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Multi-decomposition of Complete Graphs into Kites and Stars of Size Four

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Abstract. Let K_n, S_n and $K_3 + e$ respectively denote a complete graph on n vertices, a star on n edges and a kite on 4 edges. If $H_1, H_2, \ldots, H_l$ are edge disjoint subgraphs of G such that $E(G) = E(H_1) \cup E(H_2) \cup .. \cup E(H_l)$, then we say that $H_1, H_2, \ldots, H_l$ decompose G and we write this as $G = H_1 \oplus H_2 \oplus \ldots \oplus H_l$. The graph G has an $\{H_1^\alpha, H_2^\beta\}$-decomposition or (H_1, H_2)-Multi-decomposition, if α copies of H_1 and β copies of H_2 decompose G, where $H_1 \neq H_2$. In this paper, it is proved that for $n \geq 8$, the complete graph K_n admits a $\{(K_3 + e)^\alpha, S_4^\beta\}$ decomposition, if $n = 4t$ or $n = 4t + 1$ for all even t, for the pairs (α, β) satisfying $0 \leq \alpha \leq \frac{9t-4}{2}$, $\beta = \frac{1}{4}\binom{n}{2} - \alpha$, where α and β respectively denote the number of copies of $K_3 + e$ and S_4.

Keywords: Kite, Star, Complete graph, Complete bipartite graph, Decomposition, Multi-decomposition.

AMS Mathematics Subject Classification (2010): 05C70

1. Introduction

All graphs considered here are simple and finite. The complete graph on n vertices is denoted by K_n and a complete bipartite graph on m, n vertices is denoted by $K_{m,n}$. A Star S_k is the complete bipartite graph $K_{1,k}$. A Kite is a graph in which one edge is attached to a vertex of a triangle and is denoted by $K_3 + e$. For a graph G with $S \subseteq V(G)$, $\langle S \rangle$ denotes the subgraph of G induced by S. If $H_1, H_2, \ldots, H_l$ are edge disjoint subgraphs of G such that $E(G) = \bigcup_{i=1}^{l} E(H_i)$ and $E(H_i) \cap E(H_j) = \emptyset$ for $i \neq j, i, j = 1,2, \ldots, l$, then we say that $H_1, H_2, \ldots, H_l$ decompose G and we write this as $G = \oplus_{i=1}^{l} H_i$. If each $H_i \cong H$, $1 \leq i \leq l$, then we say that G admits a H-

decomposition and we denote it by $H|G$. The graph G has an $\{H_1^{\alpha}, H_2^{\beta}\}$-decomposition or a (H_1, H_2)-multi-decomposition, if α copies of H_1 and β copies of H_2 decompose G, where α and β are non-negative integers; we denote it by $\{H_1^{\alpha}, H_2^{\beta}\}|G$. For $\alpha, \beta \geq 0$, if the known necessary conditions are satisfied for the existence of a $\{H_1^{\alpha}, H_2^{\beta}\}$ of G by the pairs (α, β), then we say that (α, β) is an admissible pair.

For standard graph-theoretic terminology we refer [3, 10]. Bermond and Schonheim [2] proved the existence of a kite decomposition of K_n . The kite decomposition of certain are considered in [7, 11, 16]. Likewise, Decomposition of graphs into stars have been studied by many authors [9, 12, 13, 14, 20]. In recent years, the multi-decomposition of graphs into cycles and stars, paths and cycles etc. have been considered as potential research problems [1, 4, 8, 17, 18, 19]. Kureethara [5] studied Hamiltonian cycles in complete multipartite graphs and obtained conditions for their existence. Quadras and Sarah Surya [6] investigated wirelength in circulant and wheel related graphs. Jayawardene et al. [15] discussed star-critical Ramsey numbers involving star graphs. These works motivated the present study on the multi-decomposition of complete graphs into kites and stars of size four.

In this content, we investigate the multi-decomposition of the complete graphs into kites and stars on four edges. In this paper, we give the necessary and sufficient conditions for the existence of a $\{(K_3 + e)^{\alpha}, S_4^{\beta}\}$ - decomposition in K_n, $n \geq 8$ and $\alpha, \beta \geq 0$.

Notations:

Symbol	Meaning
K_n	Complete graph on n vertices
$K_{m,n}$	Complete bipartite graph on m, n vertices
$K_3 + e$	Kite
S_4	Star on 4 edges
α	Number of copies of kites
β	Number of copies of stars

- A kite $K_3 + e$ on 4 edges is denoted by $[(v_i, v_j, v_k); v_i v_l]$.
- A star S_4 is denoted by $S(v_i; v_j, v_k, v_l, v_m)$.

2. Some mathematical tools

In this section, we present some of the known theorems and lemmas that will be used in the proof of our main result.

Theorem 1. *[20] Let k, m and $n \in Z^+$ with $m \leq n$. Then there exists a S_k - decomposition of $K_{m,n}$ if and only if one of the following holds:*

(i) $k \leq m$ *and* $mn \equiv 0(mod\ k)$
(ii) $m < k \leq n$ *and* $n \equiv 0(mod\ k)$.

Lemma 2. *[18] If p and q are non-negative integers such that* $4(p+q) = \binom{16}{2}$ *and* $q \geq 4$, *then* K_{16} *can be decomposed into p copies of* C_4 *and q copies of* S_4.

3. Multi-decomposition of complete graphs into kites and stars of size four

3.1. Necessary conditions

In this section, we obtain the necessary condition for the existence of the multi-decomposition of the complete graph K_n into kites $K_3 + e$ and the stars S_4.

Theorem 3. *If the graph* K_n *has a* $\{(K_3+e)^\alpha, S_4^\beta\}$*-decomposition, then*

$$n \equiv 0 \text{ or } 1(mod\ 8), \text{ where } \alpha, \beta \geq 0.$$

Proof: Since K_n has $\{(K_3+e)^\alpha, S_4^\beta\}$-decomposition, by the edge divisibility condition we must have,

$$4(\alpha+\beta) = \binom{n}{2}$$

$$i.e., n(n-1) = 8k, where\ k = \alpha + \beta$$

$$i.e., n \equiv 0 \text{ or } 1(mod\ 8).$$ □

Next, we will prove a theorem on the non-existence of a kite decomposition in $K_{m,n}$.

Theorem 4. *For all positive integers* $m, n \geq 2$, $K_{m,n}$ *does not admit a kite decomposition.*

Proof. Proof follows from the definition of kite and complete bipartite graphs. □

3.2. Sufficient conditions

In this section, we have obtained some sufficient conditions for the existence of a multi-decomposition of K_n into α copies of kites $K_3 + e$ and β copies of stars S_4.
Next, we prove two lemmas that will be used to prove our main result.

Lemma 5. *The graph* K_8 *admits a* $\{(K_3+e)^\alpha, S_4^\beta\}$ *- decomposition if and only if* $\alpha + \beta = 7$, *where* $\alpha, \beta \geq 0$.

Proof: Necessity follows from Theorem 3 To prove the converse, let us assume that $V(K_8) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$. Now we can show that K_8 can be decomposed into α copies of $K_3 + e$ and β copies of S_4 for each pairs (α, β) of non-negative integers such that $\alpha + \beta = 7$. We have the following 8 cases in which the multi-decomposition of K_8 into α kites and β stars are exhibited for the all values of $\alpha, \beta \geq 0$.

Case (i). Let $\alpha = 0$ and $\beta = 7$. Hence, we have a decomposition of K_8 into 7 copies of stars:
$E(K_8) = S(v_1; v_2, v_3, v_4, v_8) \oplus S(v_2; v_3, v_4, v_5, v_8) \oplus S(v_3; v_4, v_5, v_6, v_8) \oplus$
$S(v_4; v_5, v_6, v_7, v_8) \oplus S(v_5; v_1, v_6, v_7, v_8) \oplus S(v_6; v_1, v_2, v_7, v_8) \oplus$
$S(v_7; v_1, v_2, v_3, v_8)$
.

Case (ii). Let $\alpha = 1$ and $\beta = 6$. Here, we have one kite and 6 stars in the multi-decomposition of K_8. Therefore,
$E(K_8) = [(v_5, v_7, v_8); v_4v_7] \oplus S(v_1; v_5, v_6, v_7, v_8) \oplus S(v_2; v_1, v_6, v_7, v_8) \oplus S(v_3; v_1, v_2, v_7, v_8) \oplus S(v_4; v_1, v_2, v_3, v_8) \oplus S(v_5; v_2, v_3, v_4, v_6) \oplus S(v_6; v_2, v_3, v_7, v_8)$
.

Case (iii). Let $\alpha = 2$ and $\beta = 5$. Then K_8 has a $((K_3 + e)^2, S_4^5)$-decomposition as given below:
$E(K_8) = [(v_4, v_6, v_7); v_3v_6] \oplus [(v_5, v_6, v_8); v_7v_8] \oplus S(v_1; v_5, v_6, v_7, v_8) \oplus S(v_2; v_1, v_6, v_7, v_8) \oplus S(v_3; v_1, v_2, v_7, v_8) \oplus S(v_4; v_1, v_2, v_3, v_8) \oplus S(v_5; v_2, v_3, v_4, v_6)$
.

Case (iv). For $\alpha = 3$ and $\beta = 4$.
$E(K_8) = [(v_5, v_7, v_8); v_2v_5] \oplus [(v_3, v_5, v_6); v_4v_5] \oplus [(v_4, v_6, v_7); v_7v_8] \oplus S(v_1; v_5, v_6, v_7, v_8) \oplus S(v_2; v_1, v_6, v_7, v_8) \oplus S(v_3; v_1, v_2, v_7, v_8) \oplus S(v_4; v_1, v_2, v_3, v_8)$
.

Case (v). For $\alpha = 4$ and $\beta = 3$.
$E(K_8) = [(v_2, v_4, v_5); v_1v_4] \oplus [(v_3, v_5, v_6); v_3v_4] \oplus [(v_5, v_7, v_8); v_6v_8] \oplus [(v_4, v_6, v_7); v_4v_8] \oplus S(v_1; v_5, v_6, v_7, v_8) \oplus S(v_2; v_1, v_6, v_7, v_8) \oplus S(v_3; v_1, v_2, v_7, v_8)$
.

Case (vi). For $\alpha = 5$ and $\beta = 2$.
$E(K_8) = [(v_3, v_5, v_6); v_2v_3] \oplus [(v_3, v_7, v_8); v_5v_7] \oplus [(v_4, v_6, v_7); v_6v_8] \oplus [(v_1, v_3, v_4); v_4v_8] \oplus [(v_2, v_4, v_5); v_5v_8] \oplus S(v_1; v_5, v_6, v_7, v_8) \oplus S(v_2; v_1, v_6, v_7, v_8)$
.

Case (vii). For $\alpha = 6$ and $\beta = 1$.
$E(K_8) = [(v_1, v_2, v_4); v_1v_3] \oplus [(v_5, v_7, v_8); v_3v_8] \oplus [(v_3, v_4, v_5); v_3v_6] \oplus [(v_2, v_3, v_7); v_2v_8] \oplus [(v_2, v_5, v_6); v_6v_8] \oplus [(v_4, v_6, v_7); v_4v_8] \oplus S(v_1; v_5, v_6, v_7, v_8)$
.

Case (viii). For $\alpha = 7$ and $\beta = 0$.
$E(K_8) = [(v_1, v_2, v_7); v_5v_7] \oplus [(v_1, v_3, v_4); v_1v_5] \oplus [(v_1, v_6, v_8); v_5v_8] \oplus [(v_2, v_4, v_5); v_2v_6] \oplus [(v_2, v_3, v_8); v_7v_8] \oplus [(v_3, v_5, v_6); v_3v_7]$
, $[(v_4, v_6, v_7); v_4v_8]$

Thus, there exists a $\{(K_3 + e)^{\alpha}, S_4^{\beta}\}$ - decomposition of K_8 whenever $\alpha + \beta = 7$ for all pairs (α, β) where $\alpha, \beta \geq 0$. □

Illustration 1. *For the corresponding case $\alpha = 0$ and $\beta = 7$ in K_8, we have the following MATLAB coding:*

Coding:

```
>> s=[1 1 1 1 1 1 1 2 2 2 2 2 2 3 3 3 3 3 4 4 4 4 5 5 5 6 6 7];
>>t=[2 3 4 5 6 7 8 3 4 5 6 7 8 4 5 6 7 8 5 6 7 8 6 7 8 7 8 8];
>>G=graph(s,t);
>>p=plot(G)
>> highlight(p,[1 1 1 1],[2 3 4 8],'EdgeColor','r','LineWidth',2)
>> highlight(p,[2 2 2 2],[3 4 5 8],'EdgeColor','g','LineWidth',2)
>> highlight(p,[3 3 3 3],[4 5 6 8],'EdgeColor','m','LineWidth',2)
>> highlight(p,[4 4 4 4],[5 6 7 8],'EdgeColor','y','LineWidth',2)
>> highlight(p,[5 5 5 5],[1 6 7 8],'EdgeColor','b','LineWidth',2)
>> highlight(p,[6 6 6 6],[1 2 7 8],'EdgeColor','c','LineWidth',2)
>> highlight(p,[7 7 7 7],[1 2 3 8],'EdgeColor','k','LineWidth',2)
```

Output:

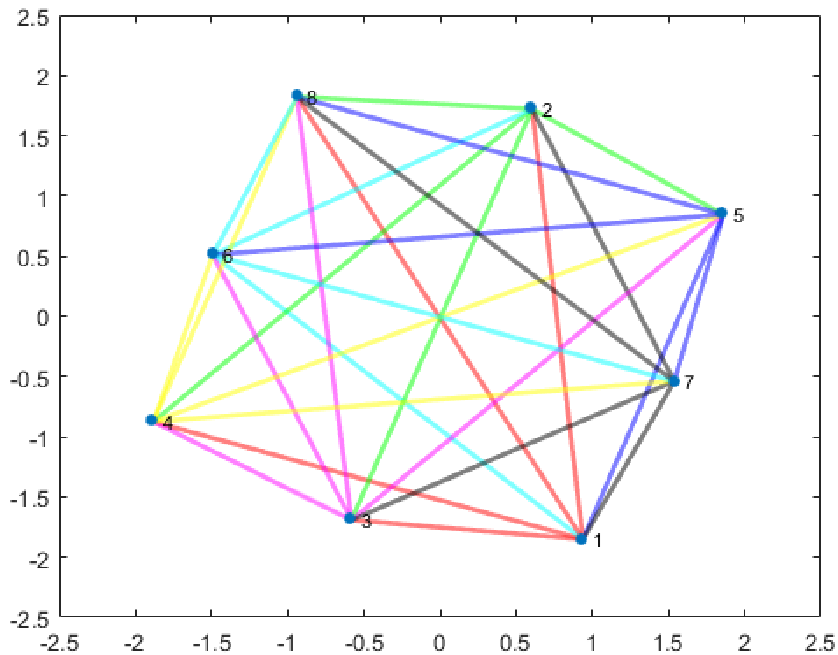

Figure1:

The above MATLAB picture shows 7 copies of stars of complete graph on 8 vertices. Each Color represents star on 4 edges.

Lemma 6. *The graph K_9 admits a $\{(K_3+e)^{\alpha}, S_4^{\beta}\}$ - decomposition if and only if $\alpha+\beta=9$, where $\alpha,\beta\geq 0$.*

Proof: Necessity follows from Theorem 3. Let $V(K_9)=\{v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9\}$. Now we exhibit the K_9 into α copies of K_3+e and β copies of S_4 for all pairs (α,β) of non-negative integers such that $\alpha+\beta=9$ in the following 10 cases.

Case (i). Let $\alpha = 0$ and $\beta = 9$. Hence, we have only 9 S_4's in the decomposition of K_9. Thus,
$E(K_9) = S(v_1; v_2, v_3, v_4, v_5) \oplus S(v_2; v_3, v_4, v_5, v_6) \oplus S(v_3; v_4, v_5, v_6, v_7) \oplus$
$S(v_4; v_5, v_6, v_7, v_8) \oplus S(v_5; v_6, v_7, v_8, v_9) \oplus S(v_6; v_1, v_7, v_8, v_9) \oplus$
$S(v_7; v_1, v_2, v_8, v_9) \oplus S(v_8; v_1, v_2, v_3, v_9) \oplus S(v_9; v_1, v_2, v_8, v_4)$

Case (ii). Let $\alpha = 1$ and $\beta = 8$. Hence, we have one kite and 8 S_4's in the decomposition of K_9. Hence,
$E(K_9) = [(v_6, v_7, v_9); v_5v_9] \oplus S(v_1; v_6, v_7, v_8, v_9) \oplus S(v_2; v_1, v_7, v_8, v_9) \oplus$
$S(v_3; v_1, v_2, v_8, v_9) \oplus S(v_4; v_1, v_2, v_3, v_9) \oplus S(v_5; v_1, v_2, v_3, v_4) \oplus$
$S(v_6; v_2, v_3, v_4, v_5) \oplus S(v_7; v_3, v_4, v_5, v_8) \oplus S(v_8; v_4, v_5, v_6, v_9)$

Case (iii). Let $\alpha = 2$ and $\beta = 7$. Then,
$E(K_9) = [(v_5, v_7, v_8); v_4v_8] \oplus [(v_6, v_8, v_9); v_5v_9] \oplus S(v_1; v_6, v_7, v_8, v_9) \oplus$
$S(v_2; v_1, v_7, v_8, v_9) \oplus S(v_3; v_1, v_2, v_8, v_9) \oplus S(v_4; v_1, v_2, v_3, v_9) \oplus$
$S(v_5; v_1, v_2, v_3, v_4) \oplus S(v_6; v_2, v_3, v_4, v_5) \oplus S(v_7; v_3, v_4, v_6, v_9)$

Case (iv). Let $\alpha = 3$ and $\beta = 6$. Now,
$E(K_9) = [(v_3, v_6, v_7); v_4v_8] \oplus [(v_6, v_8, v_9); v_5v_9] \oplus [(v_5, v_7, v_8); v_7v_9] \oplus$
$S(v_1; v_4, v_7, v_8, v_9) \oplus S(v_2; v_1, v_7, v_8, v_9) \oplus S(v_3; v_1, v_2, v_8, v_9) \oplus$
$S(v_4; v_2, v_3, v_8, v_9) \oplus S(v_5; v_1, v_2, v_3, v_4) \oplus S(v_6; v_2, v_3, v_4, v_5)$

Case (v). Let $\alpha = 4$ and $\beta = 5$. Here,
$E(K_9) = [(v_4, v_6, v_7); v_3v_4] \oplus [(v_6, v_8, v_9); v_3v_6] \oplus [(v_1, v_5, v_9); v_3v_9] \oplus$
$[(v_5, v_7, v_8); v_7v_9] \oplus S(v_1; v_2, v_6, v_7, v_8) \oplus S(v_2; v_6, v_7, v_8, v_9) \oplus$
$S(v_3; v_1, v_2, v_7, v_8) \oplus S(v_4; v_1, v_2, v_8, v_9) \oplus S(v_5; v_2, v_3, v_4, v_6)$

Case (vi). Let $\alpha = 5$ and $\beta = 4$. Then,
$E(K_9) = [(v_3, v_5, v_6); v_2v_3] \oplus [(v_1, v_5, v_9); v_2v_5] \oplus [(v_4, v_6, v_7); v_3v_4] \oplus$
$[(v_5, v_7, v_8); v_4v_5] \oplus [(v_6, v_8, v_9); v_7v_9] \oplus S(v_1; v_2, v_6, v_7, v_8) \oplus$
$S(v_2; v_6, v_7, v_8, v_9) \oplus S(v_3; v_1, v_7, v_8, v_9) \oplus S(v_4; v_1, v_2, v_8, v_9)$

Case (vii). Let $\alpha = 6$ and $\beta = 3$. Where,
$E(K_9) = [(v_4, v_8, v_9); v_1v_4] \oplus [(v_2, v_4, v_5); v_2v_3] \oplus [(v_3, v_5, v_6); v_3v_4] \oplus$
$[(v_1, v_5, v_9); v_6v_9] \oplus [(v_4, v_6, v_7); v_6v_8] \oplus [(v_5, v_7, v_8); v_7v_9] \oplus$
$S(v_1; v_2, v_6, v_7, v_8) \oplus S(v_2; v_6, v_7, v_8, v_9) \oplus S(v_3; v_1, v_7, v_8, v_9)$

Case (viii). Let $\alpha = 7$ and $\beta = 2$. Now,
$E(K_9) = [(v_3, v_5, v_6); v_2v_3] \oplus [(v_3, v_8, v_9); v_6v_8] \oplus [(v_1, v_5, v_9); v_6v_9] \oplus$
$[(v_1, v_3, v_4); v_3v_7] \oplus [(v_4, v_6, v_7); v_4v_8] \oplus [(v_5, v_7, v_8); v_7v_9] \oplus$
$[(v_2, v_4, v_5); v_4v_9] \oplus S(v_1; v_2, v_6, v_7, v_8) \oplus S(v_2; v_6, v_7, v_8, v_9)$

Case (ix). Let $\alpha = 8$ and $\beta = 1$. Here,
$E(K_9) = [(v_5, v_7, v_8); v_2v_8] \oplus [(v_4, v_6, v_7); v_3v_7] \oplus [(v_2, v_3, v_9); v_2v_6] \oplus$
$[(v_1, v_5, v_9); v_6v_9] \oplus [(v_2, v_4, v_5); v_2v_7] \oplus [(v_4, v_8, v_9); v_7v_9] \oplus$
$[(v_3, v_5, v_6); v_6v_8] \oplus [(v_1, v_3, v_4); v_3v_8] \oplus S(v_1; v_2, v_6, v_7, v_8)$

Case (x). Let $\alpha = 9$ and $\beta = 0$. Hence,
$E(K_9) = [(v_2, v_3, v_8); v_1v_8] \oplus [(v_6, v_8, v_9); v_1v_6] \oplus [(v_1, v_7, v_9); v_2v_7] \oplus$
$[(v_1, v_2, v_8); v_3v_8] \oplus [(v_1, v_3, v_4); v_1v_5] \oplus [(v_2, v_4, v_5); v_2v_6] \oplus$
$[(v_3, v_5, v_6); v_3v_7] \oplus [(v_4, v_6, v_7); v_4v_8] \oplus [(v_5, v_7, v_8); v_5v_9]$

Thus, there exists a $\{(K_3 + e)^\alpha, S_4^\beta\}$ - decomposition of K_9 whenever $\alpha + \beta = 9$ for all pairs (α, β) where $\alpha, \beta \geq 0$. □

Lemma 7. *If α and β are non-negative integers such that $4(\alpha + \beta) = \binom{16}{2}$, then K_{16} admits a $\{(K_3 + e)^\alpha, S_4^\beta\}$-decomposition.*

Proof. The proof is similar to Lemma 2. In Lemma 2, K_{16} is viewed as an edge-disjoint union of K_8, K_9 and $K_{7,8}$. From Lemma 5, K_8 can be decomposed into α copies of $K_3 + e$ and β copies of S_4 for each pair (α, β) of non-negative integers such that $\alpha + \beta = 7$. By Lemma 6, K_9 can be decomposed into α copies of $K_3 + e$ and β copies of S_4 for each pair (α, β) of non-negative integers such that $\alpha + \beta = 9$. Finally from Theorem 1, $K_{7,8}$ can be decomposed into β copies of S_4 such that $4\beta = |E(K_{7,8})| = 56$ i.e., $\beta = 14$. From these results, it is clear that that K_{16} can be decomposed into α copies of $K_3 + e$ and β copies of S_4 for the pairs (α, β) satisfying $0 \leq \alpha \leq 16$ and $\beta = 30 - \alpha$. □

Lemma 8. *If α and β are non-negative integers such that $4(\alpha + \beta) = \binom{17}{2}$, then K_{17} admits a $\{(K_3 + e)^\alpha, S_4^\beta\}$-decomposition.*

Proof: The graph K_{17} is viewed as an edge-disjoint union of K_8, K_9 and $K_{9,8}$. From Lemma 5, $\{(K_3 + e)^\alpha, S_4^\beta\}|K_8$ for all pairs of non-negative integers (α, β) such that $\alpha + \beta = 7$. Also from Lemma 6, $\{(K_3 + e)^\alpha, S_4^\beta\}|K_9$, if $\alpha + \beta = 9$, $\alpha, \beta \geq 0$. From Theorem 1, $K_{9,8}$ can be decomposed into 18 copies of S_4 such that $4\beta = |E(K_{9,8})| = 72$. From these results, it is clear that that $\{(K_3 + e)^\alpha, S_4^\beta\}|K_{17}$ for the pairs (α, β) satisfying $0 \leq \alpha \leq 16$ and $\beta = 34 - \alpha$. □

Remark 9. *Let $n = 4t$, where $t \geq 4$ be even. Then, from Lemma 7,*

$$K_{16} = K_8 \oplus K_9 \oplus K_{7,8}$$

Similarly K_{24} can be written as, $K_{24} = K_{16} \oplus K_9 \oplus K_{15,8}$ and

$$K_{32} = K_{24} \oplus K_9 \oplus K_{23,8}$$

In general, $K_{4t} = K_{4(t-2)} \oplus K_9 \oplus K_{4(t-2)-1,8}$

Similarly when $n = 4t + 1$, for even t. Then from Lemma 8, K_{17} can be written as,
$K_{17} = K_8 \oplus K_9 \oplus K_{9,8}$ *Similarly* K_{25} *and* K_{33} *can be written as,*
$K_{25} = K_{16} \oplus K_9 \oplus K_{18,8}$ *and*

$$K_{33} = K_{24} \oplus K_9 \oplus K_{27,8}$$

In general, $K_{4t+1} = K_{4(t-2)} \oplus K_9 \oplus K_{4(t-2)+\frac{1}{2}(t-2),8}$

Theorem 10. *For all even $t \geq 4$, if $n = 4t$ or $n = 4t + 1$, then K_n admits a $\{(K_3 + e)^\alpha, S_4^\beta\}$ - decomposition for the pairs (α, β) satifying $0 \leq \alpha \leq \frac{9t-4}{2}$, $\beta = \frac{1}{4}\binom{n}{2} - \alpha$.*

Proof: Let $n = 4t$. We prove this theorem by induction on t. When $t = 4$, we get $n = 16$. By Lemma 7, the graph K_{16} admits $\{(K_3 + e)^\alpha, S_4^\beta\}$ decomposition for the pairs (α, β) satisfying $0 \leq \alpha \leq 16$ such that $\alpha + \beta = 30$. Hence, the theorem is true for $t = 4$. Assume that the theorem is true for all even integers $t > 0$. We shall prove the theorem for t. By our assumption, $K_{4(t-2)} = K_{4(t-4)} \oplus K_9 \oplus K_{4(t-8),8}$. Now, $K_{4t} = K_{4(t-2)} \oplus K_9 \oplus K_{4(t-2)-1,8}$, from Remark 9. Hence, the proof follows from our assumption, Lemma 6, Lemma 7 and Theorem 1. Similarly for $n = 4t + 1$, we can prove the result from Lemma 6, Lemma 8 and Remark 9. □

4. Future work and applications

The topic of kite and star multidecomposition is a significant area in Graph Theory with various uses in cryptography, encryptions, and decryption processes. With a secure communication network system, complicated networks can be modeled using graphs like the kite and the star graph to effectively manage the flow of data. Star multidecomposition is relevant in a centralized security model that requires a server to send encryption keys to many people. Kite multidecomposition is beneficial as it helps add links that facilitate secure data transfer in a network system. The graph decomposition technique makes data encryption fast and effective by splitting big data into small chunks, which are processed simultaneously. After decrypting the data, the decomposed data is put together to reconstruct the original data. Moreover, the kite and star multidecomposition increases fault tolerance, computational efficiency, and enhances the resistance against cyber attacks.

4. Conclusion

In this paper, it is proved that the necessary condition $4(\alpha + \beta) = \binom{n}{2}$ is also sufficient for the existence of $\{(K_3 + e)^\alpha, S_4^\beta\}$-decomposition in K_n if $n = 4t$ or $n = 4t + 1$ for all even t, for the pairs (α, β) satisfying $0 \leq \alpha \leq \frac{9t-4}{2}$, $\frac{1}{4}\binom{n}{2} - \alpha = \beta$. Also, simple MATLAB coding is used to visualize the decomposition of graphs.

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