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Exact Solutions of (3 +1)-dimensional Potential-YTSF Equation by Improved (G′/G)-expansion Method and the Extended Tanh-Method

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Abstract. In the present article, we construct the exact traveling wave solutions of nonlinear evolution equations in mathematical physics via the (3+1)-dimensional potential-Yu-Toda-Sasa-Fukuyama (YTSF) equation by using two methods: namely, a further improved (G'/G)-expansion method, where $G(\xi)$ satisfies the auxiliary ordinary differential equation (ODE) $[G'(\xi)]^2 = pG^2(\xi) + qG^4(\xi) + rG^6(\xi)$; p, q and r are constants and the well known extended tanh-function method. We demonstrate that some of the exact solutions bring out by these two methods are analogous, but they are not one and the same. It is worth mentioning that the first method gives further exact solutions than the second one.

Keywords: Potential-YTSF equation, Extended tanh-function method, Improved (G'/G)-expansion method, Auxiliary equation, Traveling wave solutions

AMS Mathematics Subject Classification (2010): 35C07, 35C08

1. Introduction

Nonlinear evolution equations play a significant role in various scientific and engineering fields, such as, optical fibers, solid state physics, fluid mechanics, plasma physics, chemical kinematics, chemical physics geochemistry etc. Nonlinear wave phenomena of diffusion, reaction, dispersion, dissipation, and convection are very important in nonlinear wave equations. In recent years, the exact solutions of nonlinear PDEs have been investigated by many researchers (see [1-40]) who are concerned in nonlinear physical phenomena and many powerful and efficient methods have been offered by them. Among non-integrable nonlinear differential equations there is a wide class of equations that referred to as the partially integrable, because these equations become integrable for some values of their parameters. There are many different methods to look for the exact solutions of these equations. The most famous algorithms are the truncated Painleve expansion method [1], the Weierstrass elliptic function method [2], the tanh-function method [3-8] and the Jacobi elliptic function expansion method [9-12]. There are other methods which can be found in [13-17]. For integrable nonlinear differential equations, the inverse scattering transform method [18], the Hirota method [19], the truncated Painleve expansion method [20], the Backlund transform method [21] and the Exp-function method [22-26] are used for searching the exact solutions.

Wang et al [27] introduced a direct and concise method, called the (G'/G)-expansion method to look for traveling wave solutions of nonlinear partial differential equations, where $G = G(\xi)$ satisfies the second order linear ODE

$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$; λ and μ are arbitrary constants.

For additional references see the articles [28-34].

In this article, we bring in an alternative approach, called a improved (G'/G)-expansion method to find the exact traveling wave solutions of the potential-YTSF equation, where $G = G(\xi)$ satisfies the auxiliary

ODE $[G'(\xi)]^2 = pG^2(\xi) + qG^4(\xi) + rG^6(\xi)$; p, q and r are constants. Recently El-Wakil et al. [30] and Parkes [31] have shown that the extended tanh-function method proposed by Fan [5] and the basic (G'/G)-expansion method proposed by Wang et al [27] are entirely equivalent as they deliver exactly the same set of solutions to a given nonlinear evolution equation. This observation has also been pointed out recently by Kudryashov [35]. In this article, we assert even though the basic (G'/G)-expansion method is equivalent to the extended tanh-function method, the improved (G'/G)-expansion method presented in this article is not equivalent to the extended tanh-function method. The method projected in this article is varied to some extent from the extended (G'/G)-expansion method.

The objective of this article is to show that the improved (G'/G)-expansion method and the celebrated extended tanh-function method are not identical. Further novel solutions are achieved via the offered improved (G'/G)-expansion method. This approach will play an imperative role in constructing many exact traveling wave solutions for the nonlinear PDEs via the (3 + 1)-dimensional potential-YTSF equation.

2. The Improved (G'/G)-expansion Method

Suppose we have the following nonlinear partial differential equation,

$$H(u, u_t, u_x, u_y, u_z, u_{tt}, u_{yy}, u_{zz}, \cdots) = 0. \tag{1}$$

where $u = u(x, y, z, t)$ is an unknown function, H is a polynomial in $u = u(x, y, z, t)$ and its partial derivatives in which the highest order derivatives and the nonlinear terms are involved. The main steps of the further improved (G'/G)-expansion method are as follows:

Step 1: The traveling wave variable,

$$u\,(x, y, z, t) = u\,(\xi),\ \xi = x + y + z - V\,t\,, \tag{2}$$

where V is the speed of the traveling wave, which converts the Eq. (1) into an ODE in the form,

$$P(u, u', u'', u''', \cdots) = 0\,, \tag{3}$$

where prime denotes the derivative with respect to ξ.

Step 2: Suppose that the solution of the Eq. (3) can be expressed by means of a polynomial in (G'/G) as follows:

$$u(\xi)=\sum_{i=0}^{n}\alpha_i\left(\frac{G'}{G}\right)^i \qquad (4)$$

where α_i $(i=1,2,3,\cdots)$ are constants provided $\alpha_n\neq 0$ and $G=G(\xi)$ satisfies the following nonlinear auxiliary equation,

$$[G'(\xi)]^2=p\,G^2(\xi)+q\,G^4(\xi)+r\,G^6(\xi), \qquad (5)$$

where p, q and r are random constants to be determined later.

The general solutions of Eq. (5) are as follows [36, 37]:

No	$G(\xi)$	No	$G(\xi)$
1	$\left[\frac{-p\,q\sec h^2(\sqrt{p}\,\xi)}{q^2-p\,r\,(1\pm\tanh(\sqrt{p}\xi))^2}\right]^{\frac{1}{2}}$ or $\left[\frac{-p\,q\csc h^2(\sqrt{p}\,\xi)}{q^2-p\,r\,(1\pm\coth(\sqrt{p}\,\xi))^2}\right]^{\frac{1}{2}}$, $p>0$	6	$\left[\frac{-p\sec^2(\sqrt{-p}\xi)}{q\pm 2\sqrt{-p\,r}\,\tan(\sqrt{-p}\xi)}\right]^{\frac{1}{2}}$ or $\left[\frac{-p\csc^2(\sqrt{-p}\xi)}{q\pm 2\sqrt{-p\,r}\,\cot(\sqrt{-p}\,\xi)}\right]^{\frac{1}{2}}$, $p<0,\ r>0$
2	$\left[\frac{2\,p}{\pm\sqrt{\Delta}\cosh(2\sqrt{p}\xi)-q}\right]^{\frac{1}{2}}$, $p>0\ \Delta>0$	7	$\left[\frac{p\,e^{\pm 2\sqrt{p}\xi}}{\left(e^{\pm 2\sqrt{p}\xi}-4\,q\right)^2-64\,p\,r}\right]^{\frac{1}{2}}$, $p>0$
3	$\left[\frac{2\,p}{\pm\sqrt{\Delta}\cos(2\sqrt{-p}\,\xi)-q}\right]^{\frac{1}{2}}$ or $\left[\frac{2\,p}{\pm\sqrt{\Delta}\sin(2\sqrt{-p}\,\xi)-q}\right]^{\frac{1}{2}}$, $p<0,\ \Delta>0.$	8	$\left[-\frac{p}{q}\left(1\pm\tanh(\frac{1}{2}\sqrt{p}\xi)\right)\right]^{\frac{1}{2}}$ or $\left[-\frac{p}{q}\left(1\pm\coth(\frac{1}{2}\sqrt{p}\,\xi)\right)\right]^{\frac{1}{2}}$ $p>0,$ $\Delta=0$
4	$\left[\frac{2\,p}{\pm\sqrt{-\Delta}\sinh(2\sqrt{p}\,\xi)-q}\right]^{\frac{1}{2}}$, $p>0,\ \Delta<0$	9	$\left[\frac{\pm\,p\,e^{\pm 2\sqrt{p}\xi}}{1-64\,p\,r\,e^{\pm 4\sqrt{p}\xi}}\right]^{\frac{1}{2}}$, $p>0,\ q=0$
5	$\left[\frac{-p\sec h^2(\sqrt{p}\,\xi)}{q\pm 2\sqrt{p\,r}\,\tanh(\sqrt{p}\xi)}\right]^{\frac{1}{2}}$ or $\left[\frac{p\csc h^2(\sqrt{p}\xi)}{q\pm 2\sqrt{p\,r}\,\coth(\sqrt{p}\,\xi)}\right]^{\frac{1}{2}}$, $p>0,\ r>0$	10	$\pm\frac{1}{\sqrt{q}\,\xi}$, $p=0,\ r=0.$

where $\Delta = q^2 - 4pr$.

Step 3: In Eq. (4), n is a positive integer which is usually obtained by balancing the highest order nonlinear term(s) with the linear term(s) of the highest order come out in Eq. (3).

Step 4: Substituting Eq. (4), into Eq. (3) and utilizing Eq. (5), we obtain polynomials in $G^i(\xi)$ and $G'(\xi)\,G^i(\xi)$ $(i = 0, \pm 1, \pm 2, \pm 3, \cdots)$. Vanishing each coefficient of the resulted polynomials to zero, yields a set of algebraic equations for $\alpha_n,\ p,\ q,\ r,\ V$ and constant(s) of integration, if applicable. Suppose with the aid of symbolic computation software such as Maple, the unknown constants α_n, p, q, r and V can be found by solving these set of algebraic equations and substituting these values into Eq. (4), new and more general exact traveling wave solutions of the nonlinear partial differential equation (1) can be found.

3. Application

In this section, we apply the improved (G'/G)-expansion method to the (3 + 1)-dimensional potential-YTSF equation which is dreadfully important nonlinear evolution equations in mathematical physics and have been paid attention by a lot of researchers and the extended tanh-function method to compare the solutions obtained by the two methods.

3.1. On Solving the (3 + 1)-dimensional Potential-YTSF Equation by the Projected Method: We start with the (3 + 1)-dimensional potential-YTSF equation [38-40] in the form,

$$-4u_{xt} + u_{xxxz} + 4u_x u_{xz} + 2u_{xx}u_z + 3u_{yy} = 0. \tag{6}$$

Let us now solve the Eq. (6) by the proposed further improved (G'/G)-expansion method. Making use of the travelling wave variable (2) permits us in converting Eq. (6) into an ODE and upon integration yields:

$$(4V+3)u' + 3(u')^2 + u''' = 0, \tag{7}$$

with zero constant of integration. Considering the homogeneous balance between the highest order derivative u''' and the nonlinear term $(u')^2$ come out in Eq. (7), we deduce that $n = 1$. Therefore, the solution (4) turns out to be

$$u(\xi) = \alpha_1 (G'/G) + \alpha_0. \tag{8}$$

Substituting (8) together with Eq. (5) into (7), we obtain the following polynomial equation in G:

$$\begin{aligned} &G^2(3\alpha_1 q + 4\alpha_1 qV + 4\alpha_1 q\,p) + G^4(3\alpha_1^2 q^2 + 32\alpha_1 r\,p + 6\alpha_1 r + 8\,V\alpha_1 r + 6\alpha_1 q^2) \\ &+ G^6(12\alpha_1^2\, r\,q + 48\,\alpha_1\, r\,q) + G^8(12\alpha_1^2\, r^2 + 48\alpha_1\, r^2) = 0 \end{aligned} \tag{9}$$

Setting each coefficient of the polynomial Eq. (9) to zero, we achieve a system of algebraic equations which can be solved by using the symbolic computation software such as Maple and obtain the following two sets of solutions:

The set 1. $\alpha_1 = -4$, $\alpha_0 = \alpha_0$, $q = 2\sqrt{p\,r}$, $V = -p - \frac{3}{4}$. (10)

The set 2. $\alpha_1 = -4$, $\alpha_0 = \alpha_0$, $q = 0$, $V = -4p - \frac{3}{4}$, (11)

where α_0, p and r are arbitrary constants.
Now for the set 1, we have the following solution:

$$u(\xi) = -4\left(\frac{G'}{G}\right) + \alpha_0, \qquad (12)$$

where $\xi = x + y + z + (p + \frac{3}{4})t$.

According to the step 2 of section 2, we have the subsequent families of exact solutions:
Family 1. If $p > 0$, the solution of Eq. (5) has the form,

$$G(\xi) = \left[\frac{-p\,q\,\operatorname{csc}h^2(\sqrt{p}\,\xi)}{q^2 - p\,r\,(1 \pm \coth(\sqrt{p}\xi))^2}\right]^{\frac{1}{2}} \qquad (13)$$

In this case we have the ratio,

$$\frac{G'}{G} = \frac{\sqrt{p}\left\{q^2\sinh(\sqrt{p}\,\xi)\cosh(\sqrt{p}\,\xi) - 2pr\sinh(\sqrt{p}\,\xi)\cosh(\sqrt{p}\,\xi) \mp 2\,p\,r\cosh^2(\sqrt{p}\,\xi) \pm p\,r\right\}}{-q^2\cosh^2(\sqrt{p}\,\xi) + 2\,p\,r\cosh^2(\sqrt{p}\,\xi) \pm 2pr\sinh(\sqrt{p}\,\xi)\cosh(\sqrt{p}\,\xi) + q^2 - p\,r}$$

Since $q = 2\sqrt{p\,r}$, subsequently, we obtain the following traveling wave solutions,

$$u(\xi) = 4\sqrt{p} - \frac{8\sqrt{p}\,\sec h^2(2\sqrt{p}\,\xi)}{3\sec h^2(2\sqrt{p}\,\xi) + 2\,\tanh(2\sqrt{p}\,\xi) \mp 2} + \alpha_0, \qquad (14)$$

.

Family 2. If $p > 0, r > 0$, the solution of Eq. (5) has the form,

$$G(\xi) = \left[\frac{-p\sec h^2(\sqrt{p}\,\xi)}{q \pm 2\sqrt{p\,r}\,\tanh(\sqrt{p}\xi)}\right]^{\frac{1}{2}} \qquad (15)$$

Then we have the ratio,

$$\frac{G'}{G} = \frac{\sqrt{p}\left\{\pm q\sinh(\sqrt{p}\,\xi)\cosh(\sqrt{p}\,\xi) \mp 2\sqrt{p\,r}\cosh^2(\sqrt{p}\,\xi) \mp \sqrt{p\,r}\right\}}{\cosh(\sqrt{p}\,\xi)\left\{q\cosh(\sqrt{p}\,\xi) \pm 2\sqrt{p\,r}\sinh(\sqrt{p}\,\xi)\right\}}$$

Since $q = 2\sqrt{p\,r}$, subsequently, we obtain the following traveling wave solutions:

$$u(\xi) = \pm 2\sqrt{p} + 2\sqrt{p}\,\tanh(\sqrt{p}\,\xi) + \alpha_0 \qquad (16)$$

Family 3. If $p < 0$, $r > 0$, the solution of Eq. (5) has the form,

$$G(\xi) = \left[\frac{-p\sec^2(\sqrt{-p}\,\xi)}{q \pm 2\sqrt{-p\,r}\,\tan(\sqrt{-p}\xi)}\right]^{\frac{1}{2}} \qquad (17)$$

Then we have the ratio,

$$\frac{G'}{G}=\frac{\sqrt{-p}\left\{\sqrt{-p\,r}-2\sqrt{-p\,r}\cos^{2}(\sqrt{-p}\,\xi)\pm q\sin(\sqrt{-p}\,\xi)\cos(\sqrt{-p}\,\xi)\right\}}{\cos(\sqrt{-p}\,\xi)\left\{2\sqrt{-p\,r}\sin(\sqrt{-p}\,\xi\pm q\cos(\sqrt{-p}\,\xi)\right\}}$$

Since $q=2\sqrt{p\,r}$, subsequently, we obtain the following traveling wave solutions:

$$u(\xi)=\frac{-4\sqrt{-p}\left\{\tan(2\sqrt{-p}\,\xi)\mp 1\right\}}{1\pm\tan(2\sqrt{-p}\,\xi)+\sec(2\sqrt{-p}\,\xi)}+\alpha_0 \tag{18}$$

Family 4. If $p>0$, $\Delta=0$, the solution of Eq. (5) has the form,

$$G(\xi)=\left[-\frac{p}{q}\left\{1\pm\tanh(\frac{1}{2}\sqrt{p}\xi)\right\}\right]^{\frac{1}{2}} \tag{19}$$

Then we have the ratio,

$$\frac{G'}{G}=\frac{\sqrt{p}}{4}\left\{\pm 1-\tanh(\frac{1}{2}\sqrt{p}\,\xi)\right\}$$

Subsequently, we obtain the following traveling wave solutions:

$$u(\xi)=-\sqrt{p}\left\{\pm 1-\tanh(\frac{1}{2}\sqrt{p}\,\xi)\right\}+\alpha_0. \tag{20}$$

Family 5. If $p>0$, the solution of Eq. (5) has the form,

$$G(\xi)=\left\{\frac{p\,e^{\pm 2\sqrt{p}\,\xi}}{\left(e^{\pm 2\sqrt{p}\xi}-4q\right)^{2}-64\,p\,r}\right\}^{\frac{1}{2}} \tag{21}$$

Then we have the ratio,

$$\frac{G'}{G}=\frac{\sqrt{p}\left\{e^{\pm 4\sqrt{p}\,\xi}-16q^{2}+64\,p\,r\right\}}{\mp e^{\pm 4\sqrt{p}\,\xi}\pm 8q\,e^{\pm 2\sqrt{p}\,\xi}\mp 16q^{2}\pm 64\,p\,r}$$

Since $q=2\sqrt{p\,r}$, subsequently, we obtain the following traveling wave solutions:

$$u(\xi)=\frac{-4\sqrt{p}\,e^{\pm 2\sqrt{p}\,\xi}}{\pm 16\sqrt{p\,r}\mp e^{\pm 2\sqrt{p}\,\xi}}+\alpha_0. \tag{22}$$

where $\xi=x+y+z+(p+\frac{3}{4})t$.

For the set 2, we have the following solution:

$$u(\xi)=-4\left(\frac{G'}{G}\right)+\alpha_0,\ \text{where}\ \xi=x+y+z+(4p+\frac{3}{4})t. \tag{23}$$

According to the step 2 of section 2, we obtain the subsequent families of exact solutions:

Cohort 1. If $p>0$, $\Delta>0$, the solution of Eq. (5) has the form,

$$G(\xi)=\left[\frac{2p}{\pm\sqrt{\Delta}\cosh(2\sqrt{p}\xi)-q}\right]^{\frac{1}{2}} \tag{24}$$

Since $q=0$, then $r<0$. In this case we have the ratio,

$$\frac{G'}{G}=-\sqrt{p}\tanh(2\sqrt{p}\,\xi)$$

Therefore, we obtain the following travelling wave solution,

$$u(\xi)=4\sqrt{p}\tanh(2\sqrt{p}\,\xi)+\alpha_0. \tag{25}$$

Cohort 2. If $p>0$, $\Delta<0$, the solution of Eq. (5) has the form,

$$G(\xi)=\left[\frac{2p}{\pm\sqrt{-\Delta}\sinh(2\sqrt{p}\xi)-q}\right]^{\frac{1}{2}} \tag{26}$$

Since $q=0$, then $r>0$. In this case we have the ratio,

$$\frac{G'}{G}=-\sqrt{p}\coth(2\sqrt{p}\,\xi)$$

Therefore, we obtain the following travelling wave solutions:

$$u(\xi)=4\sqrt{p}\coth(2\sqrt{p}\,\xi)+\alpha_0. \tag{27}$$

Cohort 3. If $p<0$, $\Delta>0$, the solutions of Eq. (5) has the form,

$$G(\xi)=\left[\frac{2p}{\pm\sqrt{\Delta}\cos(2\sqrt{-p}\xi)-q}\right]^{\frac{1}{2}} \tag{28}$$

Since $q=0$, then $r>0$. Thus we have the ratio,

$$\frac{G'}{G}=\sqrt{-p}\tan(2\sqrt{-p}\,\xi)$$

Therefore, we obtain the following travelling wave solutions:

$$u(\xi)=-4\sqrt{-p}\tan(2\sqrt{-p}\,\xi)+\alpha_0. \tag{29}$$

Cohort 4. If $p>0$, $r>0$, the solutions of Eq. (5) has the form,

$$G(\xi)=\left[\frac{-p\sec h^2(\sqrt{p}\,\xi)}{q\pm2\sqrt{pr}\tanh(\sqrt{p}\xi)}\right]^{\frac{1}{2}} \tag{30}$$

Since $q=0$, we have the ratio,

$$\frac{G'}{G}=-\frac{1}{2}\sqrt{p}\left[\tanh(\sqrt{p}\,\xi)+\coth(\sqrt{p}\,\xi)\right]$$

Therefore, we obtain the following travelling wave solution

$$u(\xi)=2\sqrt{p}\left\{\tanh(\sqrt{p}\,\xi)+\coth(\sqrt{p}\,\xi)\right\}+\alpha_0. \tag{31}$$

Cohort 5. If $p<0$, $r>0$, the solutions of Eq. (5) has the form,

$$G(\xi)=\left[\frac{-p\sec^2(\sqrt{-p}\,\xi)}{q\pm 2\sqrt{-p\,r}\,\tan(\sqrt{-p}\xi)}\right]^{\frac{1}{2}} \tag{32}$$

Since $q=0$, then we have the ratio,

$$\frac{G'}{G}=-\frac{1}{2}\sqrt{-p}\left[\cot(\sqrt{-p}\,\xi)-\tan(\sqrt{-p}\,\xi)\right]$$

Therefore, we obtain the following travelling wave solution

$$u(\xi)=2\sqrt{-p}\left\{\cot(\sqrt{-p}\,\xi)-\tan(\sqrt{-p}\,\xi)\right\}+\alpha_0. \tag{33}$$

Cohort 6. If $p>0$, $q=0$, the solution of Eq. (5) has the form,

$$G(\xi)=\left[\frac{\pm p\,e^{\pm 2\sqrt{p}\,\xi}}{1-64\,p\,r\,e^{\pm 4\sqrt{p}\,\xi}}\right]^{\frac{1}{2}} \tag{34}$$

Then we have the ratio,

$$\frac{G'}{G}=\pm\frac{1}{8\sqrt{r}}\coth\left(\frac{\pm\xi}{4\sqrt{r}}\right)$$

where $64\,p\,r=1$.

Therefore, we have the solution:

$$u(\xi)=\mp\frac{1}{2\sqrt{r}}\coth\left(\frac{\pm\xi}{4\sqrt{r}}\right)+\alpha_0. \tag{35}$$

Cohort 7. If $p=0$, $r=0$, then the solution of Eq. (5) has the form,

$$G(\xi)=\pm\frac{1}{\sqrt{q}\,\xi} \tag{36}$$

Then we have the ratio,

$$\frac{G'}{G}=-\frac{1}{\xi}$$

Therefore, we have the solution:

$$u(\xi)=\frac{4}{\xi}+\alpha_0,\ \text{where}\ \xi=x+y+z+(4p+\frac{3}{4})t\,. \tag{37}$$

These are the exact solutions of the potential-YTSF equation obtained by the improved (G'/G)-expansion method.

3.2. On Solving the (3 + 1)-dimensional Potential-YTSF Equation by the Extended Tanh-function Method.

With reference to the well-known extended tanh-function method [1, 7, 8, 32, 34, 39, 46], the solution of the potential-YTSF Eq. (6) can be represented as,

$$u(\xi)=\alpha_1\,\varphi(\xi)+\alpha_0\,, \tag{38}$$

where $\varphi(\xi)$ satisfy the Riccati equation

$$\varphi'(\xi) = A + \varphi^2(\xi)\,. \qquad (39)$$

The Riccati Eq. (39) has the following solutions:

(i) If $A < 0$, then

$$\varphi(\xi) = -\sqrt{-A}\tanh(\sqrt{-A}\,\xi) \text{ or } \varphi(\xi) = -\sqrt{-A}\coth(\sqrt{-A}\,\xi)\,. \qquad (40)$$

(ii) If $A > 0$, then

$$\varphi(\xi) = \sqrt{A}\tan(\sqrt{A}\,\xi) \text{ or } \varphi(\xi) = -\sqrt{A}\cot(\sqrt{A}\,\xi)\,. \qquad (41)$$

(iii) If $A = 0$, then $\varphi(\xi) = -\dfrac{1}{\xi}$. (42)

Substituting (38) and (39) into (7), we obtain the following polynomial equation in φ as follows:

$$\begin{aligned}&(3\alpha_1^2 + 6\alpha_1)\varphi^4(\xi) + (4\alpha_1 V + 8\alpha_1 A + 6\alpha_1^2 A + 3\alpha_1)\varphi^2(\xi) + \\ &(4\alpha_1 AV + 3\alpha_1 A + 2\alpha_1 A^2 + 3\alpha_1^2 A^2) = 0.\end{aligned} \qquad (43)$$

Equating the coefficients of the polynomial to zero and solving the set of algebraic equations with the help of Maple, we obtain the following solution:

$$\alpha_1 = -2\,,\ \alpha_0 = \alpha_0\,,\ V = -\frac{3}{4} + A\,. \qquad (44)$$

where α_0 and A are arbitrary constants.

Accordingly the exact solutions of Eq. (6) are:

When $A < 0$, the solution takes the form,

$$u(\xi) = 2\sqrt{-A}\tanh(\sqrt{-A}\,\xi) + \alpha_0\,. \qquad (45)$$

When $A > 0$, the solution takes the form,

$$u(\xi) = -2\sqrt{A}\tan(\sqrt{A}\,\xi) + \alpha_0\,. \qquad (46)$$

When $A = 0$, the solution takes the form,

$$u(\xi) = \frac{2}{\xi} + \alpha_0, \text{ where } \xi = x + y + z - (-\frac{3}{4} + A)\ t\,. \qquad (47)$$

From the above results obtained by the two methods, we can draw the following remarks:

Remark 1. If we put $A = -4p$ where $p > 0$, the results arranged in Eq. (45) are identical to the result (25).

Remark 2. If we put $A = -4p$ where $p < 0$ then the results arranged in Eq. (46) are identical to the result given in (29).

Remark 3. Result given in (47) is alike to the result given in (37).

5. Conclusions

An improved (G'/G)-expansion method is suggested and applied to the (3 + 1)-dimensional potential-YTSF equation. The results obtained by the suggested method have been compared with those obtained by the celebrated extended tanh-function method. From this study, we observe that the improved (G'/G)-expansion method and the extended tanh-function method are not equivalent, although El-Wakil [30] and Parkes [31] have shown that the basic (G'/G)-expansion method and the extended tanh-function method are equivalent. We see that all the results obtained by the extended tanh-function are found by the suggested method and in addition some novel solutions are attained. The analysis shows that the proposed method is quite resourceful and practically well suited to be used in finding exact solutions of NLEEs. We expect that the suggested method might be applicable to other kinds of NLEEs in mathematical physics.

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