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An Exact Solution of Fluctuating Hydromagnetic Flow of a Dusty Fluid Between Parallel Plates

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Abstract. An initial value investigation of time-dependent flow of an incompressible conducting viscoelastic (Rivlin-Ericksen) fluid with small particles, in the presence of transverse magnetic field, through a channel with one wall is at rest and other oscillating with a with mean velocity, has been made. We have employed the separation of variable method and arrived at significant results. A comparison with the previous studies is incorporated along with the investigation of hydrodynamic and hydromagnetic flows. Effects of dust particles and other parameters appeared in the mathematical model of fluid flow are look into and presented with graphs.

Keywards: Dusty Fluid, Hydromagnet, Rivlin-Ericksen model

AMS Mathematics Subject Classification (2010): 76A25

1. Introduction

Multiphase fluid system has numerous application in various natural processes; blood flow in arteries, dust in gas cooling systems, movement of inert solid particles in atmosphere, sand or other suspended particle in sea beaches are the most common examples of multiphase fluid systems. With the use of Legendra transfor- mation, Siddabasappa et. al. [1] solved a coupled partial differential equations arises in the flow of dusty viscous fluid. it is observed that as the number density of the dust particle increases the velocity of the dust phase de- creases. Flow of an Unsteady Conducting Dusty Fluid between a Non-torsional Oscillating Plate has been by Mahesha et. al. [2] in an anholonomic coordinate system. They found the analytical solution for the velocity distribution of fluid and dust for different pressure gradients; also concluded that the effect of strength of magnetic field on the velocity at fixed time is significant and substantially retarding influence on the fluid and dust both. A system, fluids with spherical dust particle embedded by two infinite parallel plates, rotating with a constant angular velocity in the presence of transverse magnetic field under the influence of periodic impulsive pressure gradient was the purview of research of Ghosh and Ghosh [3]. Along with many major findings they found that for all values of rotation, the magnitude of drag is increased by both the magnetic field and the particles while the magnitude of the lateral stress is decreased by the magnetic field and increased by the particles. In the subsequent study of flow of Oldroyed-B fluid through a channel in presence of magnetic field under the influence of impulsive sine pressure Ghosh and Sana [3] observed that the skin-friction on the lower plate is maximum for all values of the magnetic field *M* at the sine impulse peaks; however, the skin-friction on the upper plate at small values of *M* are negative. Moreover, the increasing values of the magnetic strength diminish the negative skin-friction on the upper plate. The above mentioned papers have the research objective of astrophysics.

Keeping view of the industrial application, we adopted the present problem for investigation. Vajravelu and Sastry [1] have investigated the effects of free convection within the boundary layer, in which they considered the free stream oscillation over the flat plate. In the subsequent study, Vajravelu [2] introduced a method of periodic solution for the flow of Newtonian fluid, in the presence of transverse magnetic field, bounded by two parallel plates. Mukhopadhya and Chaudhary [3] provides a brief note on the fluctuating flow of Oldroyd type viscoelastic fluid past an infinite flat plate. Ray et al [4] investigated the viscoelastic Oldroyd fluid flow problem by using the separation of variable method. Assuming the periodic pressure gradient, Bhatnagar [5] presented the solution for the flow of viscoelastic fluid (Rivlin-Ericksen constitutive laws) with the use of perturbation method. In the present paper we are solving the time dependent viscoelastic fluid (Rivlin-Ericksen model) flow governing equation along with the constitutive relation by introducing the Vajravelu [3] 's method of solution.

2. Basic Equations

The constitutive equation of an incompressible viscoelastic fluid based on Rivlin-Ericksen model[6] is

$$S = -pI + \phi_1 A_1 + \phi_2 A_2 + \phi_3 A_1^2 + \phi_4 A_2^2 + \phi_5 (A_1 A_2 + A_2 A_1) + \phi_6 (A_1^2 A_2 + A_2 A_1^2) + \phi_7 (A_2^2 A_1 + A_1 A_2^2) + \phi_8 (A_1^2 A_2^2 + A_2^2 A_1^2) \tag{1}$$

where p is the hydrostatic pressure and ϕ_s are polynomial functions of the traces of the various tensors occurring in the representations, matrices A_1 and A_2 are defined by

$$A_{ij}^{(1)} = v_{i,j} + v_{j,i} \tag{2}$$

$$A_{ij}^{(2)} = \frac{\partial A_{ij}^{(1)}}{\partial t} + v_p A_{ij,p}^{(1)} + A_{ip}^{(1)} v_{p,j} + A_{pj}^{(1)} v_{p,i} \tag{3}$$

v_p being velocity vector. Neglecting squares and products of A_2, we have

$$S = -pI + \phi_1 A_1 + \phi_2 A_2 + \phi_3 A_1^2 \tag{4}$$

Here, ϕ_1, ϕ_2 and ϕ_3 are co-efficient of viscosity, viscoelasticity and cross viscosity respectively.

In the foregoing analysis we shall discuss the oscillatory motion of a viscoelastic dusty fluid between two plates. The either one is at rest at $y = 0.0$ and the other is in simple harmonic motion defined by a well-known equation.
The unsteady flow in $0 \le y \le d$ is governed by the coupled equations

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + (\alpha + \beta\frac{\partial}{\partial t})\frac{\partial^2 u}{\partial y^2} + \frac{k_0 N_0}{\rho}(v - u) - \frac{\sigma B_0^2}{\rho}u \tag{5}$$

and $$m\frac{\partial v}{\partial t} = k_0(u - v) \tag{6}$$

The equations of continuity and momentum equations are

$$v_{i,i} = 0 \tag{7}$$

$$\rho(\frac{\partial v_i}{\partial t} + v_i v_{i,j}) = -p_{,i} + \tau_{ij,j} \tag{8}$$

3. Formulation and Solutions

We are considering here that the upper plates are oscillatory in motion with a constant mean velocity U_d. It is assumed that the x-direction is parallel to the plate and on the lower plate and y normal to the plate. The magnetic field is applied along the transverse direction of the flow and perpendicular to the plates. Here induced magnetic field is neglected because the magnetic Reynolds number is small. As the Lorentz force acts along the x --axis, therefore the constitutive and the governing equation may be written as

$$\begin{aligned}&\frac{k_0}{m}\frac{\partial u}{\partial t} + T\frac{\partial^2 u}{\partial t^2} - \frac{k_0}{m}\frac{\partial^2 u}{\partial y^2} - T(\alpha + \beta\frac{\partial}{\partial t})\frac{\partial^3 u}{\partial t \partial y^2} - \beta\frac{k_0}{m}\frac{\partial^4 u}{\partial t^2 \partial y^2} + T\frac{k_0 N_0}{\rho}\frac{\partial u}{\partial t}\\ &+ \frac{\sigma B_0^2}{\rho}\frac{k_0}{m}u + \frac{\sigma B_0^2}{\rho}T\frac{\partial u}{\partial t} = \frac{k_0}{m}\frac{\partial U}{\partial t} + \frac{\partial^2 U}{\partial t^2} - \frac{k_0 N}{\rho}\frac{\partial U}{\partial t} - \frac{\sigma B_0^2}{\rho}U - \frac{\sigma B_0^2}{\rho}T\frac{\partial U}{\partial t}\end{aligned} \tag{9}$$

The unsteady flow with boundary and initial conditions

$$u = 0 \quad \text{at} \quad y = 0 \tag{10}$$

$$u = U(t)\ , \quad \text{at } y = d \tag{11}$$

We choose the unsteady upper plate velocity is oscillatory and its mathematical form is as follows

$$U(t) = U_0\{1 + \frac{\varepsilon}{2}(e^{i\omega^* t} + e^{-i\omega^* t})\} \tag{12}$$

Our aim is to find a periodic solution of the following form

$$u(y,t) = F_0(y) + \frac{\varepsilon}{2}(F_1(y)e^{i\omega^* t} + \overline{F}_1(y)e^{-i\omega^* t}) \tag{13}$$

We now introduce the following non-dimensional quantities

$\bar{u} = \frac{u}{U_0}$, $\bar{U} = \frac{U}{U_0}$ $\xi = \frac{y}{d}$, $\bar{t} = \frac{\mu t}{\rho d^2}$, $M^2 = \eta^2 H_0^2 \frac{\sigma}{\rho}$, $\bar{\tau} = \frac{\tau d^2}{\mu}$ into the eqn.(8) and obtain as (after omitting bar)

$$\begin{aligned}&\{(L+T-KL\omega)-iTKL\omega\}F_1'' - \{LM^2 - i\omega(L+T+TT_1+TM^2)\}F_1 \\ &= -\{(LM^2 - T\omega^2) + i\omega(L+TT_1+TM^2)\}\end{aligned} \tag{14}$$

Here, $L = \frac{k_0}{m}$, $K = \frac{\beta}{\alpha}$, $T_1 = \frac{k_0 N}{\rho}$; $\omega = \frac{\omega^* d^2 \rho}{\alpha}$

T =0, for non- dusty fluid;
=1, for dusty fluids;

The solution of the differential eqn. may be written as follows

$$u(\xi, t) = A\cosh r_1\xi + B\sinh r_1\xi + P_1 \tag{15}$$

where $A = \frac{1-P_1(1-\sinh r_1)}{\cosh r_1 - \sinh r_1}$, $B = \frac{1-P_1(1-\cosh r_1)}{\sinh r_1 - \cosh r_1}$,

$$P_1 = \frac{M^2 L - T\omega^2 + i\omega(L+TT_1+TM^2)}{|r_1 r_2|}$$

$$r_1, r_2 = \pm\sqrt{\frac{(LM^2 - T\omega^2) + i\omega(L+TM^2+TT_1)}{L+T+KL\omega^2 + iTKL\omega}}$$

The shear stress of the viscoelastic fluid can be represented as

$$\tau = (\alpha + \beta\frac{\partial}{\partial t})\frac{\partial u}{\partial y} \tag{16}$$

In the case of a periodic solution that shear stress may takes the following form $\tau = (1+iK\omega)F_1'$

Here, to assume Newtonian fluid we introduce $L = 1.0$, $T = 0.0$, $T_1 = 0.0$, $K = 0.0$ and in case of Rivlin-Erickson fluid $K \neq 0.0$ and others are same as former.

4. Discussion

The problem considered for investigation can be reduced to Rivlin-Erickson and Newtonian fluid if we introduce the value of T to 1 and 0 respectively. In the subsequent analysis we shall compare the present results with previous studies and also with different fluids appeared in this investigation. Fig. 1 is the representation of the magnitude of unsteady velocity gradient with ξ for various couple of parameter values. Graphs corresponding to A and B are meant for Newtonian fluid obtained by Vejravalue [2] for M=0.0 and 0.2 respectively at the instant t=0.0. Rivlin-Erickson (K=0.8) fluid shows that for higher values of ω, velocity gradient is constant at every point on the channel thickness (graphs C and D). On contrary the effect of magnetic field on the dusty fluid adopted in the present investigation is not significant.

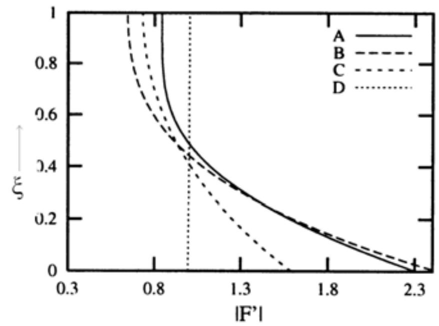

Fig. 1: Profile of the magnitude of fluctuating velocity gradient

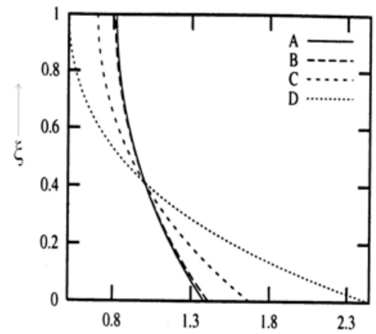

Fig. 2: Profile of the magnitude of fluctuating velocity gradient

The parameter values taken in the graphs A ($M = 0.0$ and B ($M = 2.0$) are $L = 1.0$, $T = 1.0$, $T_1 = 0.6$, $K = 0.8$, $\omega = 6.0$, $t = 0.0$ and for C ($K = 0.8$) and D ($K = 0.1$) are $L = 0.5$, $T = 1.0$, $T_1 = 0.6$, $M = 2.0, t = 0.0, \omega = 6.0$. A comparison between B and C as well as C and D put forward an important idea that the Stokes resistance k_0 and viscoelastic parameter K both increase the magnitude of the velocity gradient with their decreasing values. And this character is expected because loss of K. E. decrease with the decreasing values of k_0 and K. The distribution of skin friction amplitude with frequency parameter ω for various M has been represented in fig.2. Figure shows that the Newtonian fluid has nearly the same frictional forces on the plate $y = 0.0$ when $\omega > 15$ at $t = 30.0$ graphs A ($M = 0.0$) and B ($M = 10.0$) together with a rectilinear configuration. In contrast, graphs C ($M = 0.0$) and D ($M = 10.0$) are nonlinear and have a fluctuation in $0 < \omega < 3$; but these two take the same configuration while ω increase steadily as Newtonian case. It is to be mentioned that the viscoelastic parameter introduces some resistance and that creates more frictional force on $y = 0.0$ and that is because of orientation and elastic response of fluid particle structure. The Hartmann numbers create a resistance to the flow and varies linearly in magnitude with

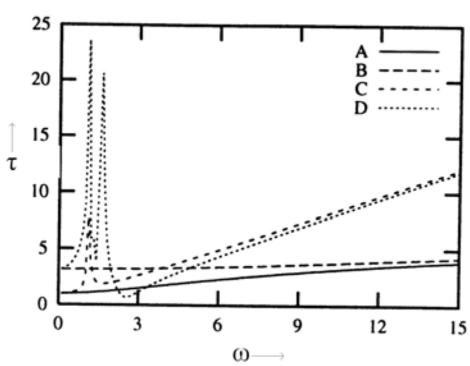

Fig. 3: Shear stress distribution on the lower plate

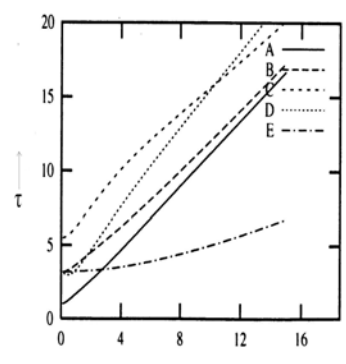

Fig. 4: Shear stress distribution of the dusty fluid on the lower plate

In a comparison between B and D $L = 0.5$, $T = 1.0$, $T_1 = 1.0$, $K = 0.8$, $t = 0.0$, $M = 10.0$ we may conclude that for the values of $M > 5$ shear stress increase rapidly with M while L decrease in a considerable magnitude.

The profiles of the phase advance $|\phi_\tau|$ with ω for both Newtonian and Rivlin-Erickson fluid have been presented in fig.5. In the presence of magnetic field, the phase angles gradually increase to 40^0 and then asymptotically proceed towards 42^0 with the increasing ω. In contrast, for the Rivlin-Erickson fluid, phase angles fluctuate within $0 < \omega < 2.5$ and then took the similar shape as for Newtonian fluid; but magnitude increases considerably. It is interesting to note that an external force which can perturb the upper plate reduce the phase angle as well as wall shear stress for the Rivlin-Erickson fluids.

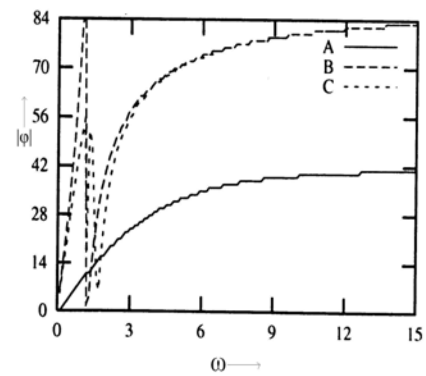

Fig. 5: Phase angles

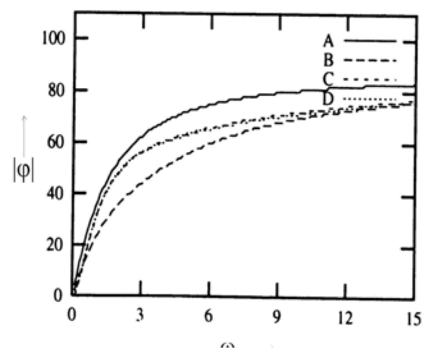

Fig. 6: Phase angles dusty fluid

On the other hand, Fig. 6 ($T = 1.0, K = 0.8, t = 10.0$) that shows the phase angle for dusty fluids put forward the idea of asymptotic convergence of $|\phi_\tau|$ to 85^0 which has resemblance with the Newtonian fluid in configurations. The particular values assumed in the formation of graphs are A ($M = 2.0$), B ($M = 10.0$) when $L = 1.0, T_1 = 0.5$) and C ($L = 0.5, T_1 = 0.5$), D ($L = 0.5, T_1 = 1.0$) when $M = 10.0$. Therefore, the study of generalized dusty fluid imply that the dust particle creates a linear change of shear stress as well as phase angle to Newtonian fluids; but for small ω it is more prominent in non dusty viscoelastic fluids.

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