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Revan Kepler Banhatti and Modified Revan Kepler Banhatti Indices of Certain Nanotubes

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Abstract. In this study, we introduce the Revan Kepler Banhatti index, the modified Revan Kepler Banhatti index and their corresponding exponentials of a graph. Furthermore, we compute these newly defined Revan Kepler Banhatti indices for some nanotubes. Also, some mathematical properties of the Revan Kepler Banhatti index are obtained.

Keywords: Revan Kepler Banhatti index, modified Revan Kepler Banhatti index, nantube

AMS Mathematics Subject Classification (2010): 05*C*10, 05*C*69

1. Introduction

In this paper, we consider only a finite, simple connected graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree d_u of a vertex u is the number of vertices adjacent to u. Let $\Delta(G)$ ($\delta(G)$) denote the maximum (minimum) degree among the vertices of G. The Revan vertex degree of a vertex u in G is defined as $r_u=\Delta(G) + \delta(G) - d_u$. The reverse edge connecting the reverse vertices u and v will be denoted by uv. We refer to [1] for undefined terms and notation.

A graph index is a numerical parameter mathematically derived from the graph structure. Several graph indices have been considered in Theoretical Chemistry and many graph indices were defined by using vertex degree concept [2]. The Zagreb, Revan, Gourava, and Banhatti, reverse indices are the most degree-based graph indices in Chemical Graph Theory. Graph indices have their applications in various disciplines in Science and Technology [3].

The first and second Revan indices were introduced by Kulli in [4]. They are defined as

$$R_1(G)=\sum_{uv\in E(G)}\left(r_u+r_v\right), \qquad R_2(G)=\sum_{uv\in E(G)} r_u r_v.$$

The product connectivity Revan index was proposed in [5] and defined as

$$PR(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{r_u r_v}}.$$

The reciprocal product connectivity Revan index is defined as

$$RPR(G) = \sum_{uv \in E(G)} \sqrt{r_u r_v}.$$

Recently, some Revan indices were studied in [6, 7, 8, 9, 10, 11].
The Kepler Banhatti index was introduced by Kulli in [12] and it is defined as

$$KB(G) = \sum_{uv \in E(G)} [(d_u + d_v) + \sqrt{d_u^2 + d_v^2}].$$

Motivated by the definition of Kepler Banhatti index, we introduce the Revan Kepler Banhatti index of a graph and it is defined as

$$RKB(G) = \sum_{uv \in E(G)} [(r_u + r_v) + \sqrt{r_u^2 + r_v^2}].$$

Considering the Revan Kepler Banhatti index, we introduce the Revan Kepler Banhatti exponential of a graph *G* and defined it as

$$RKB(G,x) = \sum_{uv \in E(G)} x^{(r_u + r_v) + \sqrt{r_u^2 + r_v^2}}.$$

We define the modified Revan Kepler Banhatti index of a graph *G* as

$$^{m}RKB(G) = \sum_{uv \in E(G)} \frac{1}{(r_u + r_v) + \sqrt{r_u^2 + r_v^2}}..$$

Considering the modified Revan Kepler Banhatti index, we introduce the modified Revan Kepler Banhatti exponential of a graph *G* and defined it as

$$^{m}RKB(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{(r_u + r_v) + \sqrt{r_u^2 + r_v^2}}}.$$

In the literature, some topological indices were studied in (13, 14, 15, 16, 17, 18).

In this paper, we determine the Revan Kepler Banhatti index, modified Revan Kepler Banhatti index and their corresponding exponentials of some nanotubes. Also, some mathematical properties of the Revan Kepler Banhatti index are obtained.

2. Mathematical properties

Theorem 1. Let *G* be a simple connected graph. Then

$$RKB(G) \geq \left(1 + \frac{1}{\sqrt{2}}\right) R_1(G)$$

with equality if *G* is regular.

Proof: By the Jensen inequality, for a concave function *f(x)*,

$$f\left(\frac{1}{n}\sum x_i\right) \geq \frac{1}{n}\sum f(x_i)$$

with equality for a strictly concave function if $x_1 = x_2 = \ldots = x_n$.
Choosing $f(x) = \sqrt{x}$, we obtain

$$\sqrt{\frac{r_u^2 + r_v^2}{2}} \geq \frac{(r_u + r_v)}{2}$$

thus
$$(r_u + r_v) + \sqrt{r_u^2 + r_v^2} \geq (r_u + r_v) + \frac{1}{\sqrt{2}}(r_u + r_v).$$

Hence, $$\sum_{uv\in E(G)}[(r_u+r_v)+\sqrt{r_u^2+r_v^2}]\geq\left(1+\frac{1}{\sqrt{2}}\right)\sum_{uv\in E(G)}(r_u+r_v).$$

Thus $$RKB(G)\geq\left(1+\frac{1}{\sqrt{2}}\right)R_1(G)$$

with equality if G is regular.

Theorem 2. Let G be a simple connected graph. Then

$$RKB(G)\leq\left(1+\sqrt{2}\right)R_1(G)-\sqrt{2}RPR(G).$$

Proof: It is known that for $1\leq x\leq y$,

$$f(x,y)=\left(x+y-\sqrt{xy}\right)-\sqrt{\frac{x^2+y^2}{2}}$$

is decreasing for each y. Thus $f(x,y)\geq f(y,y)=0$. Hence

$$x+y-\sqrt{xy}\geq\sqrt{\frac{x^2+y^2}{2}}$$

or $$\sqrt{\frac{x^2+y^2}{2}}\leq x+y-\sqrt{xy}.$$

Put $x=r_u$ and $y=r_v$, we get

$$\sqrt{\frac{r_u^2+r_v^2}{2}}\leq(r_u+r_v)-\sqrt{r_ur_v}$$

$$\sqrt{r_u^2+r_v^2}\leq\sqrt{2}[(r_u+r_v)-\sqrt{r_ur_v}]$$

which implies

$$(r_u+r_v)+\sqrt{r_u^2+r_v^2}\leq(r_u+r_v)+\sqrt{2}[(r_u+r_v)-\sqrt{r_ur_v}]$$

$$\sum_{uv\in E(G)}[(r_u+r_v)+\sqrt{r_u^2+r_v^2}]\leq\left(1+\sqrt{2}\right)\sum_{uv\in E(G)}(r_u+r_v)-\sqrt{2}\sum_{uv\in E(G)}\sqrt{r_ur_v}.$$

Thus $$RKB(G)\leq\left(1+\sqrt{2}\right)R_1(G)-\sqrt{2}RPR(G).$$

Theorem 3. Let G be a simple connected graph. Then

$$RKB(G)<2R_1(G).$$

Proof: It is known that for $1\leq x\leq y$,

$$\sqrt{x^2+y^2}<x+y$$

$$(x+y)+\sqrt{x^2+y^2}<2(x+y).$$

Setting $x=r_u$ and $y=r_v$, we get

$$(r_u+r_v)+\sqrt{r_u^2+r_v^2}<2(c_u+c_v)$$

Thus $\sum_{uv\in E(G)}[(r_u+r_v)+\sqrt{r_u^2+r_v^2}]<2\sum_{uv\in E(G)}(r_u+r_v).$

Hence $RKB(G)<2R_1(G).$

3. Results for $HC_5C_7[p,q]$ nanotubes

We consider $HC_5C_7[p,q]$ nanotubes in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. The 2-D lattice of nanotube $HC_5C_7[8,4]$ is shown in Figure 1.

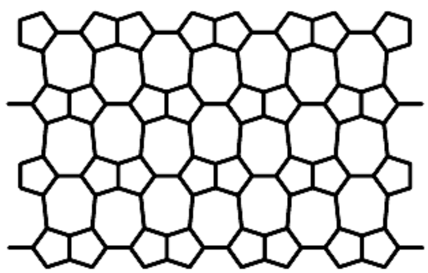

Figure 1: 2-D lattice of $HC_5C_7[8,4]$ nanotube

Let G be the graph of $HC_5C_7[p,q]$ nanotubes. We see that the vertices of G are either of degree 2 or 3. Therefore $\Delta(G) = 3$ and $\delta(G) = 2$. By the algebraic method, we obtain that G has $4pq$ vertices and $6pq - p$ edges. In G, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_{23}= \{uv \in E(G) \mid d_u = 2, d_v = 3\}$, $|E_{23}| = 4p$.

$E_{33} = \{uv \in E(G) | d_u = d_v = 3\}$, $|E_{33}| = 6pq - 5p$.

We have $r_G(u)=\Delta(G) + \delta(G) - d_u = 5 - d_u$.

Thus there are two types of Revan edges based on the degree of end Revan vertices of each Revan edge as follows:

$RE_{32} = \{uv \in E(G) \mid r_u = 3, r_v = 2\}$, $|RE_{32}| = 4p$.

$RE_{22} = \{uv \in E(G)| r_u = r_v = 2\}$, $|RE_{22}| = 6pq - 5p$.

Theorem 4. Let $G=HC_5C_7[p,q]$ be the nanotubes. Then

$$RKB(G)=12pq(2+\sqrt{2})+2p(2\sqrt{13}-5\sqrt{2}).$$

Proof: We have

$$RKB(G)=\sum_{uv\in E(G)}[(r_u+r_v)+\sqrt{r_u^2+r_v^2}]$$

$$=4p[(3+2)+\sqrt{3^2+2^2}]+(6pq-5p)[(2+2)+\sqrt{2^2+2^2}]$$

$$=12pq(2+\sqrt{2})+2p(2\sqrt{13}-5\sqrt{2}).$$

Theorem 5. Let $G=HC_5C_7[p,q]$ be the nanotubes. Then

$$RKB(G,x)=4px^{5+\sqrt{13}}+(6pq-5p)x^{4+2\sqrt{2}}.$$

Proof: We have

$$RKB(G,x)=\sum_{uv\in E(G)} x^{(r_u+r_v)+\sqrt{r_u^2+r_v^2}}$$
$$=4px^{(3+2)+\sqrt{3^2+2^2}}+(6pq-5p)x^{(2+2)+\sqrt{2^2+2^2}}$$
$$=4px^{5+\sqrt{13}}+(6pq-5p)x^{4+2\sqrt{2}}.$$

Theorem 6. Let $G=HC_5C_7[p,q]$ be the nanotubes. Then

$$RKB(G,x)=\frac{3pq}{2+\sqrt{2}}+\frac{4p}{5+\sqrt{13}}-\frac{5p}{4+2\sqrt{2}}.$$

Proof: We have

$$^{m}RKB(G)=\sum_{uv\in E(G)}\frac{1}{(r_u+r_v)+\sqrt{r_u^2+r_v^2}}$$
$$=\frac{4p}{(3+2)+\sqrt{3^2+2^2}}+\frac{6pq-5p}{(2+2)+\sqrt{2^2+2^2}}$$
$$=\frac{3pq}{2+\sqrt{2}}+\frac{4p}{5+\sqrt{13}}-\frac{5p}{4+2\sqrt{2}}.$$

Theorem 7. Let $G=HC_5C_7[p,q]$ be the nanotubes. Then

$$^{m}RKB(G,x)=4px^{\frac{1}{5+\sqrt{13}}}+(6pq-5p)x^{\frac{1}{4+2\sqrt{2}}}.$$

Proof: We have

$$^{m}RKB(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{(r_u+r_v)+\sqrt{r_u^2+r_v^2}}}$$
$$=4px^{\frac{1}{(3+2)+\sqrt{3^2+2^2}}}+(6pq-5p)x^{\frac{1}{(2+2)+\sqrt{2^2+2^2}}}$$
$$=4px^{\frac{1}{5+\sqrt{13}}}+(6pq-5p)x^{\frac{1}{4+2\sqrt{2}}}.$$

4. Results for $SC_5C_7[p,q]$ nanotubes

We consider $SC_5C_7[p,q]$ nanotubes in which p is the number of heptagons in the first row and q rows of vertices and edges are repeated alternately. The 2-D lattice of nanotube $SC_5C_7[8,4]$ is depicted in Figure 2.

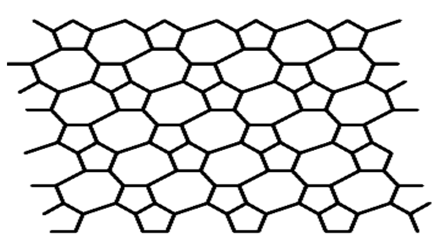

Figure 2: 2-D lattice of nanotube $SC_5C_7[8,4]$

Let H be the graph of $SC_5C_7[p,q]$ nanotubes. We see that the vertices of H are either of degree 2 or 3. Thus $\Delta(H) = 3$ and $\delta(H) = 2$. By algebraic method, we obtain that H has $4pq$ vertices and $6pq - p$ edges. In H, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_{22} = \{uv \in E(H) \mid d_u = d_v = 2\}$ $|E_{22}| = q$.

$E_{23} = \{uv \in E(H) \mid d_u = 2, d_v = 3\}$ $|E_{23}| = 6q$.

$E_{33} = \{uv \in E(H) \mid d_u = d_v = 3\}$ $|E_{33}| = 6pq - p - 7q$.

Clearly we have $r_H(u) = \Delta(H) + \delta(H) - d_u = 5 - d_u$.

Thus there are three types of Revan edges based on the degree of end revan vertices of each revan edge as follows:

$RE_{33} = \{uv \in E(H) \mid r_u = r_v(v) = 3\}$, $|RE_{33}| = q$.

$RE_{32} = \{uv \in E(H) \mid r_u = 3, r_v(v) = 2\}$, $|RE_{32}| = 6q$.

$RE_{22} = \{uv \in E(H) \mid r_u = r_v(v) = 2\}$, $|RE_{22}| = 6pq - p - 7q$.

Theorem 8. Let $H=SC_5C_7[p,q]$ be the nanotubes. Then

$$RKB(H) = (12pq - 2p - 11q)[2+\sqrt{2}] + 6q[5+\sqrt{13}].$$

Proof: We have

$$RKB(H) = \sum_{uv\in E(H)} [(r_u + r_v) + \sqrt{r_u^2 + r_v^2}]$$

$$= q[(3+3)+\sqrt{3^2+3^2}] + 6q[(3+2)+\sqrt{3^2+2^2}]$$

$$+(6pq - p - 7q)[(2+2)+\sqrt{2^2+2^2}]$$

$$= (12pq - 2p - 11q)[2+\sqrt{2}] + 6q[5+\sqrt{13}].$$

Theorem 9. Let $H=SC_5C_7[p,q]$ be the nanotubes. Then

$$RKB(H,x) = qx^{6+3\sqrt{2}} + 6qx^{5+\sqrt{13}} + (6pq - p - 7q)x^{4+2\sqrt{2}}.$$

Proof: We have

$$RKB(H,x) = \sum_{uv\in E(H)} x^{(r_u+r_v)+\sqrt{r_u^2+r_v^2}}$$

$$= qx^{(3+3)+\sqrt{3^2+3^2}} + 6qx^{(3+2)+\sqrt{3^2+2^2}} + (6pq - p - 7q)x^{(2+2)+\sqrt{2^2+2^2}}$$

$$= qx^{6+3\sqrt{2}} + 6qx^{5+\sqrt{13}} + (6pq - p - 7q)x^{4+2\sqrt{2}}.$$

Theorem 10. Let $H=SC_5C_7[p,q]$ be the nanotubes. Then

$${}^{m}RKB(H) = \frac{3pq}{2+\sqrt{2}} - \frac{p}{4+2\sqrt{2}} + \frac{6q}{5+\sqrt{13}} - \frac{19q}{6(2+\sqrt{2})}.$$

Proof: We have

$${}^{m}RKB(H) = \sum_{uv\in E(H)} \frac{1}{(r_u + r_v) + \sqrt{r_u^2 + r_v^2}}$$

$$= \frac{q}{(3+3)+\sqrt{3^2+3^2}} + \frac{6q}{(3+2)+\sqrt{3^2+2^2}} + \frac{6pq - p - 7q}{(2+2)+\sqrt{2^2+2^2}}$$

$$= \frac{q}{6+3\sqrt{2}} + \frac{6q}{5+\sqrt{13}} + \frac{6pq - p - 7q}{4+2\sqrt{2}}$$

$$= \frac{3pq}{2+\sqrt{2}} - \frac{p}{4+2\sqrt{2}} + \frac{6q}{5+\sqrt{13}} - \frac{19q}{6(2+\sqrt{2})}$$

Theorem 11. Let $H=SC_5C_7[p,q]$ be the nanotubes. Then

$$^{m}RKB(H,x) = qx^{\frac{1}{6+3\sqrt{2}}} + 6qx^{\frac{1}{5+\sqrt{13}}} + (6pq - p - 7q)x^{\frac{1}{4+2\sqrt{2}}}.$$

Proof: We have

$$^{m}RKB(H,x) = \sum_{uv\in E(H)} x^{\frac{1}{(r_u+r_v)+\sqrt{r_u^2+r_v^2}}}$$

$$= qx^{\frac{1}{(3+3)+\sqrt{3^2+3^2}}} + 6qx^{\frac{1}{(3+2)+\sqrt{3^2+2^2}}} + (6pq - p - 7q)x^{\frac{1}{(2+2)+\sqrt{2^2+2^2}}}$$

$$= qx^{\frac{1}{6+3\sqrt{2}}} + 6qx^{\frac{1}{5+\sqrt{13}}} + (6pq - p - 7q)x^{\frac{1}{4+2\sqrt{2}}}.$$

5. Conclusion

We have introduced the Revan Kepler Banhatti index, modified Revan Kepler Banhatti index and their corresponding exponentials of a graph. Furthermore, the Revan Kepler Banhatti index, modified Revan Kepler Banhatti index and their corresponding exponentials of some nanotubes are determined. Also, some mathematical properties of the Revan Kepler Banhatti index are established.

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