

Thermal Buoyancy Force Effects on Developed Flow Considering Hall and Ion-slip Current

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Abstract. One dimensional unsteady Magnetohydrodynamics fluid flow through an infinite vertical plate has been studied with heat transfer. A strong magnetic field has been taken to introduce Hall current and Ion-slip effects. Also constant suction velocities are applied perpendicular to the plate. The usual non-dimensional transformations have been used to obtain the non-similar coupled non-dimensional momentum, angular momentum, energy and concentration equations. The coupled non-dimensional equations have been solved numerically by implicit finite difference technique. The primary and secondary velocity profiles as well as temperature are discussed for the different values of dimensionless parameter verses dimensionless co-ordinate. Finally, the effects of various parameters are separately discussed and shown graphically. Also the comparisons of the present results with the published results have been shown in tabular form.

Keywords: MHD, Heat transfer, Implicit Finite difference

AMS Mathematics Subject Classification (2010): 35Q35, 37N10

1. Introduction

The principal under consideration of the study of Magnetohydrodynamics (MHD) boundary layer flow has become the basic of several industrial, scientific and engineering applications. Engineers employ MHD concept in the designs of heat exchangers, pumps and flow matters, controlling the rate of cooling etc. As a branch of plasma physics and continuum mechanics, the field of Magnetohydrodynamics (MHD) consists of the study of a continuous, electrically conducting fluid under the influence of electromagnetic fields. A transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and insulated plates has been studied by Hartmann and Lazarus (1937). Then several works have been calculated under the basic concept of MHD fluid particles in the present world. But the research work on ionized fluid with Hall effects has extended the interest of any investigations in view of its important applications in many engineering phenomena. The effects of Hall and Ion-slip currents on free convective heat generating flow in a rotating fluid have studied by Ram (1995).

Hall coefficient is defined as the ratio of the induced electric field to the product of the current density and the applied magnetic field. Under these conditions, the Hall current and Ion-slip are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. Attia (2005) has studied the unsteady Couette flow with heat transfer considering Ion-slip. Diffusion of heat can take place in solids, but is referred to separately in that case as heat conduction. Somers (1956) considered combined buoyancy mechanism for flow adjacent to a wet isothermal vertical surface in an unsaturated environment. Raptis and Kafoussias (1982) presented the analysis of free convection and mass transfer steady hydromagnetic flow, of an electrically conducting viscous incompressible fluid, through a porous medium, occupying a semi-infinite region of the space bounded by an infinite vertical and porous plate under the action of transverse magnetic field. Nithyadevi and Yang (2009) investigated numerically the effect of double-diffusive natural convection of water in a partially heated enclosure with Soret and Dufour coefficients around the density maximum. Abo-Eldahab and Ghonaim (2003) investigated convective heat transfer in an electrically conducting micropolar fluid at a stretching surface with uniform free stream.

Therefore, the present study aimed at the computation of unsteady one-dimensional Magnetohydrodynamics boundary layer flow through a vertical plate with the influence of Hall and Ion-slip currents. The one-dimensional basic governing equations of this problem contained partial differential equations; that has been non-dimensionalized under usual transformation technique and therefore solved by implicit finite difference technique. Stability and convergence test has been performed to get more stable solutions. The test calculations are not shown for brevity. The effects of shear stress in x and z direction, Nusselt number for different values of dimensionless parameters have been described.

2. Mathematical model of flow

In this paper, the unsteady one dimensional viscous ionized Magnetohydrodynamics fluid flow along finite vertical plate has been depicted. Also considered an external magnetic field applied along y -axis is normal to the plate. The infinite vertical plate is located at $y = 0$ and extends from $x = -\infty$ to $+\infty$ have been depicted in Fig. 1.

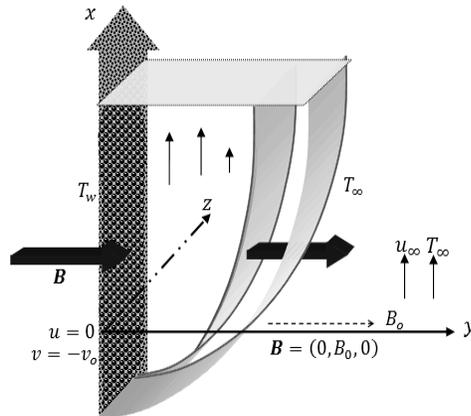


Figure 1: Fluid flow configuration and Co-ordinate system.

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A uniform magnetic field of strength B_0 is applied along the positive y -direction normal to the plate and that induced another magnetic field on the electrically conducting fluid. And therefore, electrically conducting fluid is affected by Hall and Ion-slip current. The equation of conservation of charge $\nabla \cdot J = 0$ gives $J_y = \text{constant}$. From the geometry of the problem, the plate is electrically non-conducting, this constant is zero and hence $J_y = 0$ at the plate and consequently zero everywhere. If the consideration of Hall current, ion-slip current as well as collision between electrons and neutral particles is taken into account, the generalized Ohm's law is subjected to another form (in the absence of current acceleration and diffusivity);

$$\vec{E} + (\vec{q} \wedge \vec{B}) - \frac{\vec{J} \wedge \vec{B}}{en} = \frac{\vec{J}}{\sigma_e}$$

where 3rd term on the left hand side denoted that the Hall effect (owing to Lorentz Force), σ_e , n , e are the electric conductivity, the number of density of electron, the electric charge respectively. Again rewriting the generalized Ohm's law within the presence of electric field as;

$$J_x = \frac{\sigma_e}{\alpha_e^2 + \beta_e^2} [\alpha_e(E_x - wB_0) + \beta_e(E_z + uB_0)] \quad (1)$$

$$J_z = \frac{\sigma_e}{\alpha_e^2 + \beta_e^2} [\alpha_e(E_z + uB_0) - \beta_e(E_x - wB_0)] \quad (2)$$

For the sake of definiteness, initially the plate as well as fluid are at the same temperature $T (= T_\infty)$, everywhere in the fluid is same. Also it is assumed that the fluid and the plate is at rest and then plate is moving with constant velocity in its own plane and instantaneously at dimensionless time $\tau > 0$, the temperature of the plate is raised to $T_w (> T_\infty)$. Thereafter it maintained constant, where, T_w represented the temperature at the wall.

Owing to the unsteady fluid motion and the plate is of infinite extent, all the flow variables will depend only upon y and time t . Thus accordance with the above assumptions relevant to the problem and Boussinesq's approximation, the basic boundary layer equations are given by;

The momentum equation in x -direction;

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \vartheta \left(\frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{\rho} \frac{B_0^2 \sigma_e}{(\alpha_e^2 + \beta_e^2)} (\alpha_e u + \beta_e w) \quad (3)$$

Momentum equation in z -direction;

$$\frac{\partial w}{\partial t} - v_0 \frac{\partial w}{\partial y} = \vartheta \left(\frac{\partial^2 w}{\partial y^2} \right) + \frac{1}{\rho} \frac{B_0^2 \sigma_e}{(\alpha_e^2 + \beta_e^2)} (\beta_e u - \alpha_e w) \quad (4)$$

Energy equation;

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \left(\frac{\partial^2 T}{\partial y^2} \right) + \frac{1}{c_p} \vartheta \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] + \frac{\sigma_e B_0^2}{\rho c_p} (u^2 + w^2) \quad (5)$$

The boundary conditions for the present problem are given by,

$$\begin{aligned} t \leq 0, \quad u = 0, \quad w = 0, \quad T \rightarrow T_\infty \quad \text{every where} \\ t > 0, \quad u = 0, \quad w = 0, \quad T = T_w \text{ at } y = 0 \\ \quad \quad \quad u = 0, \quad w = 0, \quad T \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \quad (6)$$

where u, v, w are the velocity components in the x, y, z direction respectively, g is the acceleration due to gravity, β is the coefficient of volume expansion, T and T_∞ are the temperature of the fluid inside the thermal boundary layer and the fluid temperature in the free stream, respectively. ϑ is the kinematic viscosity, ρ be the density, σ_e is the electrical conductivity, B_0 be the strength of the magnetic field, β_e is the Hall parameter, $\alpha_e = 1 + \beta_i \beta_e$ where, β_i is the Ion-slip. κ is the thermal conductivity of the medium, c_p is the specific heat at constant pressure.

3. Mathematical Formulation

Here, the constant v_0 a velocity, always called constant suction velocity. The following dimensionless variables have been acquainted to attain the solution of non-linear coupled partial differential equation;

$$Y = \frac{yU_0}{\vartheta}, U = \frac{u}{U_0}, W = \frac{w}{U_0}, \tau = \frac{tU_0^2}{\vartheta}, \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

The above dimensionless variables give;

$$u = U_0 U, w = U_0 W, y = \frac{Y\vartheta}{U_0}, t = \frac{\tau\vartheta}{U_0^2}, T = T_\infty + (T_w - T_\infty)\theta$$

Applying all these non-dimensional variables into the equation (1) to (3), the basic equations relevant to the problem are in dimensionless form,

$$\frac{\partial U}{\partial \tau} - S \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + G_r \theta - \frac{M}{(\alpha_e^2 + \beta_e^2)} (\alpha_e U + \beta_e W) \quad (7)$$

$$\frac{\partial W}{\partial \tau} - S \frac{\partial W}{\partial Y} = \frac{\partial^2 W}{\partial Y^2} + \frac{M}{(\alpha_e^2 + \beta_e^2)} (\beta_e U - \alpha_e W) \quad (8)$$

$$\frac{\partial \theta}{\partial \tau} - S \frac{\partial \theta}{\partial Y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial Y^2} + E_c \left[\left(\frac{\partial U}{\partial Y} \right)^2 + \left(\frac{\partial W}{\partial Y} \right)^2 \right] + M \frac{E_c}{(\alpha_e^2 + \beta_e^2)} (U^2 + W^2) \quad (9)$$

where,

$$S = \frac{v_0}{U_0}, \text{ Suction parameter,} \quad G_r = \frac{g\beta(T_w - T_\infty)\sigma^2}{U_0\vartheta}, \text{ Grashof Number}$$

$$M = \frac{\sigma_e B_0^2 \sigma^2}{\rho\vartheta}, \text{ Magnetic Parameter,} \quad P_r = \frac{\rho c_p \vartheta}{\kappa}, \text{ Prandlt Number}$$

$$\alpha_e = 1 + \beta_i \beta_e, \quad \beta_e, \text{ is the Hall parameter, } \beta_i, \text{ is the ion-slip parameter}$$

$$E_c = \frac{U_0^2}{c_p(T_w - T_\infty)} \text{ Eckert Number}$$

with the corresponding boundary conditions $\tau > 0$,

$$\begin{aligned} U = 0, W = 0, \theta = 1, & \quad \text{at } Y = 0 \\ U = 0, W = 0, \theta = 0, & \quad \text{as } Y \rightarrow \infty \end{aligned} \quad (10)$$

4. Method of Solution

This section state that, the governing second-order coupled dimensionless partial differential equations have been solved associated with the initial and boundary conditions. . The usual non-dimensional transformations have been used to obtain the non-similar coupled non-dimensional momentum, angular momentum, energy and

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concentration equations. The coupled non-dimensional equations have been solved numerically by implicit finite difference technique. After that, stability and convergence test has been carried out to get the stable conditions. The primary and secondary velocity profiles, angular velocity as well as temperature and concentrations are discussed for the different values of dimensionless parameter verses dimensionless co-ordinate. Therefore, the finite difference boundary layer equations becomes

$$\frac{\dot{U}_i - U_i}{\Delta\tau} - S \frac{U_{i+1} - U_i}{\Delta Y} = \frac{U_{i+1} - 2U_i + U_{i-1}}{(\Delta Y)^2} + G_r \theta_i - \frac{M}{(\alpha_e^2 + \beta_e^2)} (\alpha_e U_i + \beta_e W_i) \quad (11)$$

$$\frac{\dot{W}_i - W_i}{\Delta\tau} - S \frac{W_{i+1} - W_i}{\Delta Y} = \frac{W_{i+1} - 2W_i + W_{i-1}}{(\Delta Y)^2} + \frac{M}{(\alpha_e^2 + \beta_e^2)} (\beta_e U_i - \alpha_e W_i) \quad (12)$$

$$\frac{\dot{\theta}_i - \theta_i}{\Delta\tau} - S \frac{\theta_{i+1} - \theta_i}{\Delta Y} = \frac{1}{P_r} \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{(\Delta Y)^2} + E_c \left[\left(\frac{U_{i+1} - U_i}{\Delta Y} \right)^2 + \left(\frac{W_{i+1} - W_i}{\Delta Y} \right)^2 \right] + M \frac{E_c}{(\alpha_e^2 + \beta_e^2)} (U_i^2 + W_i^2) \quad (13)$$

With boundary conditions

$$\begin{aligned} U_0 = 0, \quad W_0 = 0, \quad \theta_0 = 1 & \quad \text{at} \quad \tau > 0 \\ U_L = 0, \quad W_L = 0, \quad \theta_L = 0 & \quad \text{as} \quad \tau > 0 \quad \text{as} \quad L \rightarrow \infty \end{aligned} \quad (14)$$

And the stability and convergence conditions

$$\epsilon \frac{\Delta\tau}{\Delta Y} + \frac{2}{P_r} \frac{\Delta\tau}{(\Delta Y)^2} \leq 1 \quad \text{where, } \Delta Y = 0.3, \Delta\tau = 0.001 \quad (15)$$

Here, the constant suction parameter ϵ and Prandtl number P_r are dependent arbitrary values of each other.

5. Results and Discussion

The developed model is plotted within the intermediate region of thermal boundary layer for different values of dimensionless suction parameter(S), the magnetic parameter (M), Hall current (β_e), Prandtl number (P_r), Grashof number (G_r), Ion-slip parameter (β_i) and Eckert number (E_c , not shown for brevity). The values of Grashof number G_r are varied with positive and negative numbers. i.e. $G_r < 1$ for forced or natural convection whereas $G_r \geq 1$ stands for free convection. Also the results are limited to $P_r = 0.71$ (Prandtl number for air at 20° C), $P_r = 1.0$ (Prandtl number for electrolytic solution like salt water at 20° C), $P_r = 7.0$ (Prandtl number correspond to water at 20° C). The other parameters are taken arbitrarily.

However, to get the required steady state solution for the framework, the numerical computation has been carried out dimensionless time $\tau = 80.05$. But at the present case, changes appear till $\tau = 35$. And then onward the changes are not apparent. Therefore, $\tau = 35$ is essentially a steady state solution of the problem. The transient primary and secondary velocities in terms of Shear stress in x -direction and y -direction for different values of dimensionless magnetic parameter M have been shown in Figs. 2-3. τ_x severely decrease as the Magnetic parameter increases. On the other hand, τ_z weekly increases initially in the domain $0 < \tau < 10$, and then it changes its pattern. Therefore it decreases strongly as magnetic parameter increases to meet the free stream velocity as shown in Fig. 3. The Nusselt number ($-N_u$) increases

monotonically with the increase of M as shown in Fig.4. This is because, in the presence of Magnetic field there produce a force called Lorenz force. It has a tendency to slow down the main velocity. Different values of Hall parameter β_e caused effect on shear stresses and Nusselt number. The Fig.5 depicts that, τ_x increases more strongly to get stable with the increase of Hall parameter. In Fig.6, τ_z initially decreases in the domain $0 < \tau < 10$, and after passing through the intermediate region, the profile changes its pattern and it is increases as β_e increase. As expected the Nusselt number ($-N_u$) has been decreased with the increase of Hall parameter β_e in Fig. 7. Ion-slip parameter β_i increased its flow pattern in x -direction whereas shear stress τ_z decreased its flow pattern with the same values of β_i has been expedited in Figs. 8-9. Nusselt number ($-N_u$) shows the decreasing effect with the increased parameter β_i as shown in Fig. 10.

Now a very important parameter dimensionless Grashof number G_r with its positive and negative parameter values cased different effects on Shear stresses and Nusselt number has been generated in Figs. 11-13. It is seen that, τ_x decreases for the value $G_r < 1$ and τ_x increases for the value $G_r \geq 1$. Also τ_z has follows the previous trend. But the Nusselt number ($-N_u$) has been decreased with the increase of Grashof number G_r . Different values of suction parameter S caused effects on the shear stresses as shown in Figs. 14-16. Shear stress τ_x falls drastically for large suction. This indicated that suction stabilizes the growth of the boundary layer. Also τ_z has been followed its previous flow pattern as shown in Fig. 15. Negative Nusselt number ($-N_u$) rises severely with the increase of suction has been expedited in Fig. 16. As expected both the shear stresses decrease for the increase of Prandlt number as shown in Figs. 17-19. The reason is that smaller values of P_r are subject to increasing the thermal conductivity of the fluid. Nusselt number ($-N_u$) increase its flow pattern as P_r increases. Therefore, heat is able to diffuse away more rapidly.

Finally, qualitative comparison with the previous published result has been shown in tabulated form (Table1). The accuracy of our results is qualitatively good in case of all the flow control parameters

Increased Parameter	Previous results given by Hazem A. ATTIA (2005)			Present results		
	u	w	T	τ_x	τ_z	N_u
M	Dec.	Inc.	Inc.	Dec.	Inc.	Inc.
β_e	Inc.	Dec.	Dec.	Inc.	Dec.	Dec.
β_i	Inc.	Dec.	Dec.	Inc.	Dec.	Dec.
G_r	-	-	-	Inc.	Inc.	Dec.
E_c	-	-	-	Inc.	Inc.	Dec.
S	Dec.	Dec.	Dec.	Dec.	Dec.	Inc.
P_r	-	-	-	Dec.	Dec.	Inc.

Table 1. Comparison table

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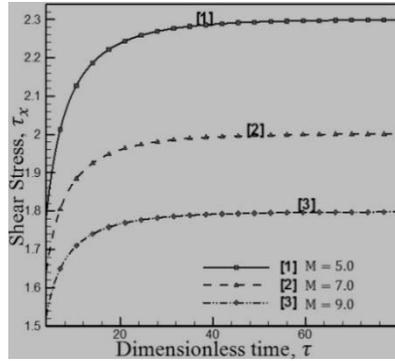


Fig. 2. Shear stress τ_x for Different values of Magnetic parameter, M at the steady state, where, $\beta_e = 3.0$, $\beta_i = 3.0$, $P_r = 0.71$, $E_c = 0.01$, $G_r = 2.0$ and $S = 0.5$.

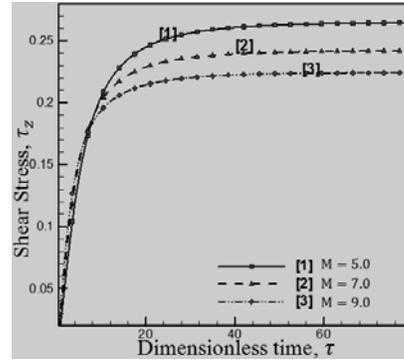


Fig. 3. Shear stress τ_z for Different values of Magnetic parameter, M at the steady state, where, $\beta_e = 3.0$, $\beta_i = 3.0$, $P_r = 0.71$, $E_c = 0.01$, $G_r = 2.0$ and $S = 0.5$.

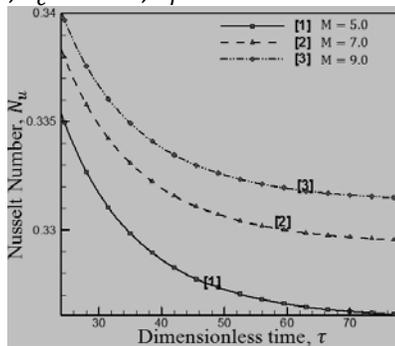


Fig. 4. Nusselt number $-N_u$ for Different values of Magnetic parameter, M at the steady state, where, $\beta_e = 3.0$, $\beta_i = 3.0$, $P_r = 0.71$, $E_c = 0.01$, $G_r = 2.0$ and $S = 0.5$.

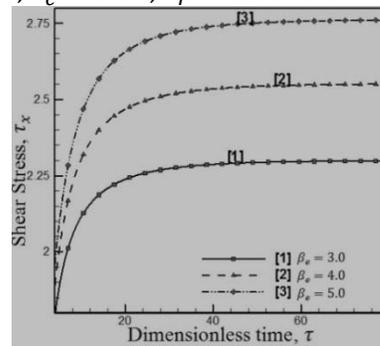


Fig. 5. Shear stress τ_x for Different values of Hall parameter, β_e at the steady state, where, $M = 5.0$, $\beta_i = 3.0$, $P_r = 0.71$, $E_c = 0.01$, $G_r = 2.0$ and $S = 0.5$.

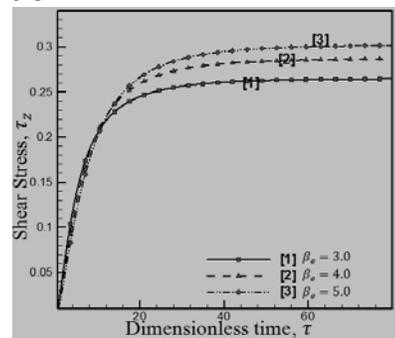


Fig. 6. Shear stress τ_x for Different values of Hall parameter, β_e at the steady state, where, $M = 5.0$, $\beta_i = 3.0$, $P_r = 0.71$, $E_c = 0.01$, $G_r = 2.0$ and $S = 0.5$.

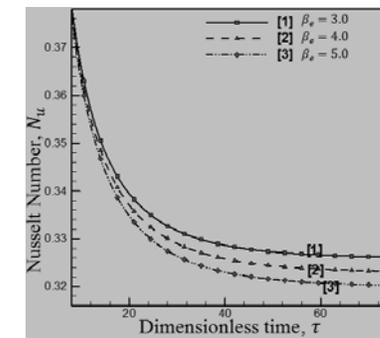


Fig. 7. Nusselt number $-N_u$ for Different values of Hall parameter, β_e at the steady state, where, $M = 5.0$, $\beta_i = 3.0$, $P_r = 0.71$, $E_c = 0.01$, $G_r = 2.0$ and $S = 0.5$.

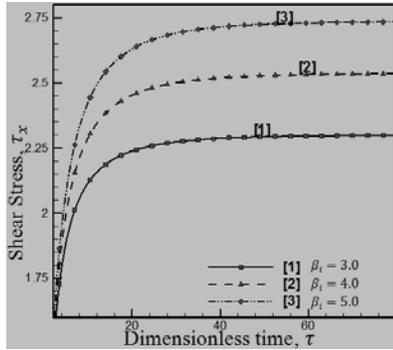


Fig. 8. Shear stress τ_x for Different values of Ion-slip parameter, β_i at the steady state, where, $M = 5.0, \beta_e = 3.0, P_r = 0.71, E_c = 0.01, G_r = 2.0$ and $S = 0.5$.

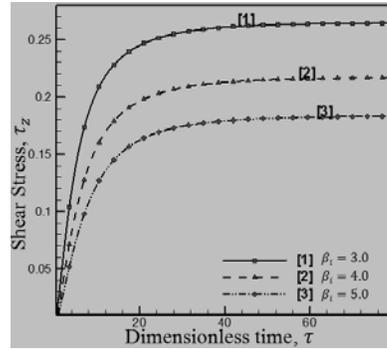


Fig. 9. Shear stress τ_z for Different values of Ion-slip parameter, β_i at the steady state, where, $M = 5.0, \beta_e = 3.0, P_r = 0.71, E_c = 0.01, G_r = 2.0$ and $S = 0.5$.

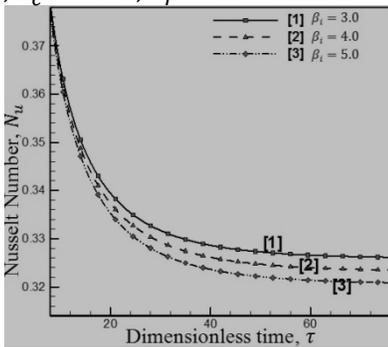


Fig. 10. Nusselt number $-N_u$ for Different values of Ion-slip parameter, β_i at the steady state, where, $M = 5.0, \beta_e = 3.0, P_r = 0.71, E_c = 0.01, G_r = 2.0$ and $S = 0.5$.

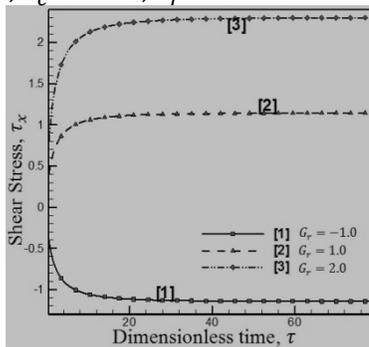


Fig. 11. Shear stress τ_x for Different values of Grashof number, G_r at the steady state, where, $M = 5.0, \beta_e = 3.0, P_r = 0.71, E_c = 0.01, \beta_i = 3.0$ and $S = 0.5$.

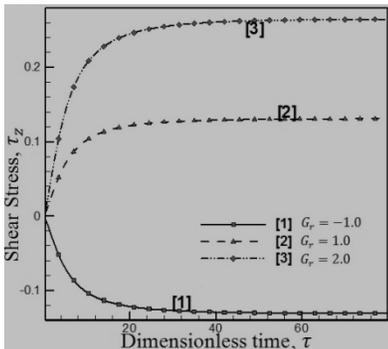


Fig. 12. Shear stress τ_z for Different values of Grashof number, G_r at the steady state, where, $M = 5.0, \beta_e = 3.0, P_r = 0.71, E_c = 0.01, \beta_i = 3.0$ and $S = 0.5$.

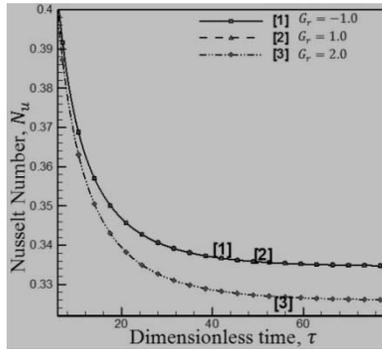


Fig. 13. Nusselt number $-N_u$ for Different values of Grashof number, G_r at the steady state, where, $M = 5.0, \beta_e = 3.0, P_r = 0.71, E_c = 0.01, \beta_i = 3.0$ and $S = 0.5$.

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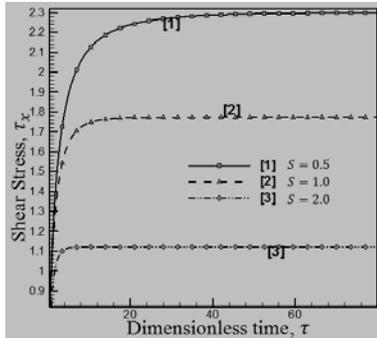


Fig. 14. Shear stress τ_x for Different values of S at the steady state, where, $M = 5.0$, $\beta_e = 3.0$, $P_r = 0.71$, $G_r = 2.0$, $\beta_i = 3.0$ and $E_c = 0.01$.

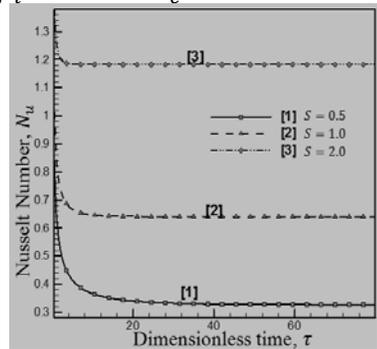


Fig. 16. Nusselt number $-N_u$ for Different values of S at the steady state, where, $M = 5.0$, $\beta_e = 3.0$, $P_r = 0.71$, $G_r = 2.0$, $\beta_i = 3.0$ and $E_c = 0.01$.

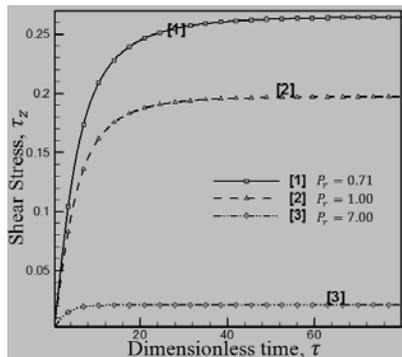


Fig.18. Shear stress τ_z for Different values of P_r at the steady state, where, $M = 5.0$, $\beta_e = 3.0$, $S = 0.5$, $G_r = 2.0$, $\beta_i = 3.0$ and $E_c = 0.01$.

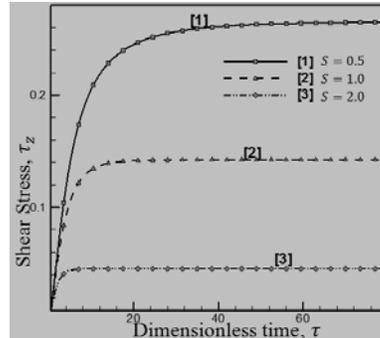


Fig. 15. Shear stress τ_z for Different values of S at the steady state, where, $M = 5.0$, $\beta_e = 3.0$, $P_r = 0.71$, $G_r = 2.0$, $\beta_i = 3.0$ and $E_c = 0.01$.

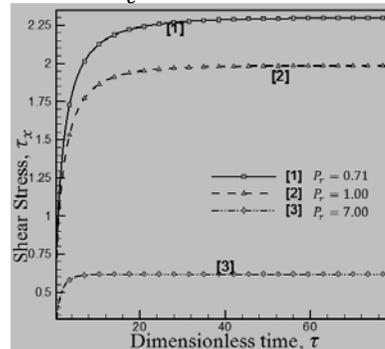


Fig. 17. Shear stress τ_x for Different values of Prandlt parameter, P_r at the steady state, where, $M = 5.0$, $\beta_e = 3.0$, $S = 0.5$, $G_r = 2.0$, $\beta_i = 3.0$ and $E_c = 0.01$.

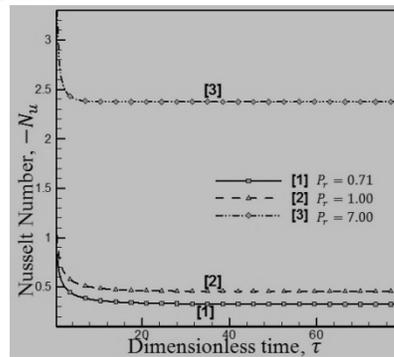


Fig. 19. Nusselt number $-N_u$ for Different values of P_r at the steady state, where, $M = 5.0$, $\beta_e = 3.0$, $S = 0.5$, $G_r = 2.0$, $\beta_i = 3.0$ and $E_c = 0.01$.

6. Conclusion

Magnetohydrodynamics fluid flow through a vertical plate with the influence of Hall and Ion-slip current has been performed in this paper. In this study, the absence of a meaningful form of mass continuity in a one-dimensional model precludes the possibility of such a model making a physically accurate prediction. In these comprehensive analyses the effect of different parameters on the shear stresses as well as Nusselt number is discussed simultaneously. In all those parameters suction parameter, Hall and Ion-slip parameter and also the Grashof number has attracted the interest of any investigations owing to its important applications in many engineering problems such as electric transformer, purification of crude oil, MHD accelerators and heating elements. Some important findings of this analysis are listed below:

1. The shear stress of fluid particle in x direction is decreases with the increase of magnetic parameter. But in the case of z direction shear stress is increased as increases the magnetic parameter.
2. The Nusselt number is increases with the increase of magnetic parameter, Suction parameter and Prandlt number. But at the same time it gives the opposite effect in the case of Hall current, Ion-slip parameter,Grashof number and Eckert number.
3. In the case of Suction parameter and Prandlt Number the shear stress of the fluid particles are increase in the both direction.
4. In the both direction the shear stress is increases with the increase of Grashof number and Eckert Number.
5. With the increase of Hall current the shear stress increases in the x direction but it shows the contrary impact in z direction. Same consequence shows for the case of Ion-slip parameter.

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