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Degree Ratio Nirmala Index of Certain Networks

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Abstract. In this study, we introduce the degree ratio Nirmala and the modified degree ratio Nirmala indices of a graph. Furthermore, we compute these degree ratios and Nirmala indices of certain networks.

Keywords: degree ratio, Nirmala index, modified degree ratio, Nirmala index, network.

AMS Mathematics Subject Classification (2010): 05*C*10, 05*C*69

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer to [1] for other undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [2].

We define the degree ratio Nirmala index of a graph as

$$DRN(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)}{d_G(v)} + \frac{d_G(v)}{d_G(u)}}.$$

Recently, some Nirmala indices were studied in [3, 4, 5, 6, 7, 8, 9].

We define the modified degree ratio Nirmala index of a graph G as

$${}^{m}DRN(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)d_G(v)}{d_G(u)^2 + d_G(v)^2}}.$$

In this paper, the degree ratio Nirmala and the modified degree ratio Nirmala indices for silicate, oxide, and honeycomb networks are computed.

2. Results for silicate networks

Silicates are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is symbolized by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A 2-dimensional silicate network is presented in Figure 1.

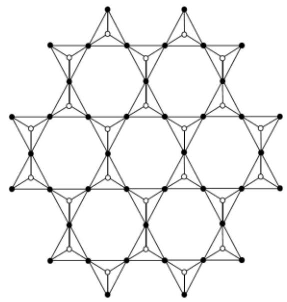

Figure 1: A 2-dimensional silicate network

Let G be the graph of a silicate network SL_n. The graph G has $15n^2 + 3n$ vertices and $36n^2$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 6n$.

$E_{36} = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_{36}| = 18n^2 + 6n$.

$E_{66} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_{66}| = 18n^2 - 12n$.

We compute the degree ratio Nirmala index of SL_n.

Theorem 1. The degree ratio Nirmala index of a silicate network SL_n is

$$DRN(SL_n) = \left(18\sqrt{\frac{5}{2}} + 18\sqrt{2}\right)n^2 + \left(6\sqrt{\frac{5}{2}} - 6\sqrt{2}\right)n.$$

Proof: Let G be the graph of a silicate network SL_n. We obtain

$$DRN(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)}{d_G(v)} + \frac{d_G(v)}{d_G(u)}}$$

$$= 6n\sqrt{\frac{3}{3} + \frac{3}{3}} + \left(18n^2 + 6n\right)\sqrt{\frac{3}{6} + \frac{6}{3}} + \left(18n^2 - 12n\right)\sqrt{\frac{6}{6} + \frac{6}{6}}$$

$$= \left(18\sqrt{\frac{5}{2}} + 18\sqrt{2}\right)n^2 + \left(6\sqrt{\frac{5}{2}} - 6\sqrt{2}\right)n.$$

We compute the modified degree ratio Nirmala index of SL_n.

Theorem 2. The modified degree ratio Nirmala index of a silicate network SL_n is

$$^{m}DRN(SL_n) = \left(18\sqrt{\frac{18}{35}} + 18\sqrt{\frac{1}{2}}\right)n^2 + \left(6\sqrt{\frac{18}{35}} - 6\sqrt{\frac{1}{2}}\right)n.$$

Proof: Let G be the graph of a silicate network SL_n. We obtain

$$^{m}DRN(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)d_G(v)}{d_G(u)^2 + d_G(v)^2}}$$

$$= 6n\sqrt{\frac{3\times 3}{3^2+3^2}} + \left(18n^2+6n\right)\sqrt{\frac{3\times 6}{3^2+6^2}} + \left(18n^2 - 12n\right)\sqrt{\frac{6\times 6}{6^2+6^2}}$$

$$= \left(18\sqrt{\frac{18}{35}} + 18\sqrt{\frac{1}{2}}\right)n^2 + \left(6\sqrt{\frac{18}{35}} - 6\sqrt{\frac{1}{2}}\right)n.$$

3. Results for oxide networks

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by OX_n. A 5-dimensional oxide network is shown in Figure 2.

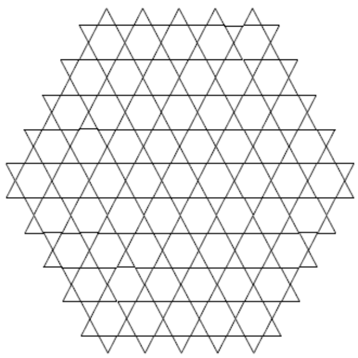

Figure 2: Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. By calculation, we obtain that G has $9n^2+3n$ vertices and $18n^2$ edges. In G, by algebraic method, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_{24} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}$, $\quad |E_{24}| = 12n$.

$E_{44} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $\quad |E_{44}| = 18n^2 - 12n$.

We compute the degree ratio Nirmala index of OX_n.

Theorem 3. The degree ratio Nirmala index of an oxide network OX_n is

$$DRN\left(OX_n\right) = 18\sqrt{2}n^2 + \left(12\sqrt{\frac{5}{2}} - 12\sqrt{2}\right)n.$$

Proof: Let G be the graph of an oxide network OX_n. We obtain

$$DRN(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)}{d_G(v)} + \frac{d_G(v)}{d_G(u)}}$$

$$= 12n\sqrt{\frac{2}{4}+\frac{4}{2}} + \left(18n^2 - 12n\right)\sqrt{\frac{4}{4}+\frac{4}{4}}$$

$$= 18\sqrt{2}n^2 + \left(12\sqrt{\frac{5}{2}} - 12\sqrt{2}\right)n.$$

We compute the modified degree ratio Nirmala index of OX_n.

Theorem 4. The modified degree ratio Nirmala index of an oxide network OX_n is

$$^{m}DRN(OX_n) = 18\sqrt{\frac{1}{2}}n^2 + \left(12\sqrt{\frac{2}{5}} - 12\sqrt{\frac{1}{2}}\right)n.$$

Proof: Let G be the graph of an oxide network OX_n. We obtain

$$^{m}DRN(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u)d_G(v)}{d_G(u)^2 + d_G(v)^2}}$$

$$= 12n\sqrt{\frac{2\times 4}{2^2+4^2}} + \left(18n^2 - 12n\right)\sqrt{\frac{4\times 4}{4^2+4^2}}$$

$$= 18\sqrt{\frac{1}{2}}n^2 + \left(12\sqrt{\frac{2}{5}} - 12\sqrt{\frac{1}{2}}\right)n.$$

4. Results for honeycomb networks

Honeycomb networks are useful in Computer Graphics and Chemistry. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 3.

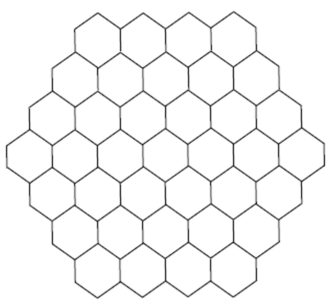

Figure 3: A 4-dimensional honeycomb network

Let G be the graph of a honeycomb network HC_n. By calculation, we obtain that G has $6n^2$ vertices and $9n^2–3n$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_{22} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_{22}| = 6$.
$E_{23} = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_{23}| = 12n – 12$.
$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_{33}| = 9n^2 – 15n + 6$.

We compute the degree ratio Nirmala index of HC_n.

Theorem 5. The degree ratio Nirmala index of a honeycomb network HC_n is

$$DRN(HC_n) = 9\sqrt{2}n^2 + \left(12\sqrt{\frac{13}{6}} - 15\sqrt{2}\right)n + 12\sqrt{2} - 12\sqrt{\frac{13}{6}}.$$

Proof: Let G be the graph of a honeycomb network HC_n. We obtain

$$DRN(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)}{d_G(v)}+\frac{d_G(v)}{d_G(u)}}$$

$$=6\sqrt{\frac{2}{2}+\frac{2}{2}}+\left(12n-12\right)\sqrt{\frac{2}{3}+\frac{3}{2}}+\left(9n^2-15n+6\right)\sqrt{\frac{3}{3}+\frac{3}{3}}$$

$$=9\sqrt{2}n^2+\left(12\sqrt{\frac{13}{6}}-15\sqrt{2}\right)n+12\sqrt{2}-12\sqrt{\frac{13}{6}}.$$

We compute the modified degree ratio Nirmala index of HC_n.

Theorem 6. The modified degree ratio Nirmala index of an oxide network HC_n is

$$^{m}DRN\left(HC_n\right)=9\sqrt{\frac{1}{2}}n^2+\left(12\sqrt{\frac{6}{13}}-15\sqrt{\frac{1}{2}}\right)n+12\sqrt{\frac{1}{2}}-12\sqrt{\frac{6}{13}}.$$

Proof: Let G be the graph of an oxide network HC_n. We obtain

$$^{m}DRN(G)=\sum_{uv\in E(G)}\sqrt{\frac{d_G(u)d_G(v)}{d_G(u)^2+d_G(v)^2}}$$

$$=6\sqrt{\frac{2\times 2}{2^2+2^2}}+\left(12n-12\right)\sqrt{\frac{2\times 3}{2^2+3^2}}+\left(9n^2-15n+6\right)\sqrt{\frac{3\times 3}{3^2+3^2}}$$

$$=9\sqrt{\frac{1}{2}}n^2+\left(12\sqrt{\frac{6}{13}}-15\sqrt{\frac{1}{2}}\right)n+12\sqrt{\frac{1}{2}}-12\sqrt{\frac{6}{13}}.$$

In this paper, the degree ratio Nirmala and modified degree ratio Nirmala indices of a graph are defined. Also, the degree ratio Nirmala and modified degree ratio Nirmala indices of certain networks are determined.

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Author's Contribution: The author is solely responsible for the conceptualization, methodology, analysis, and preparation of the manuscript.

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