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Some Special Classes of Semirings and Ordered Semirings

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Abstract. In this paper, we study the properties of Semirings satisfyingthe identity **a + ab = a**. It is proved that, **:** Let (S , +, .) be a semiring which contains multiplicative identity 1, then a + ab = a for all a, b in S if and only if (S, +) is absorbing.

Keywords: Left zero,maximal element, rectangular band, weakly separative

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1. Introduction

The theory of rings and the theory of semigroups have considerable impact on the developments of theory of semirings. Recently the semiring theory has developments in ordered semirings which are akin to ordered rings and ordered semigroups. During the last three decades, there is considerable impact of semigroup theory and semiring theory. The theory of semirings is attracting the attention of several algebraists due to its applications to Computer Science, The developments of semirings and ordered semirings require semigroup techniques.S.Gosh studied on the class of idempotent Semirings. He proved that an idempotent commutative semiring S is distributive lattice. If and only if it satisfies the absorption equality **a + ab = a** for all a, b in S. In this paper, we will have two sections. Section one deals with multiplicative and additive identity of semirings and section two ordered semirings with (S, +) is p.t.o.

2. Preliminaries

A triple (S, +, .) is called a semiring if (S, +) is a semigroup; (S, .) is semigroup; a (b + c) = ab + ac and (b + c) a = ba + ca for every a, b, c in S. (S, +) is said to be band if a + a = a for all a in S. A (S, +) semigroup is said to be rectangular band if a + b + a = a for all a, b in S. A semigroup (S, .) is said to be a band if $a = a^2$ for all a in S. A semigroup(S, .) is said to be rectangular band if aba = a. A semiring (S, +, .) is said to be Mono semiring if a + b = ab for all a, b in S.

Definition 2.1. A semiring (S, +) is said to be Absorbing if every element in S satisfies the condition $1 + a = a + 1 = 1$ for all a in S.

Theorem 2.2. Let (S , +, .) be a semiring which contains multiplicative identity 1, then $a + ab = a$ for all a, b in S if and only if (S, +) is absorbing.

Proof: Assume that S is absorbing, then we have $1 + b = 1$
If $a, b \in S$, then $a = a.1 = a(1 + b) = a + ab$
Therefore, $a + ab = a$ for all a, b in S
Conversely, assume $a + ab = a$ for all a, b in S (1)
Consider $1 + b = 1 + 1.b = 1$ ($\because$ from (1))
i.e (S, +) is absorbing

Example 2.3. S = { 1, a } the following is the example for the above Theorem 2.2.

+	1	a
1	1	1
a	1	a

.	1	a
1	1	a
a	a	a

Definition 2.4. A semigroup (S, .) is said to be left (right) singular if $ab = a$ ($ab = b$) for all a, b in S.

Definition 2.5. An element ‘x’ is said to be left (right) additive zero, if $x + a = x$ ($a + x = x$) for every ‘a’ in S.

Theorem 2.6. If (S, +, .) is a Semiring satisfying the identity $a + ab = a$ for all a, b in S, (S, +) is left cancellative and (S, +) is left zero, then

(i) (S, .)is left singular and
(ii) (S, .) is commutative.

Proof: (i) If (S, +) is left zero, then we have $b + a = b$ (1)
Consider $a + ab = a$ for all a, b in S
$\Rightarrow b + a + ab = b + a \Rightarrow b + ab = b$ (2)
from (1) and (2) we have
$b + a = b + ab \Rightarrow a = ab$ ($\because$(S, +) is left cancellative)
Similarly, we can prove that $ba = b$
Therefore, (S, .) is left singular

(ii)And also we have $b + ba = b$ for all b, a in S (3)
From (2) and (3) we have
$b + ab = b + ba \Rightarrow ab = ba$ ($\because$ (S, +) is left cancellative)
Hence (S, .) is commutative

Definition 2.7. A semigroup (S, .) is called quasi separative if $x^2 = xy = yx = y^2 \Rightarrow x = y$, for all x, y in S.

Definition 2.8. A semigroup (S, .) is said to be weakly separative if $x + x = x + y = y + y \Rightarrow x = y$, for all x, y in S.

Theorem 2.9. If (S, +, ·) is a semiring satisfying the identity $a + ab = a$, for all a, b in S and S contains a multiplicative identity 1 which is also an additive identity, then
(i) (S, .) is quasi separative
(ii) (S, +) is weakly separative.

Proof: (i) Let `e' be the multiplicative identity which is also an additive identity
i.e. $a.e = e.a = a$ & $a + e = e + a = a$.
Assume that S satisfies the condition $a + ab = a$, for all a, b in S (1)
To prove that, (S, .) is quasi separative
i.e., $a^2 = ab = ba = b^2 \Rightarrow a = b$, for all a, b in S
Let $a^2 = ab = a (e + b) = a + ab = a$
$\Rightarrow a^2 = a$ (2)
Similarly $b^2 = ba = b (e + a) = b + ba = b$ ($\because b + ba = b$)
$\Rightarrow b^2 = b$ (3)
Therefore, $a^2 = a$ and $b^2 = b$
If $a^2 = ab = ba = b^2$
$\Rightarrow a = ab = ba = b \Rightarrow a = b$
Hence, (S, .) is quasi separative.

(ii) To prove that (S, +) is weakly separative.
i.e., $a + a = a + b = b + b \Rightarrow a = b$, for all a, b in S
Consider $a + a = a + b$
$\Rightarrow (a + a) b = (a + b) b \Rightarrow ab + ab = ab + b^2$
$\Rightarrow a + ab = ab + b$ ($\because ab = a$ and $b^2 = b$)
$\Rightarrow a = (a + e) b \Rightarrow a = ab \Rightarrow a = b$ ($\because ab = b$)
Similarly, $b + b = a + b$

$\Rightarrow (b + b) a = (a + b) a \Rightarrow ba + ba = a^2 + ba$
$\Rightarrow b + ba = a + ab$ ($\because ba = b$ and $a^2 = a$)
$\Rightarrow b = a (e + b)$ ($\because b + ba = b$)
$\Rightarrow b = ab \Rightarrow b = a$ ($\because a b = a$)
Therefore, $a + a = a + b = b + b \Rightarrow a = b$
Hence, (S, +) is weakly separative.

Definition 2.10. A semiring (S, +, .) with additive identity zero is said to be zerosumfree semiring if $x + x = 0$ for all x in S.

Theorem 2.11. If (S, +, .) is a Semiring satisfying the identity $a + ab = a$ for all a, b in S and S contains a multiplicative identity which is also an additive identity.If (S, +, .) is a zerosumfree semiring,. then (S, .) is right singular.

Proof: Consider $a + ab = a$ for all a, b in S (1)
$\Rightarrow a + ab + b = a + b \Rightarrow a + (a + e) b = a + b \Rightarrow a + ab = a + b$ ($\because a + e = a$)

$\Rightarrow$ a + a + ab = a + a + b $\Rightarrow$ 0 + ab = 0 + b

ab = b ($\because$ zerosumfree semiring) (2)

Again from (1)

b + ba = b $\Rightarrow$b + ba + a = b + a

$\Rightarrow$ b + (b + e) a = b + a $\Rightarrow$ b + ba = b + a

$\Rightarrow$ b + b + ba = b + b + a $\Rightarrow$ 0 + ba = 0 + a

$\Rightarrow$ ba = a (3)

From (2) and (3)

(S, .) is right singular

3. Ordered Semirings

Definition 3.1. A totally ordered semigroup (S, ·) is said to be non-negatively (non-positively) ordered if every one of its elements is non-negative (non-positive). (S, ·) is positively (negatively) ordered in strict sense if xy $\geq$ x and xy $\geq$ y (xy $\leq$ x and xy $\leq$ y) for every x and y in S.

Theorem 3.2. Let (S, +, .) be a t.o semiring satisfying the identity a + ab = a for all a, b in S and PRD. If (S, +) is p.t.o (n.t.o), then 1 is the maximum(minimum) element.

Proof : Consider a + ab = a for all a, b in S (1)

Taking b = a in (1) we get

a + a^2 = a $\Rightarrow$ a(1 + a) = a.1 $\Rightarrow$ 1 + a = 1

$\Rightarrow$ 1 = 1 + a $\geq$ a

$\Rightarrow$ 1 $\geq$ a $\forall$ a $\in$ S

Therefore, 1 is the maximum element

Similarly, we can prove that 1 is the minimum element if (S, +) is n.t.o

Theorem 3.3. Let (S, +, .) be a t.o.zerosumfreesemiring satisfying the identity a + ab = a for all a, b in S.If (S, +) is p.t.o, then (S, .) is n.t.o.

Proof: Since (S, +) is p.t.o, 0 is the minimum element

$\Rightarrow$ ab = 0 $\leq$ a, b

$\Rightarrow$ ab$\leq$ a, b

Hence, (S, .) is n.t.o

Theorem 3.4. If (S, +, .) is a t.o mono semiring satisfying the identity a + ab = a for all a, b in S and (S, +) is p.t.o. Then a = a + b.

Proof: Consider a + ab = a for all a, b in S (1)

$\Rightarrow$ a = a + ab $\geq$ ab ($\because$ (S, +) is p.t.o)

$\Rightarrow$a $\geq$ a + b (by (1)) ($\because$ S is monosemiring)

(S, +) is p.t.o $\Rightarrow$ a + b $\geq$ a (2)

From (1) and (2) we get a + b = a.

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