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A Note on Equivalence Relations on Ternary Semigroups

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Abstract. The awareness *of* ideals obviously affects the concern of certain equivalence relations over a ternary semi-group. These equivalence relations, early studied by J.A. Green (1951), have played a basic role in the progress of semigroup theory. The major application of this paper is to find some results on Green Relations and their conditions on ternary semigroups.

Keywords: Ternary semi-groups, Commutative, Idempotent, Green's Relations and Congruence's

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1. Introduction and preliminary concepts

In 1932, Lehmer initiated the conception of ternary algebraic structures [4]. But now that Kasner has reexamined that system, he has given some thought to the n-ary algebraical structure—the ternary semigroup- as a universal algebra with one associative ternary operation. Banach has presented the view of ternary semi-groups. He showed by the illustration that a ternary semigroup can not necessarily be compressed to a standard semigroup. Los investigated several characters of ternary semi-groups and derived that any ternary semi-group shall be embedded in a semi-group in 1955. Santiago [8] improved the generalization of ternary semi-groups and semi-heaps in 1983. On the other hand, Green's relations distinguish that in terms of the principal ideals generated by the components of a semigroup in mathematics. Howie [3], an eminent semigroup theoretician, delineated this area as so that it is all-pervading that, on confronting a fresh semigroup. In 1997, Dewanand and Dixit [2] started the study of left, lateral and right congruences over ternary semigroups, and they argued Green's relations $\mathcal{L}, \mathcal{M}, \mathcal{R}, \mathcal{H}, \mathcal{D}$ and $\mathcal{J}$ over ternary semigroups. After that, Rabah Kellil investigated Green's relations over ternary semigroups in 2013. This article establishes certain results of Green's equivalence relation over ternary semi-groups.

Definition 1.1. A non-empty set with the operation among triplets of members is stated as a triplex when the following premises are satisfied.

Premise I.

$$\begin{aligned}(a.b.c).d.e &= d.(a.b.c).e = d.be.(a.b.c)\\ &= (a.b.d).c.e = (a.b.e).c.d = (a.c.d)b.e\\ &= (a.c.e).b.d = (a.d.e).b.c = (b.c.d).a.e\\ &= (b.c.e).a.d = (b.d.e).a.c = (c.d.e).a.b\end{aligned}$$

for any $a,b,c,c,d,e \in S$.

Premise II. If the members a and b in S , then there exists a member x in S such that $a.b.x = c.$

The number of members in S is calling as order of triplex and is given when essential by adding one of the premises:

Premise III₁. S contains ‘ n ’ members.

Premise III₂ . S contains extremely many elements.
According as III_1 or III_2 holds, the triplex is called finite or infinite.

Examples:
1. Set of all natural numbers is a triplex under ‘ $\cdot$ ’ multiplication.
2. $Z = \{\pm 1, \pm 2, \pm 3, \ldots\ldots\ldots\ldots\}$ is a triplex under ‘ $\cdot$ ’ multiplication.
3. If S is the collection of matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ of order 2×2 matrices where a and b are in N , set of natural numbers. Thus S is a triple with general matrix multiplication.

Definition 1.2. A nonempty set T be called ternary semi-group if there is one ternary operator $\circ : T \times T \times T \to T$ describe by
$(a,b,c) \to a \circ b \circ c$ holds the below condition
$(a.b.c).d.e = a.(b.c.d).e = a.b.(c.d.e)$ for any $a,b,c,d,e \in T$.

Example 1. Let $T = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$. Then T is a ternary semigroup with respect to matrix multiplication.

Example 2. We take $T = \{0,1,2,3,4,5\}$ and‘ $*$ ’ is act as in the following table such as $abc = (a*b)*c$ for any $a,b,c \in T$.
Accordingly (T , $*$) is a ternary semigroup.

*	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	1	1	1	1
2	0	1	2	2	1	1
3	0	1	1	1	2	2
4	0	1	4	5	1	`1
5	0	1	1	1	4	5

Example 3. Z^- is denote as the collection of negative integers and '$*$' is general ternary multiplication on Z^-. Hence Z^- is a ternary semigroup.

Note 1: In generally $a.b.c$ will write as abc.
2: Every semigroup can be reduced to a ternary semigroup.
3: Every ternary semigroup need not be a semigroup.

Example 4: Consider $M = \{-i,\ i\}$ and '.' is complex multiplication then M bean ternary semigroup. But it is not a semigroup by the following table

.	$-i$	i
$-i$	-1	1
i	1	-1

Definition 1.3. A ternary semigroup T is called abelian (commutative) if $xyz = yzx = zxy = yxz = zyx = xzy$, for any $x, y, z \in T$.

Example: Let $T = \{0, -i, i\}$ be a commutative ternary semigroup with the usual ternary multiplication.

Definition 1.4. An element 'a' of a ternary semigroup T is said to be left identity or left unit (lateral identity or lateral unit, right identity or right unit) if $aat = t$ ($ata = t$, $taa = t$) for all $t \in T$.

Definition 1.5. An element 'x' of ternary semigroup T is said to be an idempotent if $x^3 = x$.

Note: The set of all idempotent elements in a ternary semigroup T is denoted by $E(T)$.

Definition 1.6. An element 'a'in ternary semigroup T is said to be a non-trivial 'a' is idempotent, but not an identity in T if identity is there.

A ternary semigroup T is said to be an idempotent ternary semigroup or a ternary band if all elements T are idempotent.

2. Green's relations and its properties

Definition 2.1. Let T be a ternary semigroup. From T^1 we denote the set $T \cup \{1\}$ where 1 is identity for the ternary operation. We are defining equivalence relations on T^1 which we define Green's relations by:

$a\mathcal{L}^T b \Leftrightarrow \exists\, x, y, u, v \in T^1$ *such that* $a = xyb$ and $b = uva$ $\forall\, a, b \in T^1$.

$a\mathcal{R}^T b \Leftrightarrow \exists\, x, y, u, v \in T^1$ *such that* $a = bxy$ and $b = auv$ $\forall\, a, b \in T^1$.

$a\mathcal{J}^T b \Leftrightarrow \exists\, x, y, u, v \in T^1$ *such that* $a = xby$ and $b = uav$ $\forall\, a, b \in T^1$.

$a\mathcal{H}^T b \Leftrightarrow a\mathcal{L}^T b$ and $a\mathfrak{R}^T b\ \forall\, a, b \in T^1$.

Now we define the relation $\mathcal{D}^T$ to be the least equivalence relation containing both $\mathcal{L}^T$ and $\mathcal{R}^T$. For any $a\epsilon T^1$, $\mathcal{L}_a^T$, $\mathcal{R}_a^T$, $\mathcal{H}_a^T$, $\mathcal{D}_a^T$ and $\mathcal{J}_a^T$ will denote the equivalence classes of madulo 'a' respectively $\mathcal{L}^T$, $\mathcal{R}^T$, $\mathcal{H}^T$, $\mathcal{D}^T$ and $\mathcal{J}^T$.
The relations will be denoted if there is no confusion on ternary semigroups by $\mathcal{L}$, $\mathcal{R}$, $\mathcal{H}$, $\mathcal{D}$, and $\mathcal{J}$. The corresponding class of an element $a\epsilon T$ will be denoted by $\mathcal{L}_a$, $\mathcal{R}_a$, $\mathcal{H}_a$, $\mathcal{D}_a$ and $\mathcal{J}_a$.

Definition 2.2. An equivalence relation ρ in a ternary semigroup T is said to be
(i) Left congruence if $a\rho b \Rightarrow (sta)\rho(stb)$
(ii) Right congruence if $a\rho b \Rightarrow (ast)\rho(bst)$
(iii) Lateral congruence if $a\rho b \Rightarrow (sat)\rho(sbt)$
(iv) Congruence if
$a\rho a', b\rho b'$ and $c\rho c' \Rightarrow (abc)\rho(a'b'c')$ for any $a, b, c, a', b', c' \epsilon T$ and $s, t \epsilon T$.

Theorem 2.3. If $\mathcal{L}$ *and* $\mathcal{R}$ are Green's equivalence relations on ternary semigroup T, then $\mathcal{L}$ and $\mathcal{R}$ are respectively right and left congruence's on T.

Proof: Given that $\mathcal{L}$ and $\mathcal{R}$ be equivalence relations over T .Then $a\mathcal{L}^T b \Leftrightarrow \exists\, u, v, x, y \in T^1$ such as $a = xyb$ and $b = uva$ and $a\mathcal{R}^T b \Leftrightarrow \exists u, v, x, y \in T^1$ such as $a = bxy$ and $b = auv$ for all $a, b\epsilon T^1$.
(i)We have to prove that $\mathcal{L}$ is right congruence on T.
i.e., if $a\mathcal{L}b \Rightarrow$ (ast)$\mathcal{L}$(bst).
Consider $a, b\epsilon T^1$ and $a\mathcal{L}b \Leftrightarrow \exists u, v, x, y \in T^1$
such as $a = xyb$ and $b = uva$ for any $a, b\epsilon T^1$.

$\Rightarrow ast = xybst$ and $bst = uvast$ for any $a, b, s, t\epsilon T^1$ $\Rightarrow ast\,\mathcal{L}bst$
Therefore $a\mathcal{L}b \Rightarrow ast\,\mathcal{L}bst$.
(ii) To prove that $\mathcal{R}$ be a left congruence on T.
i.e., $a\mathcal{R}b \Rightarrow$ (sta)$\mathcal{R}$(stb).
Since $\mathcal{R}$ is an equivalence relation. Then
$a\mathcal{R}b \Leftrightarrow \exists x, y, u, v \in T^1$ such that $a = bxy$ and $b = auv\ \forall a, b\epsilon T^1$.

$\Rightarrow$ $ta = tbxy$ and $tb = tauv$ for any $a, b, t \epsilon T^1$.
$\Rightarrow$ $sta = stbxy$ and $stb = stauv$ for any $a, b, s, t \epsilon T^1$
$a\mathcal{R}b \Rightarrow$ $sta\mathcal{R}$ stb.

Theorem 2.4. Let T be a commutative ternary semigroup and $\mathcal{J}$ be a Green's equivalence relation on T. Then $\mathcal{J}$ is lateral congruence on T.

Proof: Let T be a commutative ternary semigroup and let $\mathcal{J}$ be a Green's equivalence relation on T.

i,e., we define $\mathcal{J}$ by $a\mathcal{J}b \Leftrightarrow \exists x, y, u, v \in T^1$

such that $a = xby$ and $b = uav$ for any $a, b \epsilon T^1$.

We have to prove that $\mathcal{J}$ is lateral congruence on T.

Consider $a\mathcal{J}b \Leftrightarrow \exists x, y, u, v \in T^1$ such that $a = xby$ and $b = uav$ for any $a, b \epsilon T^1$

$$\Rightarrow sat = (sxb)yt \text{ and } sbt = (sua)vt \text{ for any } a, b, s, t \epsilon T^1$$

$\Rightarrow$ $sat = xs(byt)$ and $sbt = us(avt)$ (since T is commutative)

$\Rightarrow$ $sat = x(sbt)y$ and $sbt = u(sat)v$ $for\ any$ $a, b, s, t \in T^1$

$\Rightarrow (sat)\mathcal{J}(sbt)$ for all $a, b, s, t \epsilon T^1$.

Hence $\mathcal{J}$ is a lateral congruence relation on T.

Definition 2.5. The relation $\mathcal{D} = (\mathcal{L} \circ \mathcal{R}) = (\mathcal{R} \circ \mathcal{L})$ is the smallest equivalence relation contains both $\mathcal{R}$ and $\mathcal{L}$. i.e., $\mathcal{D} = (\mathcal{L} \cup \mathcal{R}) = (\mathcal{R} \cup \mathcal{L})$

Definition 2.6. A relation $\mathcal{H} = \mathcal{L} \cap \mathcal{R} = \mathcal{R} \cap \mathcal{L}$ is one equivalence relation in a ternary semi-group T containing in both $\mathcal{L}$ and $\mathcal{R}$.

We denote $\mathcal{D}_x$ and $\mathcal{H}_x$ the corresponding equivalence class that contains the element $x \epsilon T$. Clearly, for $x \epsilon T$, $\mathcal{H}_x = \mathcal{L}_x \cap \mathcal{R}_x$.

Theorem 2.7. Let T be a ternary semigroup. Then

(i) $x\mathcal{D}y \Leftrightarrow \mathcal{L}_x \cap \mathcal{R}_y \neq \emptyset \Leftrightarrow \mathcal{R}_x \cap \mathcal{L}_y \neq \emptyset$.

(ii) $\mathcal{D}_x = \bigcup_{y \in D_x} \mathcal{L}_y = \bigcup_{y \in D_x} \mathcal{R}_y$.

Proof: Let T be a ternary semi-group.

Since $\mathcal{D}$ smallest equivalence relation over T.

We have $\mathcal{D} = \mathcal{R} \cup \mathcal{L} = \mathcal{L} \cup \mathcal{R}$.

(i) For any x and $y \epsilon T$, $x\mathcal{D}y \Leftrightarrow x(\mathcal{R} \cup \mathcal{L})y \Leftrightarrow x(\mathcal{L} \cup \mathcal{R})y$

$\Leftrightarrow$ there exists $z \in T$ such that $x\mathcal{L}z$ and $z\mathcal{R}y \Leftrightarrow x\mathcal{R}z$ and $z\mathcal{L}y$

$\Leftrightarrow z \in \mathcal{L}_x$ and $z \in \mathcal{R}_y \Leftrightarrow z \in \mathcal{R}_x$ and $z \in \mathcal{L}_y$

$\Leftrightarrow z \in \left(\mathcal{L}_x \cap \mathcal{R}_y\right) \Leftrightarrow z \in (\mathcal{R}_x \cap \mathcal{L}_y)$

$\Leftrightarrow$ $(\mathcal{L}_x \cap \mathcal{R}_y) \neq \emptyset \Leftrightarrow (\mathcal{R}_x \cap \mathcal{L}_y) \neq \emptyset$.

Hence $x\mathcal{D}y \Leftrightarrow \mathcal{L}_x \cap \mathcal{R}_y \neq \emptyset \Leftrightarrow \mathcal{R}_x \cap \mathcal{L}_y \neq \emptyset$.

(ii) For any $x \in T$,

Let $\mathcal{D}_x = \{ y \in T : x\mathcal{D}y \}$
$= \{ y : x(\mathcal{L} \cup \mathcal{R})y = x(\mathcal{R} \cup \mathcal{L})y \}$
$= \{ y : \exists\, z \in T \text{ such that } x\mathcal{L}z \text{ and } z\mathcal{R}y = x\mathcal{R}z \text{ and } z\mathcal{L}y \}$
$= \bigcup_{y \in D_x} \mathcal{L}_y = \bigcup_{y \in D_x} \mathcal{R}_y$

Therefore $\mathcal{D}_x = \bigcup_{y \in D_x} \mathcal{L}_y = \bigcup_{y \in D_x} \mathcal{R}_y$.

Theorem 2.8. Let T be an idempotent ternary semigroup. Then for all $a, b \in T$

(i) $a\mathcal{L}b \Leftrightarrow a = abb$ and $b = baa$.

(ii) $a\mathcal{R}b \Leftrightarrow a = bba$ and $b = aab$.

Proof: Let T be a idempotent ternary semigroup.

Thus all elements of T are idempotent.

i.e., $a^3 = a$ for any $a \epsilon T$.

Suppose $\mathcal{L}$ and $\mathcal{R}$ are equivalence classes on T.

$i.e.,\ a\mathcal{L}b \Leftrightarrow \exists s, t, s', t' \in T^1$ such as $a = stb$ and $b = s't'a$

and $a\mathcal{R}b \Leftrightarrow \exists s, t, s', t' \in T^1$ such that $a = bst$ and $b = as't'$ for any $a, b \epsilon T^1$.

(i) Consider $a\mathcal{L}b \Leftrightarrow a = stb$ and $b = s't'a$, for any $a, b \in T^1$

$\Leftrightarrow abb = (stb)bb$ and $baa = (s't'a)aa$,
$\Leftrightarrow abb = st(bbb)$ and $baa = s't'(aaa)$
$\Leftrightarrow abb = stb$ and $baa = s't'a$
$\Leftrightarrow abb = a$ and $baa = b$

Hence $a\mathcal{L}b \Leftrightarrow abb = a$ and $baa = b$.

(ii) Consider $a\mathcal{R}b \Leftrightarrow a = bst$ and $b = as't'$, $\forall a, b \epsilon T$

$\Leftrightarrow bba = bb(bst)$ and $aab = aa(as't')$
$\Leftrightarrow bba = (bbb)st$ and $aab = (aaa)s't'$
$\Leftrightarrow abb = bst$ and $baa = as't'$
$\Leftrightarrow bba = a$ and $aab = b$.

Therefore $a\mathcal{R}b \Leftrightarrow bba = a$ and $aab = b$.

Definition 2.9. Let T be a ternary semigroup. Then for each $s, t \epsilon T$ we define (as before) the mapping $\rho : T^1 \to T^1$ by $\rho(z) = stz \ \ \forall\ z \epsilon T^1$.

Theorem 2.10. Let T be a ternary semi-group. For any $x, y \epsilon T$, let $x\mathcal{L}y$. We have

(i) $\rho : \mathcal{R}_x \to \mathcal{R}_y$ is bijection and $\eta : \mathcal{R}_y \to \mathcal{R}_x$ is bijection.

(ii) $\eta = \rho^{-1}$ inverse function of ρ restricted to $\mathcal{R}_x$.

(iii) $\rho(z)$ fixes the $\mathcal{L}$- class, that is $z\mathcal{L}\rho(z)$ for all $z \epsilon \mathcal{R}_x$.

(iv) ρ preserves $\mathcal{H}$ - class, that is for all $u, v \in \mathcal{R}_x; u\mathcal{H}v \Leftrightarrow \rho(u)\mathcal{H}\rho(v)$.

Proof: Let T be a ternary semi-group. Since $x\mathcal{L}y$. There have $s, t, s', t' \in T^1$ such that that $stx = y$ and $s't'y = x$ (1)

(i) Define $\rho(z) : \mathcal{R}_x \to \mathcal{R}_y$ nd $\eta: \mathcal{R}_y \longrightarrow \mathcal{R}_x$ by $\rho(z) = stz \ \forall \ z \in \mathcal{R}_x$, $s, t \in T^1$ and $\eta(z) = s't'z, \ \forall \ z \in \mathcal{R}_y$ an $s', t' \in T^1$ respectively.

First to prove that $\rho : \mathcal{R}_x \to \mathcal{R}_y$ bijective mapp.

Let $z \in \mathcal{R}_x$. Then $z\mathcal{R}x \Leftrightarrow xT^1T = zT^1T$

Consider $xT^1T = zT^1T \Rightarrow stxT^1T = stzT^1T$

$$\Rightarrow stzT^1T = stxT^1T$$
$$\Rightarrow stzT^1T = yT^1T \text{(from equation (1))}$$
$$\Rightarrow stz\mathcal{R}y$$
$$\Rightarrow \rho(z)\mathcal{R}y$$

Thus $\rho(z) \in \mathcal{R}_y$ for any $s, t \in T$ and $z \in \mathcal{R}_x$. Hence ρ is onto.

Next to show that ρ is one-one.

For any $z \in \mathcal{R}_x$, let $\rho(x) = \rho(z)$

$$\Rightarrow stx = stz \text{ for some } s, t \in T^1.$$
$$\Rightarrow stx - stz = 0$$
$$\Rightarrow st(x - z) = 0$$
$$\Rightarrow x - z = 0 \text{ and } st \neq 0$$
$$\Rightarrow x = z$$

Therefore ρ is one – one.

Hence $\rho : \mathcal{R}_x \to \mathcal{R}_y$ is bijective map.

Similarly, we can show that $\eta : \mathcal{R}_y \to \mathcal{R}_x$ is bijective mapp.

(ii) To claim that $\eta = \rho^{-1}$.

Consider $z \in \mathfrak{R}_x \Rightarrow z\mathcal{R}x \Leftrightarrow \exists u, v, u', v' \in T^1$ such that $z = xuv$ and $x = zu'v'$.

Consider $(s't')(stz) = (s't')(st)xuv$

$$= (s't')(stx)uv$$
$$= (s't')(y)uv$$
$$= (s't'y)uv$$
$$= xuv$$
$$= z$$
$$(s't')(stz) = z \qquad (2)$$

Now $\eta\rho(z) = \eta(\rho(z)) = \eta(stz) = s't'(stz) = z$.

Here $\eta(\rho(z))$ is the identity mapping of $\mathcal{R}_x$ and $\rho(\eta(z))$ is the identity mapping of $\mathcal{R}_y$. Hence $\rho\eta = \eta\rho = I \Rightarrow \eta = \rho^{-1}$.

(iii) Assume that $z\epsilon\mathcal{R}_x \Rightarrow stz = \rho(z)$ and $s't'\rho(z) = z$.

From equation (2), then z and stz ($or \rho(z)$) are in the same $\mathcal{L}$ - class.

Therefore $\rho(z)$ fixes the $\mathcal{L}-$ class, that is $z\mathcal{L}\rho(z)$ for all $z\epsilon\mathcal{R}_x$.

(iv) Let $u, v\epsilon\mathcal{R}_x \Rightarrow u\epsilon\mathcal{R}_x$ and $v\epsilon\mathcal{R}_x$ $\Rightarrow$ by (iii) u, v are in the $\mathcal{L}-$ class.

Here $u\mathcal{L}v \Leftrightarrow stu = v$ and $u = s't'v \Rightarrow \rho(u) = v$ and $\rho(v) = u$.

Consider $\quad u\mathcal{H}v \Leftrightarrow u(\mathcal{L}\cap\mathcal{R})v$

$$\Leftrightarrow u\mathcal{L}v \text{ and } u\mathcal{R}v$$
$$\Leftrightarrow \rho(u)\mathcal{L}\rho(v) \text{ and } \rho(u)\mathcal{R}\rho(v)$$
$$\Leftrightarrow \rho(u)\mathcal{H}\rho(v).$$

Definition 2.11: Let T be a ternary semigroup. For $p, r\epsilon T$, we define a mapping $f: T^1 \to T^1$ by $\rho(z) = zpr$ for any $z\epsilon T^1$.

Theorem 2.12. Let T be a ternary semigroup. Define green's equivalence $\mathcal{R}$ on T by $y\mathcal{R}z \Leftrightarrow ypr = z$ and $zp'r' = y$ for any $p, r, p', r'\epsilon T^1$, *then*

(i) f: $\mathcal{L}_y \to \mathcal{L}_z$ bijection and $g: \mathcal{L}_z \to \mathcal{L}_y$ bijection.

(ii) $g = f^{-1}$ is an inverse function of f restricted to $\mathcal{L}_y$.

(iii) f preserves $\mathcal{R}-class$, *that is* $w\mathcal{R}f(w)$, for any $w\epsilon\mathcal{L}_y$.

Proof: Simply we can prove this proof following by theorem 2.10.

Theorem 2.13. Let E_T be the collection of idempotent elements of ternary semigroup T and let $e\epsilon E$. If $x\mathcal{L}e$ and $x\mathcal{R}e$ then respectively $x = xee$ and $x = eex$.

Proof: Let T be a ternary semigroup and given $e\epsilon E_T$ and $x\epsilon T$.

(i) Consider $x\mathcal{L}e \Leftrightarrow \exists\, s, t, s', t' \in T^1$ such that $stx = e$ and $s't'e = x$ $s't'e = x$.

$$\Rightarrow x = s't'e$$
$$= s't'e^3$$
$$= s't'eee$$
$$x = xee$$

(ii) Consider $x\mathcal{R}e \Leftrightarrow \exists s, t, s', t' \in T^1$ such that $xst = e$ and $es't' = x$.

$$\Rightarrow x = es't'$$
$$= e^3s't'$$
$$= eees't'$$

Hence $x = eex$.

3. Conclusions

We introduced the notion of Green's Relations congruences over ternary semigroups, investigated their properties, and described their extension. In the course of this work, we plan to study Green's relations over Ternary semigroups.

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