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A Generalised Cubature Rule Using Simpson Cubature Rule

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Abstract. This paper presents a higher-degree precision cubature rule for the numerical evaluation of double integrals. The proposed method is developed using a generalised quadrature technique that combines the Gauss–Legendre two-point cubature rule with Simpson's 1/3 cubature rule. The formulation aims to enhance accuracy while maintaining computational efficiency. The performance and dominance of the proposed rule are examined through numerical experiments on a set of standard test problems. Furthermore, error bounds are derived and analysed under both adaptive and non-adaptive environments. The results demonstrate that the proposed cubature rule provides improved precision and reliability compared to existing methods.

Keywords: Generalised Cubature rule; Gauss-Legendre Cubature rule; Simpson Cubature rule

AMS Mathematics Subject Classification (2010): 65D30, 65D32

1. Introduction

Numerical analysis is a fundamental branch of mathematics concerned with developing approximate methods for solving problems where exact analytical solutions are difficult or impossible to obtain. Among its major areas, numerical integration plays a crucial role in evaluating definite integrals, especially when the integrand is complex, multidimensional, or not expressible in closed form.

Numerical integration techniques are broadly classified into quadrature rules for single integrals and cubature rules for multiple integrals. The effectiveness of any quadrature or cubature rule is measured by its degree of precision, which indicates the highest degree of polynomial that the rule can integrate exactly. In general, a rule with higher precision is considered superior, as it yields more accurate results for a wider class of functions.

Over the years, several methodologies have been developed to enhance the precision of numerical integration rules. Notable among these are Richardson extrapolation [9,10], Kronrod extensions [1,2], and mixed quadrature rules [3,4,7,8,9,10,13,14], each aiming to improve accuracy without significantly increasing computational cost.

More recently, S.K. Mohanty and R.B. Dash [5,6,8] introduced the concept of generalised quadrature technique, where two or more existing quadrature rules are combined in a systematic manner to construct a new rule of higher precision. These generalised rules are designed to satisfy the SR-condition, ensuring improved stability and efficiency.

Motivated by this approach, the present work extends the idea of generalised quadrature to multiple integrals. In this paper, we construct a generalised cubature rule $SM_4^2(f)$ of degree of precision five by suitably combining two well-known cubature rules namely, the Gauss–Legendre two-point cubature rule $G_2^2(f)$ and the Simpson's 1/3rd cubature rule $S_{\frac{1}{3}}^2(f)$, each having precision three. The resulting rule achieves higher accuracy while maintaining computational simplicity. The formulation of the rule is systematic, and its performance is further validated through numerical examples and error analysis.

2. Base cubature rules

In this section we discuss the two base cubature rules.

2.1. Gauss–Legendre 2-point cubature rule

The Gauss–Legendre 2-point cubature rule for a double integral [1,2,15] is obtained by applying the 2-point Gauss–Legendre quadrature in both variables.
Consider the integral over the square $[-1,1] \times [-1,1]$:

$$I = \int_{-1}^{1} \int_{-1}^{1} f(x,y)\, dx\, dy. \tag{1}$$

The 2-point Gauss–Legendre nodes are $x_1 = -\frac{1}{\sqrt{3}}, x_2 = \frac{1}{\sqrt{3}}$, with weights $w_1 = w_2 = 1$

Applying the rule in both directions, $I \approx \sum_{i=1}^{2} \sum_{j=1}^{2} w_i\, w_j\, f(x_i, y_j)$.

Since $w_1 = w_2 = 1$, the gauss-legendre 2-point cubature rule for approximation of I, is given by

$$G_2^2(f) = f\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \tag{2}$$

Using Maclaurin series expansion on (1), we get

$$G_2^2(f) = 4f(0,0) + \frac{2}{3}\left[f_{2,0}(0,0) + f_{0,2}(0,0)\right] + \frac{1}{54}\left[f_{4,0}(0,0) + f_{0,4}(0,0)\right]$$
$$+\frac{1}{9} f_{2,2}(0,0) + \frac{1}{4860}\left[f_{6,0}(0,0) + f_{0,6}(0,0)\right] + \frac{1}{324}\left[f_{4,2}(0,0) + f_{2,4}(0,0)\right] + \cdots \tag{3}$$

Exact value of the integral

Due to Maclaurin series expansion, the exact value of the integral

$$I(f) = 4f_{0,0}(0,0) + \frac{2}{3}\left[f_{2,0}(0,0) + f_{0,2}(0,0)\right] + \frac{1}{30}\left[f_{4,0}(0,0) + f_{0,4}(0,0)\right] + \frac{1}{9}f_{2,2}(0,0) + \frac{4}{7!}\left[f_{6,0}(0,0) + f_{0,6}(0,0)\right] + \frac{1}{180}\left[f_{4,2}(0,0) + f_{2,4}(0,0)\right] \quad (4)$$

2.1.1. Error due to Gauss-Legendre's 2-point cubature rule

The truncation error due to Gauss-Legendre 2-point cubature rule $EG_2^2(f)$ is given by

$$EG_2^2(f) = I(f) - G_2^2(f) \quad (5)$$

Using (3), (4) and (5), we get

$$EG_2^2(f) = \frac{2}{135}\left[f_{4,0}(0,0) + f_{0,4}(0,0)\right] + \frac{1}{1701}\left[f_{6,0}(0,0) + f_{0,6}(0,0)\right] + \frac{1}{404}\left[f_{4,2}(0,0) + f_{2,4}(0,0)\right] + \cdots \quad (6)$$

2.2. Simpson's 1/3rd cubature rule

The Simpson's $\frac{1}{3}$rd cubature rule [1,2,12,13] for double integration over the square region $[-1,1] \times [-1,1]$is given by

$$S_{\frac{1}{3}}^{2}(f) = \frac{1}{9}[[f(-1,-1) + 4f(0,-1) + f(1,-1)] + 4[f(-1,0) + 4f(0,0) + f(1,0)] + [f(-1,1) + 4f(0,1) + f(1,1)]]. \quad (7)$$

Applying Maclaurin's series expansion on (7)

$$S_{\frac{1}{3}}^{2}(f) = 4f_{0,0}(0,0) + \frac{2}{3}[f_{2,0}(0,0) + f_{0,2}(0,0)] + \frac{1}{18}[f_{4,0}(0,0) + f_{0,4}(0,0)] + \frac{1}{9}f_{2,2}(0,0) + \frac{11}{6480}[f_{6,0}(0,0) + f_{0,6}(0,0)] + \frac{1}{108}[f_{4,2}(0,0) + f_{2,4}(0,0)] + \cdots. \quad (8)$$

2.2.1. Error due to

The truncation error due to Simpson's $\frac{1}{3}$-rule cubature rule $EG_2^2(f)$ is given by

$$ES_{\frac{1}{3}}^{2}(f) = I(f) - S_{\frac{1}{3}}^{2}(f) \quad (9)$$

Using (4), (8) and (9), we get

$$ES_{\frac{1}{3}}^{2}(f) = -\frac{1}{45}[f_{4,0}(0,0) + f_{0,4}(0,0)] - \frac{41}{45360}[f_{6,0}(0,0) + f_{0,6}(0,0)] - \frac{1}{270}[f_{4,2}(0,0) + f_{2,4}(0,0)] + \cdots. \quad (10)$$

Remark 2.1: From the error term, it is clear that both the rules $G_2^2(f)$ and $S_{\frac{1}{3}}^{2}(f)$ integrates exactly all polynomials of degree up to 3 in each variable. Hence, the degree of precision of both the Rules is 3.

3. Construction of higher precision cubature rules

Definition: (**SR-Conditions**) A set of quadrature rules $R_1, R_2, \cdots R_n$ are said to satisfy SR-Conditions if

1. The degree of precision of the quadrature rules are in monotonically increasing order.

2. The degree of precision of first two rules must be equal, that is dR_1 = dR_2.
3. The degrees precision of 2nd, 3rd, 4th, …,nth quadrature rules are in an arithmetic progression with common difference 2

Since both the rules $G_2^2(f)$ and $S_{\frac{1}{3}}^2(f)$ satisfy the SR-condition, by the generalised quadrature technique [4,6,8] the proposed higher precision (precision 5) cubature rule can be expressed as

$$SM_4^2(f) = a.G_2^2(f) + b.S_{\frac{1}{3}}^2(f) \tag{11}$$

where a and b are rational constants satisfying the normality condition

$$a + b = 1. \tag{12}$$

The truncation due to the corresponding $SM_4^2(f)$ rule is given by

$$ESM_4^2(f) = a.EG_2^2(f) + b.ES_{\frac{1}{3}}^2(f) \tag{13}$$

Using values from

$$ESM_4^2(f) = \left(\frac{2a}{135} - \frac{b}{45}\right)\left[f_{4,0}(0,0) + f_{0,4}(0,0)\right] + \left(\frac{a}{1701} - \frac{41b}{45360}\right)\left[f_{6,0}(0,0) + f_{0,6}(0,0)\right] + \left(\frac{a}{404} - \frac{b}{270}\right)\left[f_{4,2}(0,0) + f_{2,4}(0,0)\right] + \cdots \tag{14}$$

Since the degree of precision of the rule $SM_4^2(f)$ is five, from (14), we get

$$\frac{2a}{135} - \frac{b}{45} = 0 \tag{15}$$

On solving (12) and (15), we get $a = \frac{3}{5}$ and $b = \frac{2}{5}$.

Using the value of a and b on (11) and (13), we have the following theorem.

Theorem 3.1. Let $f(x, y)$be a sufficiently differentiable function in the closed region $[-1,1] \times [-1,1]$. Then the Generalised cubature rule $SM_4^2(f)$ formed out by $G_2^2(f)$ and $S_{\frac{1}{3}}^2(f)$ is given by $SM_4^2(f) = \frac{1}{5}\left[3.G_2^2(f) + 2.S_{\frac{1}{3}}^2(f)\right]$

4. Analysis of error

The Error analysis gives the following theorems.

Theorem 4.1. Let $f(x, y)$be a sufficiently differentiable function in the closed region $[-1,1] \times [-1,1]$. Then truncation error due to the Generalised cubature rule $SM_4^2(f)$ is given by $ESM_4^2(f) = \frac{1}{5}\left[3.EG_2^2(f) + 2.ES_{\frac{1}{3}}^2(f)\right]$.

Proof: Using the value of a and b on (13), we get the desired result. □

Theorem 4.2. Let $f(x, y)$be a sufficiently differentiable function in the closed region $[-1,1] \times [-1,1]$. Then truncation error bounds due to the Generalised cubature rule $SM_4^2(f)$ is $ESM_4^2(f) = -\frac{1}{11340}\left[f_{6,0}(0,0) + f_{0,6}(0,0)\right] + \frac{1}{272700}\left[f_{4,2}(0,0) + f_{2,4}(0,0)\right] + \cdots$.

Proof: Using the values from of a and b on (14), we get the result.

□

Remark 4.1: From theorem 4.2, it is clear that the rules $SM_4^2(f)$ integrates exactly all polynomials of degree up to 5 in each variable. Hence, the degree of precision of the Rules is 5.

Theorem 4.3. Let $f(x,y)$be a sufficiently differentiable function in the closed region $[-1,1] \times [-1,1]$. Then truncation error due to the Generalised cubature rule $ESM_4^2(f)$ is less than the base rules.
Proof: From Theorem 4.2 and (6), we get $|SM_4^2(f)| < |EG_2^2(f)|$.
From Theorem 4.2 and (10), we get $|SM_4^2(f)| < |ES_{1/3}^2(f)|$ ☐

5. Numerical verification

To establish the dominance and effectiveness of the proposed cubature rule, we consider several test examples of different types.

1. **Polynomial Function** $I_1 = \int_{-1}^{1} \int_{-1}^{1} (x^4 + y^4 + x^2y^2 + 1)\ dx\, dy$

Table 1: Approximate values and Absolute errors obtained using the base rules $G_2^2(f)$ and $S_{\frac{1}{3}}^2(f)$, and the formulated Cubature rule $SM_4^2(f)$.

Exact value 6.0444444446	Rule $G_2^2(f)$	Rule $S_{\frac{1}{3}}^2(f)$	Rule $SM_4^2(f)$
Approximation value	5.3333333333	7.1111111111	6.0444444444
Absolute Error	7.111111×10^{-1}	1.066666×10^{0}	2.0000×10^{-10}

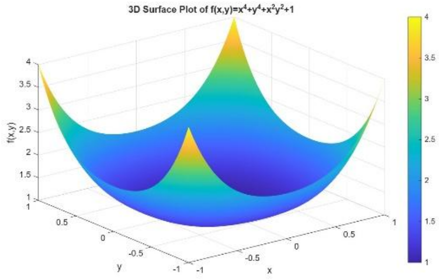

Figure 1a: Surface plot for the integral I_1.

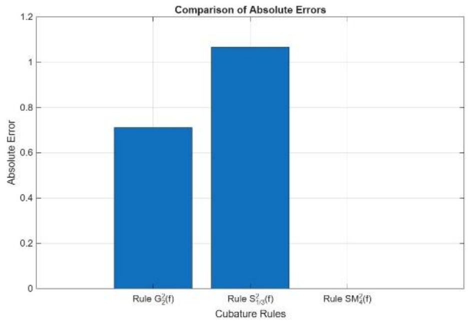

Figure 1b: Error comparison plot for the integral $I_1(f)$

2. **Exponential Function** $I_2 = \int_{-1}^{1} \int_{-1}^{1} e^{-(x^2+y^2)}\ dx\, dy$

Table 2: Approximate values and Absolute errors obtained using the base rules $G_2^2(f)$ and $S_{\frac{1}{3}}^2(f)$, and the formulated Cubature rule $SM_4^2(f)$.

Exact value 2.2309851405	Rule $G_2^2(f)$	Rule $S_{\frac{1}{3}}^2(f)$	Rule $\|SM_4^2(f)$
Approximation value	2.0536684761	2.4919346880	2.2289749609
Absolute Error	1.773167×10^{-1}	2.609495×10^{-1}	2.01018×10^{-3}

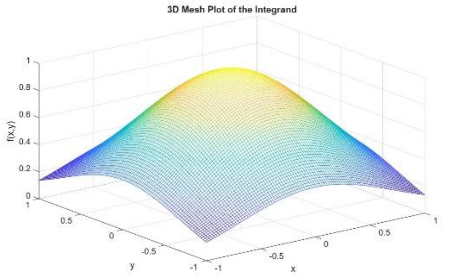

Figure 2a: Surface plot for the integra I_2.

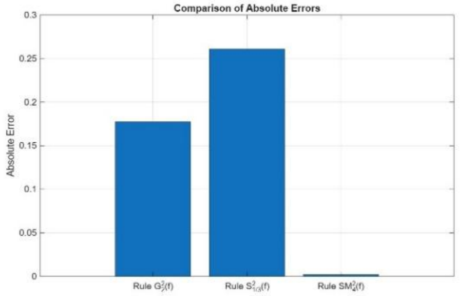

Figure 2b: Error comparison plot for the integral I_2.

3. **Logarithmic Function** $I_3 = \int_{-1}^{1} \int_{-1}^{1} \ln(1 + x^2 + y^2)\, dx\, dy$

Table 3: Approximate values and Absolute errors obtained using the base rules $G_2^2(f)$ and $S_{\frac{1}{3}}^2(f)$, and the formulated Cubature rule $SM_4^2(f)$.

Exact value 1.9158485734	Rule $G_2^2(f)$	Rule $S_{\frac{1}{3}}^2(f)$	Rule $\|SM_4^2(f)$
Approximation value	2.0433024951	1.7205337826	1.9141950101
Absolute Error	1.274539×10^{-1}	1.9531478×10^{-1}	1.6535633×10^{-3}

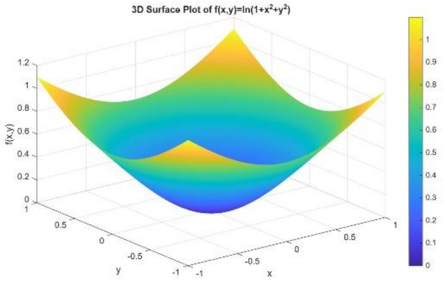

Figure 3a: Surface plot for the integral I_3

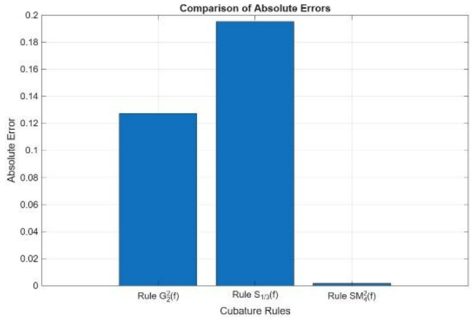

Figure 3b: Error comparison plot for the integra I_3.

Table 4: Results Obtained Using the Gauss–Legendre 3-Point Cubature Rule $G_3^2(f)$): Absolute Values and Error Comparison with Rule $SM_4^2(f)$.

Integral	Approximation value	|Error $G_3^2(f)$|	$\|ESM_4^2(f)\|$	Remark
I_1	6.0444444443	3.0000×10^{-10}	2.0000×10^{-10}	$\|ESM_4^2(f)\|$ is least
I_2	2.24604	$0.01.51\times 10^{-3}$	2.01018×10^{-3}	$\|ESM_4^2(f)\|$ is least
I_3	1.914176871	1.67170×10^{-3}	1.65356×10^{-3}	$\|ESM_4^2(f)\|$ is least

6. Application in adaptive routine

An efficient cubature has been designed [4,5,6,7,8,12,13,16]

A Generalised Cubature Rule Using Simpson Cubature Rule

Algorithm (Adaptive Cubature)

The proposed cubature routine is based on a recursive adaptive subdivision strategy:

1. Initial Approximation
   A base quadrature rule (tensor-product Simpson-type rule) is applied over the rectangular domain.
2. Subdivision Strategy
   The region is divided into four equal subregions:
   $$[a_x, b_x] \times [a_y, b_y] \rightarrow 4 \text{ subrectangles}$$
3. Refined Approximation
   The integral is recomputed over each subregion and summed to obtain a refined estimate.
4. Error Estimation
   The local error is estimated as: Error $= | I_{\text{refined}} - I_{\text{coarse}} |$

Stopping Criterion

The adaptive process continues until: $| I_2 - I_1 | < \text{Tolerance} = 10^{-6}$

- If the condition is satisfied → accept the result
- Otherwise → subdivide further and repeat recursively

To ensure global accuracy, the tolerance is distributed among subregions (tol/4 for each subdivision).

<u>MATLAB Program</u>

```
function I = adaptive_cubature(f, ax, bx, ay, by, tol)
% Adaptive cubature over rectangular region [ax,bx] × [ay,by]
% f   : function handle f(x,y)
% tol : prescribed tolerance (e.g., 1e-6)

  % Initial approximation
  I = cubature2D(f, ax, bx, ay, by);

  % Recursive refinement
  I = adapt_rec(f, ax, bx, ay, by, tol, I);
end

%-----------------------------------------------------------
function I = cubature2D(f, ax, bx, ay, by)
% Basic 2D quadrature rule (tensor Simpson-like rule)

  mx = (ax + bx)/2;
  my = (ay + by)/2;

  I = (bx-ax)*(by-ay)/36 * ( ...
    f(ax,ay) + f(bx,ay) + f(ax,by) + f(bx,by) + ...
    4*(f(mx,ay) + f(mx,by) + f(ax,my) + f(bx,my)) + ...
    16*f(mx,my) );
end
```

```
%-----------------------------------------------------------
function I = adapt_rec(f, ax, bx, ay, by, tol, I1)
% Recursive adaptive refinement

   mx = (ax + bx)/2;
   my = (ay + by)/2;

   % Subdivide into 4 subregions
   I2 = cubature2D(f, ax, mx, ay, my) + ...
        cubature2D(f, mx, bx, ay, my) + ...
        cubature2D(f, ax, mx, my, by) + ...
        cubature2D(f, mx, bx, my, by);

   % Error estimate
   err = abs(I2 - I1);

   % Stopping criterion
   if err < tol
      I = I2;
   else
      % Recursive refinement with reduced tolerance
      I = adapt_rec(f, ax, mx, ay, my, tol/4, cubature2D(f, ax, mx, ay, my)) + ...
          adapt_rec(f, mx, bx, ay, my, tol/4, cubature2D(f, mx, bx, ay, my)) + ...
          adapt_rec(f, ax, mx, my, by, tol/4, cubature2D(f, ax, mx, my, by)) + ...
          adapt_rec(f, mx, bx, my, by, tol/4, cubature2D(f, mx, bx, my, by));
   end
end
```

Application of the Adaptive routine

The results obtained by the said test examples ($I_1(f), I_2(f)$ and $I_3(f)$.) are reflects in the following Table.

Table 4: Adaptive Cubature Results with Prescribed Tolerance 1×10^{-6}, Showing Approximate Values, Number of Steps, and Final Errors for the Integrals $I_1(f)$, $I_2(f)$, and $I_3(f)$.

Integrals	**Approximation value**	**No. of steps**	**Final Error**
For the cubature rule Rule $G_2^2(f)$			
$I_1(f)$	6.0444435665	30	$8.781100513\times 10^{-7}$
$I_2(f)$	2.2309842949	14	$8.4552808710\times 10^{-7}$
$I_3(f)$	1.9158495361	13	$9.6268665972\times 10^{-7}$
For the cubature rule Rule $S_{1/3}^2(f)$			
$I_1(f)$	6.0444453439	33	$8.9924647195\times 10^{-7}$
$I_2(f)$	2.2309861052	15	$9.6470122335\times 10^{-7}$
$I_3(f)$	1.9158477575	15	$8.1585961165\times 10^{-7}$

For the generalised cubature rule $SM_4^2(f)$			
$I_1(f)$	6.0444444444	01	$2.0000000000\times 10^{-10}$
$I_2(f)$	2.2309860565	03	$9.1604867869\times 10^{-7}$
$I_3(f)$	1.9158483905	04	$1.8286015058\times 10^{-7}$

Remark 5.1: From the above adaptive cubature Table 4, it is observed that the proposed rule $SM_4^2(f)$ converges more rapidly than the base rules $G_2^2(f)$ and $S_{1/3}^2(f)$. The number of steps required by rule $SM_4^2(f)$ to achieve the prescribed tolerance is comparatively smaller, which indicates its higher computational efficiency and better convergence behavior. On the other hand, rules $G_2^2(f)$ and $S_{1/3}^2(f)$ require more iterative steps to attain the same level of accuracy and hence converge more slowly. Therefore, the numerical results establish the dominance and superiority of the proposed cubature rule over the base rules.

Limitations

Extending numerical integration to higher-dimensional problems presents significant challenges in terms of computational complexity and practical implementation. The required number of function evaluations and subdivisions increases rapidly, making manual computation extremely lengthy, tedious, and time-consuming, a phenomenon known as the "curse of dimensionality," which greatly reduces efficiency. Despite these limitations, higher-dimensional integrals are essential in many scientific and engineering applications. Hence, efficient methods such as adaptive cubature and generalised quadrature rules become crucial. Future work may focus on higher-dimensional extensions using adaptive and parallel techniques, as well as applications to irregular and non-rectangular domains.

6. Conclusion

From the above theoretical results, error analysis, numerical tables, and graphical representations, it is concluded that the proposed cubature rule $SM_4^2(f)$ provides more accurate and efficient results in comparison with its constituent rules $G_2^2(f)$ and $S_{1/3}^2(f)$. The superiority of the rule $SM_4^2(f)$ is clearly established in both non-adaptive and adaptive computational environments.

Therefore, the proposed higher precision cubature rule can be effectively used for the numerical evaluation of surface integrals and general double integrals arising in applied mathematics, engineering, and scientific computations.

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