Annals of Pure and Applied Mathematics
Vol. 31, No. 1, 2025, 23-29
ISSN: 2279-087X (P), 2279-0888(online)
Published on 13 February 2025
www.researchmathsci.org
DOI: http://dx.doi.org/10.22457/apam.v31n1a03958

Annals of
Pure and Applied Mathematics

Domination Kepler Banhatti and Modified Domination Kepler Banhatti Indices of Graphs

V.R.Kulli

Department of Mathematics
Gulbarga University, Gulbarga 585 106, India
E-mail: vrkulli@gmail.com

Received 31 December 2024; accepted 12 February 2025

Abstract. In this study, we introduce the domination Kepler Banhatti and modified domination Kepler Banhatti indices and their corresponding exponentials of a graph. Furthermore, we compute these indices for some standard graphs, French windmill graphs. Also we obtain some properties of domination Kepler Banhatti index.

Keywords: domination Kepler Banhatti index, modified domination Kepler Banhatti index, graphs.

AMS Mathematics Subject Classification (2010): 05*C*07, 05*C*09

1. Introduction

In this paper, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree d_u of a vertex u is the number of vertices adjacent to u. We refer [1], for other undefined notations and terminologies.

Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [2].

The domination degree $d_d(u)$ [3] of a vertex u in a graph G is defined as the number of minimal dominating sets of G which contains u.

The modified first domination Zagreb index [3] of a graph is defined as

$$DM_1^*(G) = \sum_{uv \in E(G)} \left(d_d(u) + d_d(v)\right).$$

Ref. [3] was soon followed by a series of publications [4, 5, 6, 7, 8, 9].
The domination Sombor index was introduced in [10] and it is defined as

$$DSO(G) = \sum_{uv \in E(G)} \sqrt{d_d(u)^2 + d_d(v)^2}.$$

The reciprocal domination product connectivity index [11] of a graph G *is* defined as

$$RDP(G) = \sum_{uv \in E(G)} \sqrt{d_d(u) d_d(v)}.$$

The Kepler Banhatti index was introduced by Kulli in [12] and it is defined as

$$KB(G)=\sum_{uv\in E(G)}[(d_u+d_v)+\sqrt{d_u^2+d_v^2}].$$

Motivated by the definition of Kepler Banhatti index, we introduce the domination Kepler Banhatti index of a graph and it is defined as

$$DKB(G)=\sum_{uv\in E(G)}[(d_d(u)+d_d(v))+\sqrt{d_d(u)^2+d_d(v)^2}].$$

Considering the domination Kepler Banhatti index, we introduce the domination Kepler Banhatti exponential of a graph G and defined it as

$$DKB(G,x)=\sum_{uv\in E(G)}x^{(d_d(u)+d_d(v))+\sqrt{d_d(u)^2+d_d(v)^2}}.$$

We define the modified domination Kepler Banhatti index of a graph G as

$${}^{m}DKB(G)=\sum_{uv\in E(G)}\frac{1}{(d_d(u)+d_d(v))+\sqrt{d_d(u)^2+d_d(v)^2}}.$$

Considering the modified domination Kepler Banhatti index, we introduce the modified domination Kepler Banhatti exponential of a graph G and defined it as

$${}^{m}DKB(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{(d_d(u)+d_d(v))+\sqrt{d_d(u)^2+d_d(v)^2}}}.$$

Recently, some Kepler Banhatti indices were studied in [13, 14, 15].
In this paper, the domination Kepler Banhatti index, modified domination Kepler Banhatti index and their corresponding exponentials of certain graphs are computed.

2. Results for some standard graphs

Proposition 1. If K_n is a complete graph with n vertices, then

$$DKB(K_n)=\frac{(2+\sqrt{2})n(n-1)}{2}.$$

Proof: If K_n is a complete graph, then $d_d(u)=1$. From definition, we have

$$DKB(K_n)=\frac{n(n-1)}{2}[(1+1)+\sqrt{1^2+1^2}]=\frac{(2+\sqrt{2})n(n-1)}{2}.$$

Proposition 2. Let $K_{m,n}$ be a complete bipartite graph with $2\le m\le n$. Then

$$DKB(K_{m,n})=mn[(m+n+2)+\sqrt{(m+1)^2+(n+1)^2}].$$

Proof: Let $G=K_{m,n}$, $m, n\geq 2$ with $d_d(u)=m+1$
$= n+1$, for all $u\in V(G)$.

From definition, we obtain

$$DKB(K_{m,n})=mn[(m+n+2)+\sqrt{(m+1)^2+(n+1)^2}].$$

We obtain the domination Kepler Banhatti exponentials of K_n and $K_{m,n}$.

Proposition 3. The domination Kepler Banhatti exponentials of K_n and $K_{m,n}$ are given by

(i) $DKB(K_n, x) = \frac{n(n-1)}{2} x^{(2+\sqrt{2})}$.

(ii) $DKB(K_{m,n}, x) = mnx^{[(m+n+2)+\sqrt{(m+1)^2+(n+1)^2}]}$.

3. Mathematical properties

Theorem 1. Let *G* be a simple connected graph. Then

$$DKB(G) \geq \left(1+\frac{1}{\sqrt{2}}\right) DM_1^*(G)$$

with equality if *G* is regular.

Proof: By the Jensen inequality, for a concave function *f(x),*

$$f\left(\frac{1}{n}\sum x_i\right) \geq \frac{1}{n}\sum f(x_i)$$

with equality for a strict concave function if $x_1 = x_2 = \ldots = x_n$. Choosing $f(x) = \sqrt{x}$, we obtain

$$\sqrt{\frac{d_d(u)^2 + d_d(v)^2}{2}} \geq \frac{(d_d(u)+d_d(v))}{2}$$

thus

$$(d_d(u)+d_d(v)) + \sqrt{d_d(u)^2 + d_d(v)^2} \geq (d_d(u)+d_d(v)) + \frac{1}{\sqrt{2}}(d_d(u)+d_d(v)).$$

Hence

$$\sum_{uv\in E(G)} [(d_d(u)+d_d(v)) + \sqrt{d_d(u)^2 + d_d(v)^2}] \geq \left(1+\frac{1}{\sqrt{2}}\right) \sum_{uv\in E(G)} (d_d(u)+d_d(v)).$$

Thus

$$DKB(G) \geq \left(1+\frac{1}{\sqrt{2}}\right) DM_1^*(G)$$

with equality if *G* is regular.

Theorem 2. Let *G* be a simple connected graph. Then

$$DKB(G) \leq (1+\sqrt{2}) DM_1^*(G) - \sqrt{2}RDP(G).$$

Proof: It is known that for $1 \leq x \leq y$,

$$f(x,y) = (x+y-\sqrt{xy}) - \sqrt{\frac{x^2+y^2}{2}}$$

is decreasing for each *y*. Thus $f(x,y)^3 \; f(y,y) = 0$. Hence

$$x+y-\sqrt{xy}\,^3 \sqrt{\frac{x^2+y^2}{2}}$$

or $$\sqrt{\frac{x^2+y^2}{2}} \le x+y-\sqrt{xy}.$$

Put $x = d_d(u)$ and $y = d_d(v)$, we get

$$\sqrt{\frac{d_d(u)^2+d_d(v)^2}{2}} \le \left(d_d(u)+d_d(v)\right)-\sqrt{d_d(u)d_d(v)}$$

$$\sqrt{d_d(u)^2+d_d(v)^2} \le \sqrt{2}[\left(d_d(u)+d_d(v)\right)-\sqrt{d_d(u)d_d(v)}]$$

which implies

$$\left(d_d(u)+d_d(v)\right)+\sqrt{d_d(u)^2+d_d(v)^2} \le \left(d_d(u)+d_d(v)\right)$$
$$+\sqrt{2}[\left(d_d(u)+d_d(v)\right)-\sqrt{d_d(u)d_d(v)}]$$

$$\sum_{uv\in E(G)}[\left(d_d(u)+d_d(v)\right)+\sqrt{d_d(u)^2+d_d(v)^2}] \le \left(1+\sqrt{2}\right)\sum_{uv\in E(G)}\left(d_d(u)+d_d(v)\right)$$
$$-\sqrt{2}\sum_{uv\in E(G)}\sqrt{d_d(u)d_d(v)}$$

Thus $$DKB(G) \le \left(1+\sqrt{2}\right)DM_1^*(G)-\sqrt{2}RDP(G).$$

Theorem 3. Let G be a simple connected graph. Then

$$DKB(G) \le 2DM_1^*(G).$$

Proof: It is known that for $1 \le x \le y$,

$$\sqrt{x^2+y^2} < x+y$$

$$(x+y)+\sqrt{x^2+y^2} < 2(x+y).$$

Setting $x = d_d(u)$ and $y = d_d(v)$, we get

$$\left(d_d(u)+d_d(v)\right)+\sqrt{d_d(u)^2+d_d(v)^2} < 2\left(d_d(u)+d_d(v)\right).$$

Thus $$\sum_{uv\in E(G)}[\left(d_d(u)+d_d(v)\right)+\sqrt{d_d(u)^2+d_d(v)^2}] < 2\sum_{uv\in E(G)}\left(d_d(u)+d_d(v)\right).$$

Hence $$DKB(G) \le 2DM_1^*(G).$$

Theorem 4. Let G be a simple connected graph. Then

$$DKB(G) = DM_1^*(G)+DKS(G).$$

Proof: We have

$$\sum_{uv\in E(G)}[\left(d_d(u)+d_d(v)\right)+\sqrt{d_d(u)^2+d_d(v)^2}] = \sum_{uv\in E(G)}\left(d_d(u)+d_d(v)\right)$$
$$+\sum_{uv\in E(G)}\sqrt{d_d(u)^2+d_d(v)^2}$$

Hence $$DKB(G) = DM_1^*(G)+DKS(G).$$

4. Results for French Windmill graphs

The French windmill graph F_n^m is the graph obtained by taking $m \geq 3$ copies of $K_{n,n} \geq 3$ with a vertex in common. The graph F_n^m is presented in Figure 1. The French windmill graph F_3^m is called a friendship graph.

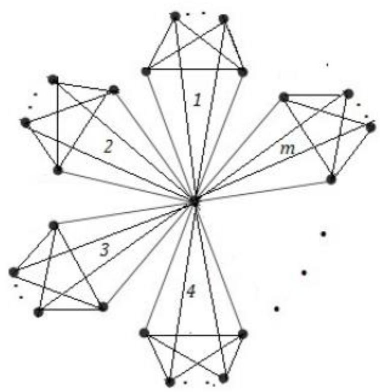

Figure 1: French windmill graph F_n^m

Let F be a French windmill graph F_n^m. Then

$$d_d(u) = 1, \quad \text{if } u \text{ is the center vertex,}$$

$$= (n-1)^{m-1}, \quad \text{otherwise.}$$

Theorem 5. Let F be a French windmill graph F_n^m. Then

$$DKB(F) = m(n-1)[\left(1+(n-1)^{(m-1)}\right) + \sqrt{1+(n-1)^{2(m-1)}}]$$
$$+[(mn(n-1)/2) - m(n-1)][\left(2+\sqrt{2}\right)(n-1)^{(m-1)}].$$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the centre vertex and E_2 be the set of all edges of the complete graph. Then

$$DKB(F) = \sum_{uv\in E(F)} [\left(d_d(u)+d_d(v)\right) + \sqrt{d_d(u)^2 + d_d(v)^2}]$$
$$= m(n-1)[\left(1+(n-1)^{(m-1)}\right) + \sqrt{1^2+(n-1)^{2(m-1)}}]$$
$$+[(mn(n-1)/2) - m(n-1)]$$
$$[\left((n-1)^{(m-1)} + (n-1)^{(m-1)}\right) + \sqrt{(n-1)^{2(m-1)} + (n-1)^{2(m-1)}}]$$
$$= m(n-1)[\left(1+(n-1)^{(m-1)}\right) + \sqrt{1+(n-1)^{2(m-1)}}]$$
$$+[(mn(n-1)/2) - m(n-1)][\left(2+\sqrt{2}\right)(n-1)^{(m-1)}].$$

Corollary 5.1. Let F_3^m be a friendship graph. Then

$$DKB\left(F_3{}^m\right) = 2m[\left(1+2^{(m-1)}\right) + \sqrt{1+2^{2(m-1)}}] + m\left(2+\sqrt{2}\right)2^{(m-1)}.$$

5. Conclusion

In this paper, the domination Kepler Banhatti index, modified domination Kepler Banhatti index and their corresponding exponentials are defined and studied.

Acknowledgment. I thank the reviewers for their report on the improvement of the work.

Author's Contributions. The author solely prepared the paper.

Conflicts of interest. The author declares that there are no conflicts of interest regarding the publication of this paper. No financial, personal, or professional relationships influenced this work's research, analysis, or conclusions.

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, Graph indices, in *Handbook of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2019) 66-91.
3. A.M.H.Ahmed, A.Alwardi and M.Ruby Salestina, On domination topological indices of graphs, *International Journal of Analysis and Applications*, 19(1) (2021) 47-64.
4. V.R.Kulli, Domination augmented Banhatti, domination augmented Banhatti sum indices of certain chemical drugs, *International Journal of Mathematics and Computer Research*, 11(7) (2023) 3558-3564.
5. V.R.Kulli, Gourava domination indices of graphs, *International Journal of Mathematics and Computer Research*, 11(8) (2023) 3680-3684.
6. V.R.Kulli, Sum and product connectivity Gourava domination indices of graphs, *International Journal of Mathematics and Statistics Invention*, 11(4) (2023) 35-43.
7. V.R.Kulli, On hyper Gourava domination indices, *International Journal of Engineering Sciences & Research Technology*, 12(10) (2023) 12-20.
8. V.R.Kulli, Modified domination Sombor index and its exponential of a graph, *International Journal of Mathematics and Computer Research*, 11(8) (2023) 3639-3644.
9. S.Raju, Puttuswamy and S.R.Nayaka, On the second domination hyper index of graph and some graph operations, *Advances and Applications in Discrete Mathematics*, 39(1) (2023) 125-143.
10. A.A.Shashidhar. H.Ahmed, N.D.Soner and M.Cancan, Domination version: Sombor index of graphs and its significance in predicting physicochemical properties of butane derivatives, *Eur. Chem. Commun.*, 5 (2023) 91-102.
11. V.R.Kulli, Domination product connectivity indices of graphs, *Annals of Pure and Applied Mathematics*, 27(2) (2023) 73-78.
12. V.R.Kulli, Kepler Banhatti and Modified Kepler Banhatti indices, *Inter. Journal of Mathematics and Computer Research*, 12(6) (2024) 4310-4314.

13. V.R.Kulli, Revan Kepler Banhatti and modified revan Kepler Banhatti indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 30(2) (2024) 129-136.
14. V.R. Kulli, Reverse Kepler Banhatti and modified reverse Kepler Banhatti indices, *International Journal of Mathematical Archive*, 15(7), (2024) 1-8.
15. V.R.Kulli, Product connectivity E-Banhatti indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 27(1) (2023) 7-12.