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Using Intuitionistic Fuzzy Success Tree Analysis for Car Engine Reliability System (IFST)

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Abstract. In this research, we will study the most important electrical faults in gasoline-powered car engines, we will try to limit these faults as much as possible, which lead to the engine not working properly or it breaks down and stops working, whether at the beginning of operation or during work and operation, Then we present some important definitions used in this, with the help of previous research that addressed this problem and Citing the findings of these studies, we identify the most important major and minor faults, which is of great importance in identifying the components of the failure tree and also the success tree, which is the focus of our current research, where we calculate the reliability of these faults using the failure tree sometimes and the success tree other times, so that in our work we use triangular Intuitionistic Fuzzy number to Fault Tree Analysis of car engine safety system as an application.

Keywords: Reliability, Fault tree, Success tree, fuzzy set, intuitionistic**,** cut set, path set, minimal cut set.

AMS Mathematics Subject Classification (2010): 94D05

1. Introduction

For many years, many researchers have studied the concept of fuzzy set theory which is produced by Zadeh [1] in 1965 since various problems in our real life contains different kinds of uncertainty and ambiguity Zadeh’s thoughts concentrated on the idea of grade of degree of membership which is represent the backbone concept of fuzzy set theory, later Atanassov [2] presented Intuitionistic Fuzzy sets and Keiser[3] studied Local Area Networks, also Masuli, Mitra and Pasi [10] study applications of fuzzy sets theory also Gau and Buehrer [4] study Vague Sets and Chen and Pham [7] studied fuzzy sets, fuzzy logic, and fuzzy control system, [6] discussion history of fault tree, after that Lata [15] studied analysis of fuzzy fault tree using intuitionstic fuzzy numbers then Mahapatra, and Roy [13] studied reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations and Subrie, Musleh introduced [18] evaluating fuzzy reliability system using intuitionistic fuzzy sets. Recently, Salman [19] introduced the success trees technique to evaluate system reliability in design. In this study, we will use the theory of fuzziness and the concept of intuition to calculate reliability using a success tree for electrical faults in gasoline-powered car engines.

We must mention here some of the valuable contributions made by other researchers, such as Mitra, Datta and Chanda [15], who introduced. On some exact solutions for a generalised form of Yang's Euclidean R-Gauge equations and their relation with Painleve 's property, Arunkumar, Agilan and Ramamoorthy [16]. Solution and generalised Ulam-Hyers stability of an n-dimensional additive functional equation in Banach space and Banach algebra.

2. Preliminaries

Some definitions and concepts**:**

Definition 1.1. [6] Suppose that R be any set and B is a subset of R, now the connection between an element $x \in R$ and set B is either $x\in$ B or $x\notin$ B, Such a fact can be substantive or indicated by the function below :

$$\mu_B(x) = \begin{cases} 1 & if \;\; x \in B \\ 0 & if \;\; x \notin B \end{cases}$$

Definition 1.2. [5] Let a set X be fixed.An intuitionistic fuzzy set (shortly IFS) $\tilde{A}$ in X is an object having the form

$\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \rho_{\tilde{A}}(x) >: x \in X\}$

where $\mu_{\tilde{A}}(x): X \to [0,1]$ and $\rho_{\tilde{A}}(x): X \to [0,1]$ represent the stage of membership and the stage of non-membership respectively of the element $x \in X$ to the set $\tilde{A}, \tilde{A} \subseteq X, \forall x \in X, 0 \le \mu_{\tilde{A}}(x) + \rho_{\tilde{A}}(x) \le 1$

Definition 1.3. **[2]** An intuitionistic fuzzy number(briefly IFN) $\tilde{A}$ is recognized as below :

i. Intuitionistic fuzzy subset of the real line.
ii. $Normal: \; there \; is \; any \; x_\circ \in R \ni \mu_{\tilde{A}} \; (x_\circ) = 1, \nu_{\tilde{A}} \; (x_\circ) = 0$
iii. The membership function $\mu_{\tilde{A}} \; (x_\circ)$ is convex
iv. The non-membership function $\nu_{\tilde{A}} \; (x_\circ)$ is concave

Definition 1.4. [15] T$riangula$r Int$uition$istic fuzzy numbers (TIFN):

The TIFN $\tilde{A}$ is an intuitionistic fuzzy set in R with five real numbers $(\alpha_1, \alpha_2, \alpha_3, \alpha', \alpha'')$ (i.e. TFFN) with $(\alpha' \le \alpha_1 \le \alpha_2 \le \alpha_3 \le \alpha'')$ and the membership function and non-membership function two triangular functions:

$$\mu_{\tilde{A}} \; (x) = \begin{cases} \dfrac{x - \alpha_1}{\alpha_2 - \alpha_1} & \alpha_1 \le x \le \alpha_2 \\ \dfrac{\alpha_3 - x}{\alpha_3 - \alpha_2} & \alpha_2 \le x \le \alpha_3 \\ 0 & elsewhere \end{cases}$$

And

$$\nu_{\tilde{A}}(x) = \begin{cases} \dfrac{\alpha_2 - x}{\alpha_2 - \alpha'} & \alpha' \le x \le \alpha_2 \\ \dfrac{x - \alpha_2}{\alpha'' - \alpha_2} & \alpha_2 \le x \le \alpha'' \\ 1 & elsewhere \end{cases}$$

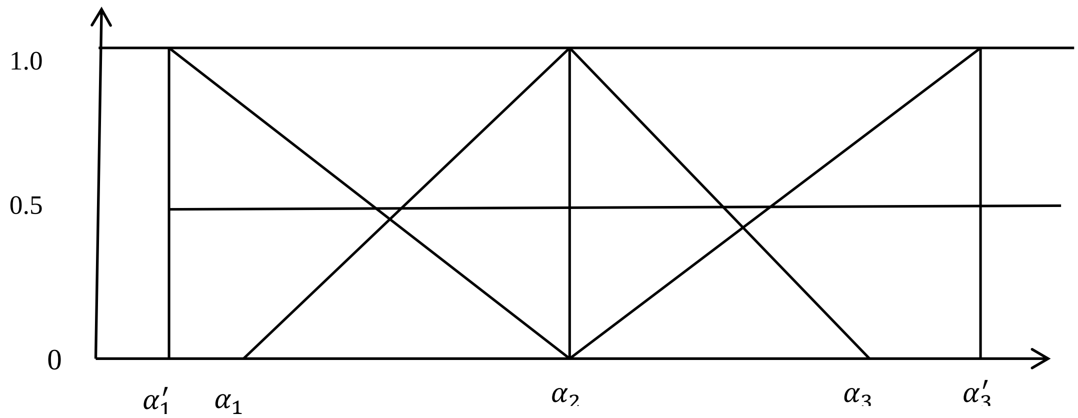

Figure 1: Shows Membership function $M_A(x)$ and non-membership function $V_A(x)$ of TIFN.

Intuitionistic Fuzzy mathematics has the ability to deal with like cases. For operations of addition and multiplication, see [16]. Noted that we deal with component rates as TIFN, for the sake of the analysis of system reliability, to analyse (TFFT) in our study, a numerical solution of the failure of the car engine system.

Definition 1.5. [9] A cut set is a combination of vehicles such that if the parts are separated from the framework, then there is no other path than another vehicle.

Definition 1.6. [19] A fault tree is a graphical representation of a system failure that breaks down a system hazard backwards to its possible causes.

Definition 1.7. [9] A framework is the total construction that is conceived, which, in function, consists of subordinate constructions called subsystems, which are called vehicles.

Definition 1.8. [19] Success Trees (ST) is a path collection of an event or summation of events whose nonoccurrence ensures the non-occurrence of top–level. A path set is smaller if it is inestimable, becomes even more miniature, and still remains a path set. For more details, see [5], FT and ST hold the relation

$$\mathrm{St} = 1 - \mathrm{FT} \tag{1}$$

Definition 1.9. [17] Safety is defined as those methods that lead to avoiding failure or avoiding making any of those mistakes that cause danger to the lives of workers and those around the system. Which negatively affects human lives, negatively affects the surrounding environment, or leads to losses in property or equipment.

3. Fault tree analysis

In the quantitative analysis of an intuitionistic fuzzy fault tree, this fuzzy information about singular vehicles has been included within the tree at the lower pyramidal level and combined, utilising the reasoning of the tree with intuitionistic Fuzzy mathematics to allow the failure estimation of the total framework under study.

The fault tree of the gas engine safety system, which is shown in the figure below, is taken for analysis. The failure of the gas engine safety system consists of distinct factors, such as human errors, car battery failure, Ignition system (spark) failure, and Electronic systems failure.

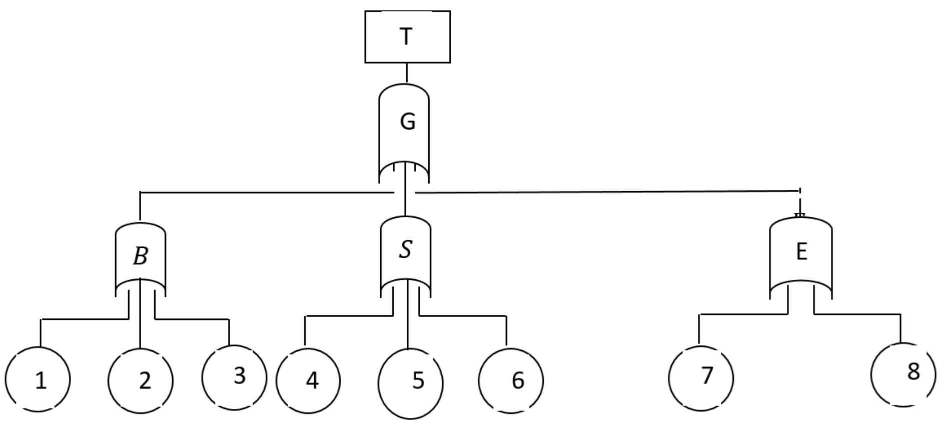

Figure 2: fault tree of a system in example (1)

The system failure (top event) T is:

$T = B \cup S \cup E$

where $B = 1 \cup 2 \cup 3, S = 4 \cup 5 \cup 6$ *and* $E = 7 \cup 8$,

$T = B \cup S \cup E = 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 7 \cup 8$

3.1. Three major factors are

The car engine system failure each of them has three-sub factors, Battery problems, including battery drain, weak charging, and battery damage, where Charging system faults, such as a faulty alternator or charging cables, Ignition system malfunctions, such as problems with the spark plugs, cables, or ignition control module, Electronic system faults such as faulty electronic control units and sensor problems.

2.2. The intuitionistic fuzzy failure

The intuitionistic fuzzy failure of the framework It could be evaluated where the failures for a basic fault events appearance are known, the failure of the safety system is evaluated as follows:

$\tilde{f}_{CE} = 1 \ominus (1 \ominus \tilde{f}_B)(1 \ominus \tilde{f}_S)(1 \ominus \tilde{f}_E)$
where $\tilde{f}_B = 1 \ominus \left(1 \ominus \tilde{f}_1\right)\left(1 \ominus \tilde{f}_2\right)\left(1 \ominus \tilde{f}_3\right)$,
$\tilde{f}_S = 1 \ominus (1 \ominus \tilde{f}_4)(1 \ominus \tilde{f}_5)(1 \ominus \tilde{f}_6)$,
$\tilde{f}_{CG} = 1 \ominus (1 \ominus \tilde{f}_7)(1 \ominus \tilde{f}_8)$

where:
$\tilde{f}_{CE}$ represent the failure of car engine safety system.
$\tilde{f}_B$ represent the failure of Battery system.
$\tilde{f}_E$ represent the failure of Electronic system.
$\tilde{f}_S$ represent the failure of Ignition system.
$\tilde{f}_1$ represent the failure of battery drain.
$\tilde{f}_2$ represent the failure of weak charging.
$\tilde{f}_3$ represent the failure of battery damage.
$\tilde{f}_4$ represent the failure of the spark plugs.
$\tilde{f}_5$ represent the failure of the cables.
$\tilde{f}_6$ represent the failure of ignition control module.
$\tilde{f}_7$ represent the failure of ignition electronic control units.
$\tilde{f}_8$ represent the failure sensors.
By converting the FT in figure (2) into Success Tree as in bellow

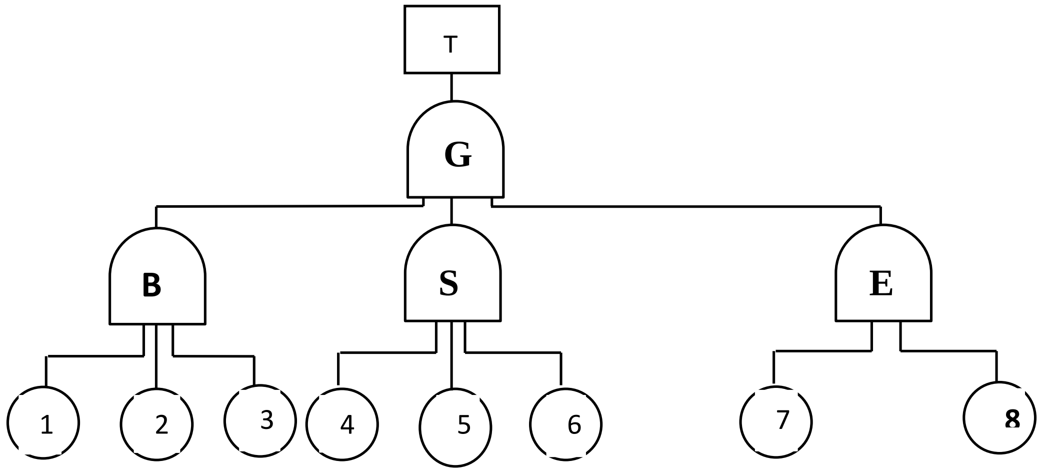

Figure 3: Success tree of a system in example (1)

The reliability and the reliability of each case are :

$\tilde{R}_{CE} = B \cap S \cap E$

where $B = 1 \cap 2 \cap 3$, $S = 4 \cap 5 \cap 6$ and $E = 7 \cap 8$,

$$\tilde{R}_{CE} = (1 \cap 2 \cap 3) \cap (4 \cap 5 \cap 6) \cap (7 \cap 8)$$

$\tilde{R}_{CE}$ represent the reliability of car engine safety system.
$\tilde{R}_{B}$ represent the reliability of Battery system.
$\tilde{R}_{E}$ represent the reliability of Electronic system.
$\tilde{R}_{S}$ represent the reliability of Ignition system.
$\tilde{R}_{1}$ represent the reliability of battery drain.
$\tilde{R}_{2}$ represent the reliability of weak charging.
$\tilde{R}_{3}$ represent the reliability of battery damage.
$\tilde{R}_{4}$ represent the reliability of the spark plugs.
$\tilde{R}_{5}$ represent the reliability of the cables.
$\tilde{R}_{6}$ represent the reliability of ignition control module.
$\tilde{R}_{7}$ represent the reliability of ignition electronic control units.
$\tilde{R}_{8}$ represent the reliability sensors.

1. $\tilde{R}_{CS} = \tilde{R}_{G} \otimes \tilde{R}_{S} \otimes \tilde{R}_{E}$

2. $\tilde{R}_{B} = \tilde{R}_{1} \otimes \tilde{R}_{2} \otimes \tilde{R}_{3}$, $\tilde{R}_{S} = \tilde{R}_{4} \otimes \tilde{R}_{5} \otimes \tilde{R}_{6}$ $and \tilde{R}_{E} = \tilde{R}_{7} \otimes \tilde{R}_{8}$ and the reliability of the system is $R_s = 1 - F_s$

Thus $\tilde{R}_{CS} = \tilde{R}_{1} \otimes \tilde{R}_{2} \otimes \tilde{R}_{3} \otimes \tilde{R}_{4} \otimes \tilde{R}_{5} \otimes \tilde{R}_{6} \tilde{R}_{7} \otimes \tilde{R}_{8}$

Now by applying (α, β) definitions of cuts and the operations of IF number for more detail see [18] :

$\tilde{R}_{1}$ =(0.75, 0.85, 0.8, 0.9, 0.8, 0.7),
$\tilde{R}_{2}$ =(0.8, 0.75, 0.85, 0.8, 0.9, 0.8),
$\tilde{R}_{3}$ = (0.9, 0.85, 0.8, 0.82, 0.91, 0.73),
$\tilde{R}_{4}$ = (0.85,0.75, 0.88, 0.87, 0.91, 0.83),
$\tilde{R}_{5}$ = (0.77, 0.75, 0.85, 0.85, 0.94, 0.83),
$\tilde{R}_{6}$ = (0.66, 0.75, 0.66, 0.8, 0.77, 0.86)
$\tilde{R}_{7}$ = (0.81,0.75, 0.85, 0.77, 0.93, 0.81),
$\tilde{R}_{8}$ = (0.74, 0.75, 0.85, 0.86, 0.92, 0.76)

$\tilde{R}_{CE}$ is calculated as in below:

$$\tilde{R}_{CE} = \tilde{R}_{G} \otimes \tilde{R}_{S} \otimes \tilde{R}_{E} = \tilde{R}_{1} \otimes \tilde{R}_{2} \otimes \tilde{R}_{3} \otimes \tilde{R}_{4} \otimes \tilde{R}_{5} \otimes \tilde{R}_{6} \tilde{R}_{7} \otimes \tilde{R}_{8}$$
$$\tilde{R}_{CE} = (0.1398, 0.1285, 0.194, 0.2312, 0.3692, 0.149)$$

4. Conclusions

For the sake of system safety and reliability, it is necessary to determine the causes of success in the car engine, as mentioned above, since there is a close relationship among them. One of the important causes is human error, which leads to one or more failures. As mentioned above, an active tool success tree analysis technique is used. It is possible to list the cut sets which refer to the failure of a system. The success tree technique is used to analyse the reliability of a car engine, which is related to machine safety. There is a benefit of employing IF sets over fuzzy sets in that IF sets disjoint the positive and negative evidence for a membership of an element in the set.

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REFERENCES

1. L.A. Zadeh, K.S. Fu, K.Tanaka and M. Shimura eds., Fuzzy Sets and Their Applications to Cognitive and Decision Processes, Academic Press Inc. New York. (1975).
2. K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20 (1986) 87-96.
3. E.G. Keiser, Local Area Networks, McGraw-Hill, Inc., New York, 1989.
4. W.L. Gau and D.J.B. uehrer, Vague Sets. *IEEE Transactions on Systems Man, and Cybernetics*, 23 (1993) 610-614.
5. K.T.Atanassov, New operations defined over the intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 61 (1994) 137-142.
6. Clifton A. Ericson II, History of Fault Tree, 1999, The Boeing Company Seattle, Washington.
7. G. Chen and T.T. Pham, *Introduction to fuzzy sets, Fuzzy logic, and Fuzzy Control System*, CRC Press LLC. (2001).
8. G.J. Klir and Bo Yaun, *Fuzzy Logic Theory and Applications*, Prentice-Hall, 6th edition (2002).
9. L.S. Srinat, Reliability Engineering, 4th Edition, Affiliated East, Wast Press Private Limited -New Delhi, (2005).
10. F.Masuli, S.Mitra and G.Pasi, *Applications of Fuzzy Sets Theory*, Springer–Verlag Berlin Heidelberg (2007).
11. G.S. Mahapatra, and T.K. Roy, Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations, *World Academy of Science, Engineering and Technology International Journal of Computer and Information Engineering*, 3(2) 2009.
12. Neeraj Lata, Analysis of fuzzy fault tree using intuitionistic fuzzy numbers, IJCSET, 4(7) (2013) 918-924.

13. I. Mitra, D.P. Datta and P.K. Chanda, On some exact solutions for a generalized form of Yang's Euclidean R-Gauge equations and their relation with Painleve' s property, *Journal of Physical Sciences*, 19 (2014) 51-58.
14. M.Arunkumar, P.Agilan and S.Ramamoorthy, Solution and generalized ulam-hyers stability of a n dimensional additive functional equation in Banach space and banach algebra: direct and fixed point methods, *Annals of Pure and Applied Mathematics*, 15(1) (2017) 25-40.
15. M.S.Abdol and S.K.Panchal, Some new uniqueness results of solutions to nonlinear fractional integro-differential equations, *Annals of Pure and Applied Mathematics*, 16(2) (2018) 345-352.
16. A.Subrie and A.Musleh Evaluating fuzzy Reliability system using Intuitionistic Fuzzy sets, *Journal of Babylon University*, 6(26) (2018) 313-321.
17. A. Salman, Success trees technique to evaluate system reliability in design, *Samarra Journal of Pure and Applied Science*, 2nd Samarra Local Scientific Conference Proceedings, 3-4 (2023) 377 -385.