}\rangle$ respectively.

Let $\rho_{\tilde{\mathrm{s}}}(\mathrm{x}) = 1 - \mathrm{w}_{\tilde{\mathrm{s}}}(\mathrm{x}) - \mathrm{u}_{\tilde{\mathrm{s}}}(\mathrm{x})$, which is called hesitation degree or intuitionistic fuzzy index of whether x belongs to $\tilde{\mathrm{s}}$. If the hesitation degree is small then knowledge whether x belongs to $\tilde{\mathrm{s}}$ is more certain, while if the hesitation degree is large then knowledge on that is more uncertain.

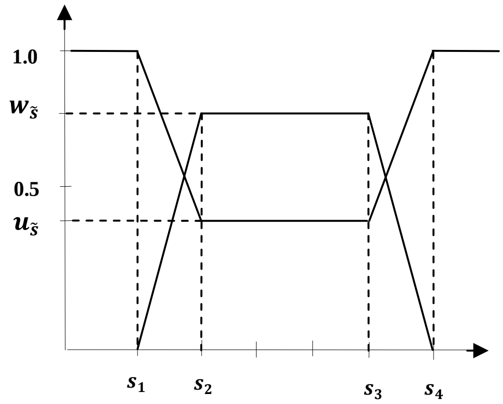

Figure 1: Membership degree and non-membership degree of TrIFN

2.3. Arithmetic operations of two TrIFN

Let $\tilde{s} = \langle (s_1, s_2, s_3, s_4); w_{\tilde{s}}, u_{\tilde{s}} \rangle$ and $\tilde{t} = \langle (t_1, t_2, t_3, t_4); w_{\tilde{t}}, u_{\tilde{t}} \rangle$ be the two TrIFN and η be a real number. Then,

$\tilde{s} \oplus \tilde{t} = \langle (s_1 + t_1, s_2 + t_2, s_3 + t_3, s_4 + t_4); w_{\tilde{s}} + w_{\tilde{t}} - w_{\tilde{s}} w_{\tilde{t}}, u_{\tilde{s}} u_{\tilde{t}} \rangle$

$\tilde{s} \otimes \tilde{t} = \langle (s_1 t_1, s_2 t_2, s_3 t_3, s_4 t_4); w_{\tilde{s}} w_{\tilde{t}}, u_{\tilde{s}} + u_{\tilde{t}} - u_{\tilde{s}} u_{\tilde{t}} \rangle$

$\eta \tilde{s} = \langle (\eta s_1, \eta s_2, \eta s_3, \eta s_4); 1 - (1 - w_{\tilde{s}})^{\eta}, {u_{\tilde{s}}}^{\eta} \rangle$

2.4. Definition Support of TrIFN

The support of a TrIFN $\tilde{s} = \langle (s_1, s_2, s_3, s_4); w_{\tilde{s}}, u_{\tilde{s}} \rangle$ for the membership function and non-membership function are defined as $\sup_w(\tilde{s}) = \{x | w_{\tilde{s}}(x) \geq 0\}$ and $\sup_u(\tilde{s}) = \{x | u_{\tilde{s}}(x) \leq 1\}$ respectively.

2.5. Definition Ordered Weighted Averaging (OWA) Operator [7]

An OWA operator of dimension m is a function τ, $\tau: \Re^{+m} \to \Re^{+}$ which has associated a set of weights or weighting vector $\omega = [\omega_1, \omega_2, \dots, \omega_m]^T\ \forall \omega_i \in [0,1]$ and $\sum_i \omega_i = 1.$ Furthermore, $\tau(y_1, y_2, \dots, y_m) = \omega^T . B = \sum_{j=1}^{m} \omega_j b_j$ where B is the associated ordered value vector and each element $b_j \in B$, is the j^{th} largest value among y_j $(j = 1,2,3, \dots, m)$.

2.6. Definition Geometric Mean Operator [7]

A geometric mean operator of dimension m is a function ρ, $\rho: \Re^{+m} \to \Re^{+}$ such that $\rho(y_1, y_2, \dots, y_m) = \prod_{k=1}^{m} (y_k)^{\frac{1}{m}}$.

2.7. Definition Ordered Weighted Geometric (OWG) operator [8]

The OWG operator is developed based on the concept of OWA operator and the geometric mean operator so this operator inherits the advantages of both the OWA operator and the geometric mean operator. Mathematically we defined an OWA operator of dimension m is a function δ, $\delta: \Re^{+m} \to \Re^{+}$ that has associated with it a weight vector, $\omega = [\omega_1, \omega_2, \dots, \omega_m]^T\ \forall \omega_i \in [0,1]$ and $\sum_i \omega_i = 1$ such that $\delta(y_1, y_2, \dots, y_m) =$

$\prod_{k=1}^{m}(c_k)^{\omega_k}$ where c_k is the k^{th} largest value among $y_k(k = 1,2,3,\dots,m)$. One of the most important factor in the OWG operator is to determine its associated weights. Xu [4] develop a normal distribution based method which is defined as $\omega_i = \frac{e^{-\frac{(i-\mu_m)}{2\sigma_m^2}}}{\sum_{k=1}^{m} e^{-\frac{(k-\mu_m)}{2\sigma_m^2}}}$, $k = 1,2,\dots,m$

where μ_m is the mean of the collection of $1,2,\dots,m$ and $\sigma_m(> 0)$ is the standard deviation of the collection of $1,2,\dots,m$. So mathematically we define $\mu_m = \frac{1+n}{2}$ and $\sigma_m = \sqrt{\frac{1}{m}\sum_{l=1}^{m}(l-\mu_m)^2}$.

2.8. Definition Induced Intuitionistic Trapezoidal Fuzzy Ordered Weighted Geometric Operator [4]

If $\widetilde{\varphi_i}(i = 1,2,\dots,n)$ be a collection intuitionistic trapezoidal fuzzy numbers. An induced intuitionistic trapezoidal fuzzy ordered weighted geometric (I-ITFOWG) operator of dimension n is a mapping $I - ITFOWG: \varepsilon^n \to \varepsilon$ which has an associated vector $\omega = [\omega_1,\omega_2,\dots,\omega_n]^T$ such that $\forall\omega_i \in [0,1]$ and $\sum_{i=1}^{n}\omega_i = 1$.

Moreover, $I - ITFOWG_\omega(\langle u_1,\tilde{\varphi}_1\rangle,\langle u_2,\tilde{\varphi}_2\rangle,\dots,\langle u_n,\tilde{\varphi}_n\rangle) = \tilde{\varphi}_{\sigma(1)}^{\omega_1} \otimes \tilde{\varphi}_{\sigma(2)}^{\omega_2} \dots \otimes \tilde{\varphi}_{\sigma(n)}^{n}$ where $\omega = [\omega_1,\omega_2,\dots,\omega_n]^T$ is the weighted vector of $\tilde{\varphi}_j(j = 1,2,\dots,n)$ $\forall\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$.Let $[\sigma(1),\sigma(2),\dots,\sigma(j-1),\sigma(j),\sigma(j+1),\dots,\sigma(n)]$ is a permutation of $[1,2,3,\dots,n]$ such that $u_{\sigma(j-1)} \geq u_{\sigma(j)}$. $\tilde{\varphi}_{\sigma(j)}$ is the $\tilde{\varphi}_i$value of the ITFOW pair $\langle u_i,\tilde{\varphi}_i\rangle$ having the j^{th} largest u_i ($u_i \in [0,1]$), where u_i in $\langle u_i,\tilde{\varphi}_i\rangle$ is referred to as the order inducing variable and $\tilde{\varphi}_i$as the intuitionistic trapezoidal fuzzy numbers. Now if we consider $\tilde{s}_i(i = 1,2,\dots,n)$ as a collection intuitionistic trapezoidal fuzzy numbers then their aggregated value by using the $I - ITFOWG$ operator is also an intuitionistic fuzzy number and is expressed as,

$$I - ITFOWG_\omega(\tilde{s}_1,\tilde{s}_2,\dots,\tilde{s}_n) = \left\langle\left(\prod_{k=1}^{n} a_{\sigma(j)}^{\omega_k},\prod_{k=1}^{n} b_{\sigma(j)}^{\omega_k},\prod_{k=1}^{n} c_{\sigma(j)}^{\omega_k},\prod_{k=1}^{n} d_{\sigma(j)}^{\omega_k}\right);\prod_{k=1}^{n} w_{\tilde{s}_{\sigma(k)}}^{\omega_k}, 1-\prod_{k=1}^{n}(1-u_{\tilde{s}_{\sigma(k)}})^{\omega_k}\right\rangle$$

where $\omega = [\omega_1,\omega_2,\dots,\omega_n]^T$ is the weight vector of $I - ITFOWG$ operator $\forall\omega_i \in [0,1]$ and $\sum_{i=1}^{n}\omega_i = 1$.

3. Ranking intuitionistic fuzzy numbers

Let $\tilde{s} = \langle(s_1,s_2,s_3,s_4);w_{\tilde{s}},u_{\tilde{s}}\rangle$ be a trapezoidal intutionistic fuzzy number and the area covered by it's membership and non-membership grade is depicted in Fig. 2. The total area is divided into five parts namely, $x_1x_3p'z_1$, $z_1p'y_4y_1$, $y_1y_4x_4x_2$, $x_2x_4q'z_2$ and $z_2q'y_3y_2$. The coordinates of the corner point of the plotted graph are listed below:

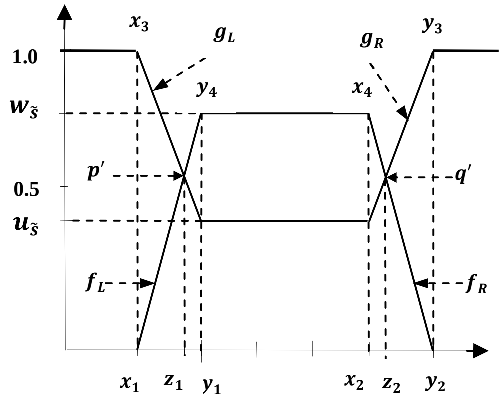

Fig. 2: Representation of area covered under membership and non-membership function of TrIFN

$x_1 = (s_1, 0), y_1 = (s_2, 0), x_2 = (s_3, 0), y_2 = (s_4, 0), x_3 = (s_1, 1), y_3 = (s_4, 1), x_4 = (s_3, w_{\tilde{s}}), y_4 = (s_2, w_{\tilde{s}})$
$p' = \left(\frac{(s_1 w_{\tilde{s}} - s_1 u_{\tilde{s}} + s_2)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}, \frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}\right), q' = \left(\frac{(s_4 w_{\tilde{s}} - s_4 u_{\tilde{s}} + s_3)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}, \frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}\right), z_1 = \left(\frac{(s_1 w_{\tilde{s}} - s_1 u_{\tilde{s}} + s_2)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}, 0\right), z_2 = \left(\frac{(s_4 w_{\tilde{s}} - s_4 u_{\tilde{s}} + s_3)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}, 0\right)$. Das and Guha [9] proposed the centroid point$(X_{\tilde{s}}, Y_{\tilde{s}})$ of TrIFN such that $X_{\tilde{s}} = \frac{\int xf(x)dx}{\int f(x)dx}, Y_{\tilde{s}} = \frac{\int yf(y)dy}{\int f(y)dy}$. Now, $X_{\tilde{s}} = \frac{x_1}{x_2}$ and $Y_{\tilde{s}} = \frac{y_1}{y_2}$ where,

$$x_1 = \int_{s_1}^{\frac{(s_1 w_{\tilde{s}} - s_1 u_{\tilde{s}} + s_2)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}} x g_L dx + \int_{\frac{(s_1 w_{\tilde{s}} - s_1 u_{\tilde{s}} + s_2)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}}^{b} x f_L dx + \int_b^c x w_{\tilde{s}} dx + \int_c^{\frac{(s_4 w_{\tilde{s}} - s_4 u_{\tilde{s}} + s_3)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}} x f_R dx + \int_{\frac{(s_4 w_{\tilde{s}} - s_4 u_{\tilde{s}} + s_3)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}}^{s_4} x g_R dx$$

$$x_2 = \int_{s_1}^{\frac{(s_1 w_{\tilde{s}} - s_1 u_{\tilde{s}} + s_2)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}} g_L dx + \int_{\frac{(s_1 w_{\tilde{s}} - s_1 u_{\tilde{s}} + s_2)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}}^{b} f_L dx + \int_b^c w_{\tilde{s}} dx + \int_c^{\frac{(s_4 w_{\tilde{s}} - s_4 u_{\tilde{s}} + s_3)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}} f_R dx + \int_{\frac{(s_4 w_{\tilde{s}} - s_4 u_{\tilde{s}} + s_3)}{w_{\tilde{s}} - u_{\tilde{s}} + 1}}^{s_4} g_R dx$$

$$y_1 = \int_0^{w_{\tilde{s}}} y(h_R - h_L)\, dy + \left[\int_0^1 y s_4 dy - \int_0^{\frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}} y h_R dy - \int_{\frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}}^{1} y I_R dy\right] + \left[\int_0^{\frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}} y h_L\, dy + \int_{\frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}}^{1} y I_L dy - \int_0^1 y s_1 dy\right]$$

$$y_2 = \int_0^{w_{\tilde{s}}} (h_R - h_L)\, dy + \left[\int_0^1 s_4 dy - \int_0^{\frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}} h_R dy - \int_{\frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}}^{1} y t_R dy\right] + \left[\int_0^{\frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}} h_L\, dy + \int_{\frac{w_{\tilde{s}}}{w_{\tilde{s}} - u_{\tilde{s}} + 1}}^{1} t_L dy - \int_0^1 s_1 dy\right]$$

and $f_L: [s_1, s_2] \to [0, w_{\tilde{s}}]$, $f_L: [s_3, s_4] \to [0, w_{\tilde{s}}]$ are leftmost and rightmost part of membership grade and $[s_1, s_2] \to [0, u_{\tilde{s}}]$, $g_R: [s_3, s_4] \to [0, u_{\tilde{s}}]$ are leftmost and rightmost part of non-membership grade of TrIFN respectively depicted in

Fig. 2. $h_L: [0, w_{\tilde{s}}] \rightarrow [s_1, s_2]$, $h_R: [0, w_{\tilde{s}}] \rightarrow [s_3, s_4]$ are the inverse function of $f_L$and f_R respectively. $t_L: [0, u_{\tilde{s}}] \rightarrow [s_1, s_2]$, $t_R: [0, u_{\tilde{s}}] \rightarrow [s_3, s_4]$ are the inverse function of $g_L$and $g_R$respectively. The analytical expression of each of these above functions are given below:

$f_L(x) = \frac{w_{\tilde{s}}(x-s_1)}{(s_2-s_1)}\ ; s_1 \le x \le s_2$ $\qquad f_R(x) = \frac{w_{\tilde{s}}(x-s_4)}{(s_3-s_4)}\ ; s_3 \le x \le s_4$

$g_L(x) = \frac{(x-s_2)+u_{\tilde{s}}(s_1-x)}{(s_1-s_2)}\ ; s_1 \le x \le s_2$ $\quad g_L(x) = \frac{(x-s_3)+u_{\tilde{s}}(s_4-x)}{(s_4-s_3)}\ ; s_3 \le x \le s_4$

$h_L(y) = s_1 + \frac{(s_2-s_1)y}{w_{\tilde{s}}}\ ; 0 \le y \le w_{\tilde{s}}$ $\quad h_R(y) = s_4 - \frac{(s_4-s_3)y}{w_{\tilde{s}}}\ ; 0 \le y \le w_{\tilde{s}}$

$t_L(y) = \frac{(s_1-s_2)y+(s_2-s_1u_{\tilde{s}})}{(1-u_{\tilde{s}})}; u_{\tilde{s}} \le y \le 1$ $\quad t_R(y) = \frac{(s_4-s_3)y+(s_3-s_4u_{\tilde{s}})}{(1-u_{\tilde{s}})}; u_{\tilde{s}} \le y \le 1$

Now the ranking of any two TrIFN $\tilde{r}$ and $\tilde{s}$ **[9]** is done based on following criteria :

$$\begin{aligned}
&\boldsymbol{if}\ X_{\tilde{r}} > X_{\tilde{s}}\ ,\boldsymbol{then}\ \tilde{r} > \tilde{s}\\
&\boldsymbol{if}\ X_{\tilde{r}} < X_{\tilde{s}}\ ,\boldsymbol{then}\ \tilde{r} < \tilde{s}\\
&\boldsymbol{if}\ X_{\tilde{r}} = X_{\tilde{s}}\ ,\boldsymbol{then}\\
&\boldsymbol{if}\ Y_{\tilde{r}} > Y_{\tilde{s}}\ ,\boldsymbol{then}\ \tilde{r} > \tilde{s}\\
&\boldsymbol{else\ if}\ Y_{\tilde{r}} < Y_{\tilde{s}}\ ,\boldsymbol{then}\ \tilde{r} < \tilde{s}\\
&\boldsymbol{else}\ Y_{\tilde{r}} = Y_{\tilde{s}}\ ,\boldsymbol{then}\ \tilde{r} = \tilde{s}
\end{aligned}$$

4. Proposed Shortest Path Algorithm

We consider a connected acyclic network having a source vertex u and and a sink vertex z. Each edge $i - j$ of the network represents the cost (or distance) parameter between vertices i and j. We consider these parameters to determine the shortest path in a network. The edges of our network are associated with a pair of ordered inducing variable and TrIFN, $\langle u_j, \tilde{\varphi}_j \rangle$. The significance of such a pair in the context of the connected network is as follows:

The order inducing variable, $u_i (\subseteq [0,1])$ determines the j^{th} largest edge $\langle u_i, \tilde{\varphi}_i \rangle$ which is included in a path from source to sink of the network such that for any two edges included in a path, $u_{\sigma(j-1)} \ge u_{\sigma(j)}$; whereas for each TrIFN $\tilde{\varphi}_j$, its membership grade represents the acceptance degree to which an edge $i - j$ will be included in the shortest path between source to sink and it's non-membership grade represents the rejection degree to which an edge $i - j$ will be accepted from the shortest path between source to sink. The new algorithm is detailed below:

Input: Let $G = (V, E)$ be a directed acyclic network having intuitionistic fuzzy costs associated with its edges.

Output: A shortest path between the source and the sink vertices of the Network is traced out based on comparison of centroid point of the aggregated trapezoidal intuitionistic fuzzy numbers for all the paths that exists between the source and sink of the network under consideration.

Step-1: Trace all existing paths between source and sink of an acyclic connected network having intuitionistic fuzzy costs associated with its edges.

Step-2: Rearrange every edges of a path between two terminal vertices in non-increasing order with respect to their order inducing variable.

Step-3: Derive the associated weight vector [8] $\omega = [\omega_1, \omega_2, \ldots, \omega_m]^T$ for $I - ITFOWG$ operator, where each $\omega_i > 0$, $\sum_{i=1}^{m} \omega_i = 1$ and m is the number of edges of the network path under consideration. Calculate the aggregated TrIFN,
$\mathrm{I - ITFOWG}_\omega\left(\tilde{\varphi}_{\sigma_{p_i}(1)}, \tilde{\varphi}_{\sigma_{p_i}(2)}, \ldots, \tilde{\varphi}_{\sigma_{p_i}(m)}\right)$ for path p_i. Repeat Step-2 and Step-3 for all the traced paths.

Step-4: Determine the centroid pont of all the aggregated TrIFN. The network path corresponding to the aggregated TrIFN having largest X or Y value of its centroid point gives the shortest path of the network.

Step-5: Add $\tilde{\varphi}_k$'s associated with the edges of the selected shortest path to get the shortest path length.

5. Numerical Explanation

A connected acyclc network has been considered in Fig. 3 with vertices u, v, x, y, w and z, with u the source vertex and the z the sink vertex. Every edge weights of the network are expressed as a pair of ordered inducing variable and TrIFN $\langle u_j, \tilde{\varphi}_j \rangle$ that are listed in Table 1. Our main objective is to search the shortest path between u and z by applying our algorithm illustrated in section 4.

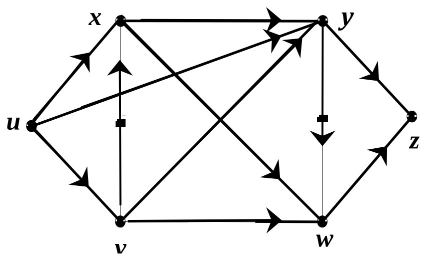

Fig. 3: Connected acyclic network

Edges $i - j$	$\langle u_j, \tilde{\varphi}_j \rangle$
$u - x$	$\langle 0.90, \{(0.3,0.4,0.6,0.8)\}; (0.8,0.1) \rangle$
$u - v$	$\langle 0.60, \{(0.2,0.3,0.5,0.7)\}; (0.5,0.1) \rangle$
$x - y$	$\langle 0.35, \{(0.2,0.3,0.5,0.6)\}; (0.6,0.4) \rangle$
$v - w$	$\langle 0.55, \{(0.2,0.3,0.5,0.6)\}; (0.6,0.4) \rangle$

$v-x$	$\langle 0.14,\{(0.1,0.2,0.7,0.8)\};(0.4,0.5)\rangle$
$v-y$	$\langle 0.38,\{(0.15,0.35,0.45,0.72)\};(0.3,0.42)\rangle$
$x-w$	$\langle 0.42,\{(0.6,0.7,0.8,0.9)\};(0.5,0.27)\rangle$
$y-w$	$\langle 0.30,\{(0.1,0.3,0.5,0.8)\};(0.4,0.3)\rangle$
$u-y$	$\langle 0.82,\{(0.4,0.6,0.7,0.8)\};(0.7,0.2)\rangle$
$y-z$	$\langle 0.75,\{(0.4,0.7,0.8,0.9)\};(0.6,0.2)\rangle$
$w-z$	$\langle 0.40,\{(0.2,0.5,0.6,0.7)\};(0.5,0.3)\rangle$

Table 1: List of edges and their weights

Now applying our algorithm we trace out all the paths between the source and sink of the considered network which are listed below:

$p_1 \coloneqq u-x-y-z$; $p_2 \coloneqq u-x-w-z$; $p_3 \coloneqq u-v-w-z$; $p_4: u-y-w-z$; $p_5 \coloneqq u-v-y-w-z$; $p_6 \coloneqq u-v-x-y-z$; $p_7 \coloneqq u-v-x-w-z$; $p_8 \coloneqq u-v-y-z$; $p_9 \coloneqq u-x-y-w-z$; $p_{10} \coloneqq u-y-z$; $p_{11} \coloneqq u-v-x-y-w-z$.

Considering p_1 which consist of edges $u-x, x-y$ and $y-z$. We rearrange these edges in non-increasing order based on their respective order inducing variable. Therefore, $u-x, x-y$ and $y-z$ are arranged to $u-x, y-z$ and $x-y$.

So the rearranged edges are,

$\tilde{\varphi}_{\sigma_{p_1}(1)} = [u-x] = \langle 0.90,\{(0.3,0.4,0.6,0.8)\};(0.8,0.1)\rangle$;$\tilde{\varphi}_{\sigma_{p_1}(2)} = [y-z] = \langle 0.75,\{(0.4,0.7,0.8,0.9)\};(0.6,0.2)\rangle$ and

$\tilde{\varphi}_{\sigma_{p_1}(3)} = [x-y] = \langle 0.35,\{(0.2,0.3,0.5,0.6)\};(0.6,0.4)\rangle$

The normal distribution based weight vector for path p_1 is calculated as $\omega = [0.2429,0.5142,0.2429]^T$.

Now calculating aggregated trapezoidal intuitionistic fuzzy number using $I-ITFOWG$ operator we get,

$\mathrm{I-ITFOWG}\left(\widetilde{\varphi}_{\sigma_{p_1}(1)},\widetilde{\varphi}_{\sigma_{p_1}(2)},\widetilde{\varphi}_{\sigma_{p_1}(3)}\right)=$
$\langle\left(\prod_{k=1}^{3}\mathrm{a}_{\sigma_{p_1}(k)}^{\omega_k},\prod_{k=1}^{3}\mathrm{b}_{\sigma_{p_1}(k)}^{\omega_k},\prod_{k=1}^{3}\mathrm{c}_{\sigma_{p_1}(k)}^{\omega_k},\prod_{k=1}^{3}\mathrm{d}_{\sigma_{p_1}(k)}^{\omega_k}\right);\prod_{k=1}^{3}\mathrm{w}_{\widetilde{\varphi}_{\sigma_{p_1}(k)}}^{\omega_k},1-$
$\prod_{k=1}^{3}(1-\mathrm{u}_{\widetilde{\varphi}_{\sigma_{p_1}(k)}})^{\omega_k}\rangle$

$=\langle\{(0.3^{0.2429}\times 0.4^{0.5142}\times 0.2^{0.2429}),(0.4^{0.2429}\times 0.7^{0.5142}\times 0.3^{0.2429}),(0.6^{0.2429}\times 0.8^{0.5142}\times 0.5^{0.2429}),(0.8^{0.2429}\times 0.9^{0.5142}\times 0.6^{0.2429})\};(0.8^{0.2429}\times 0.6^{0.5142}\times 0.6^{0.2429}),(1-\{(1-0.1)^{0.2429}\times(1-0.2)^{0.5142}\times(1-0.2)^{0.2429}\})\rangle$

$=\langle(0.315204,0.497373,0.665520,0.792583);0.6434,0.2323\rangle.$

The aggregated trapezoidal intuitionistic fuzzy numbers for the all the paths are listed in Table 2.

Paths from source($\boldsymbol{u}$) to sink($\boldsymbol{z}$)	**Aggregated $\boldsymbol{TrIFN}$ using $\boldsymbol{I-ITFOWG}$ operator**
$u-x-y-z$	$\langle(0.315204,0.497373,0.665520,0.792583);0.6434,0.2323\rangle$
$u-x-w-z$	$\langle(0.388274,0.563079,0.695656,0.822822);0.5604,0.2397\rangle$
$u-v-w-z$	$\langle(0.246363,0.393778,0.621358,0.749751);0.5,0.2030\rangle$
$u-y-w-z$	$\langle(0.2,0.461653,0.595908,0.7469138);0.5139,0.2769\rangle$
$u-v-y-w-z$	$\langle(0.162654,0.377359,0.513453,0.721618);0.404959,0.206274\rangle$
$u-v-x-y-z$	$\langle(0.2,0.321265,0.566576,0.704543);0.5291,0.2985\rangle$
$u-v-x-w-z$	$\langle(0.262408,0.450108,0.659718,0.779366);0.483002,0.299104\rangle$
$u-v-y-z$	$\langle(0.220700,0.382616,0.546306,0.749171);0.461653,0.213916\rangle$
$u-x-y-w-z$	$\langle(0.191277,0.374131,0.547722,0.691797);0.553229,0.309889\rangle$
$u-y-z$	$\langle(0.4,0.648074,0.748331,0.848528);0.648074,0.2\rangle$
$u-v-x-y-w-z$	$\langle(0.157174,0.323594,542089,0.699827);0.489029,0.338282\rangle$

Table 2: List of aggregated TrIFN for different paths

Now, we find out the centroid point **[9]** of each of the aggregated $TrIFN$listed in Table 2.

Aggregated $TrIFN$ $(\varphi_{agg(p_i)})$ of all paths from source to sink	**Centroid Point ith path**
$\langle(0.315204, 0.497373, 0.665520, 0.792583); 0.6434, 0.2323\rangle$	(0.551259,0 .345403)
$\langle(0.388274, 0.563079, 0.695656, 0.822822); 0.5604, 0.2397\rangle$	(0.601941,0 .332687)
$\langle(0.246363, 0.393778, 0.621358, 0.749751); 0.5, 0.2030\rangle$	(0.496098,0 308039)
$\langle(0.2, 0.461653, 0.595908, 0.7469138); 0.5139, 0.2769\rangle$	(0.463530,0 335227)
$\langle(0.162654, 0.377359, 0.513453, 0.721618); 0.404959, 0.206274\rangle$	(0.441321,0.319675)
$\langle(0.2, 0.321265, 0.566576, 0.704543); 0.5291, 0.2985\rangle$	(0.453922,0.318196)
$\langle(0.262408, 0.450108, 0.659718, 0.779366); 0.483002, 0.299104\rangle$	(0.513091,0.318800)
$\langle(0.220700, 0.382616, 0.546306, 0.749171); 0.461653, 0.213916\rangle$	(0.489474,0.317353)
$\langle(0.191277, 0.374131, 0.547722, 0.691797); 0.553229, 0.309889\rangle$	(0.438326,0.335173)
$\langle(0.4, 0.648074, 0.748331, 0.848528); 0.648074, 0.2\rangle$	(0.616970,0.347077)
$\langle(0.157174, 0.323594, 542089, 0.699827); 0.489029, 0.338282\rangle$	(0.427513,0.324039)

Table 3: Centroid points of aggregated TrIFNs

Hence, applying ranking algorithm of TrIFN we rank the TrIFN in the following order:

$\varphi_{agg(p_{11})} \preccurlyeq \varphi_{agg(p_9)} \preccurlyeq \varphi_{agg(p_5)} \preccurlyeq \varphi_{agg(p_6)} \preccurlyeq \varphi_{agg(p_4)} \preccurlyeq \varphi_{agg(p_8)} \preccurlyeq \varphi_{agg(p_3)} \preccurlyeq \varphi_{agg(p_7)} \preccurlyeq \varphi_{agg(p_1)} \preccurlyeq \varphi_{agg(p_2)} \preccurlyeq \varphi_{agg(p_{10})}$. Since $X_{\varphi_{agg(p_{10})}}$ value of the centroid point of $\varphi_{agg(p_{10})}$ is largest so the path corresponding to $\varphi_{agg(p_{10})}$ is, $u - y - z$ which is recognized to be the shortest path from source to sink of the network and its length is $\langle(0.8, 1.3, 1.5, 1.7); 0.88, 0.04\rangle$.

6. Conclusion

In this paper we have proposed an algorithm to find the shortest paths of a network. The edge weights of the network is represented as a pair of ordered inducing variable and TrIFN, $\langle u_j, \tilde{\varphi}_j\rangle$. The TrIFNs used in this paper can be consider as the more generalized version of TrFNs since TrIFN considers belongingness and non-belongingness of an event as a matter of which the uncertain information can be expressed in a much better way with respect to TrFN. In our algorithm of shortest path problem we have used $\mathrm{I-ITFOWG}$ operator to find the aggregated TrIFNs and have ranked them based on the centroid values of those TrIFNs and finally find the shortest path having largest centroid point value and have also calculated its path length.

From the view point of computational complexity this problem can be well considered as NP −hard. Execution of Step-1, Step-2 and Step-4 of the proposed algorithm may be a hard task in a large scale network, since these steps of the algorithm may take most of the computational time. In the future, improvements can be done in terms of efficient comparisons of TrIFN and data structure for this algorithm.

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