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A Modified Approach on Fuzzy Time Series Forecasting

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Abstract. During the last decade different time series models have been designed and developed. The subject matter of this research is uncertain forecasting based on uncertain observed data. In this paper nearest symmetric trapezoidal fuzzy numbers are used to further enhance the forecasting accuracy. An illustrative example for enrollment forecasting is used to verify the effectiveness of the proposed model and compare with some of the existing methods to confirm the potential benefits of the proposed approach with a very small error.

Keywords: Fuzzy time series, Symmetric trapezoidal fuzzy numbers, Uncertain data,Forecast, Ranking function.

AMS Mathematics Subject Classifications (2010): 03E72, 62M10

1. Introduction

Fuzzy numbers play a significant role among all fuzzy sets. Many researchers have proved that fuzzy set theory let us effectively model in transforming imprecise transformation. There is a difficulty to assign membership functions corresponding to fuzzy numbers which gives the variation as well as imprecise terms. Recently, researchers focused on the computation of the triangular and trapezoidal approximation of fuzzy numbers. Delegado et al. [4] proposed the nearest symmetric trapezoidal approximation which preserves the value and ambiguity. Adrian Ban [1] considered approximation of fuzzy numbers by trapezoidal fuzzy numbers preserving the expected interval. Ma et.al [12] have used the concept of the symmetric triangular fuzzy numbers and they have introduced a new method to defuzzify a general fuzzy quantity. The basic idea of their method was obtaining the nearest symmetric triangular fuzzy number for each fuzzy quantity. In 2005, Grzegorzewski et al. [6] have suggested a new approach to trapezoidal approximation of fuzzy numbers, called the nearest trapezoidal approximation operator preserving expected interval and possesses many desired properties. In some situations their operator may fail to lead a trapezoidal fuzzy number. In 2007, Grzegorzewski [7] proposed a corrected version of the trapezoidal approximation operator.

Veeramani et al. [20] proposed a method of converting any fuzzy number in to a symmetric trapezoidal fuzzy number by using metric distance. In a defuzzification method every fuzzy set is converted to a real number, we generally lose too much important information. Nearest symmetric trapezoidal fuzzy number is said to defuzzify the fuzzy number and at the same time fuzziness of original quantity. In this paper, we used the concept of any trapezoidal fuzzy number may be converted in to nearest symmetric trapezoidal fuzzy number to forecast the fuzzy time series.

An ordered sequence of observed values is known as time series. If the observed values represent measured values, it is often not possible to assign precise numerical values to the observed data, they then possess data uncertainty. This paper concerns with the time series comprised of imprecise i.e., uncertain observed values. In the case of time series the uncertainty of the individual observed values as well as the interpretation of a sequence of uncertain observed values are of interest. The uncertain observed value is thus modeled as a fuzzy variable. Fuzzy sets represent concepts such as low etc. are called fuzzy variables. Modeling of the individual observed values as fuzzy variables results in fuzzy time series. Forecasting using fuzzy time series has been widely used in many activities. It arises in forecasting the weather, earthquakes, stock fluctuations and any phenomenon indexed by variables that change unpredictably in time The traditional time series forecasting methods cannot be used for forecasting problems in which the historical data are linguistic values. Song and Chissom (1993, 1994) proposed time variant and time invariant fuzzy time series models and fuzzy forecasting to model and forecast processes whose observation are linguistic values. Instead of complicated maximum minimum composition operations Chen (1996) used a simple arithmetic operation for time series forecasting. Thereafter a number of related research works have been reported that follow their framework and aim to improve forecasting accuracy and reduce the computational overhead. These works include enrollments, length of intervals, temperature prediction, weighted method, stock price, hidden Markov model, genetic algorithm, neural – fuzzy system, bulk shipping, seasonal and heuristic models.

Why we need symmetric trapezoidal fuzzy number? The linguistic assessments are just approximate assessments, given by experts and accepted by decision maker whenever obtaining more accurate value is impossible or unnecessary. Whatever method is used one should be aware that in an environment of uncertainty it is reasonable to leave some area for variations in estimating membership functions. When operating with fuzzy numbers, the result of our calculations strongly depend on the shape of the membership functions of these numbers. Less regular membership functions lead to more complex calculations. For the sake of simplicity, symmetric trapezoidal fuzzy numbers are used. Here we used symmetric trapezoidal membership function for fuzzy number and labels of linguistic variables.

2. Fuzzy time series and related definitions

In the following, we briefly review some basic concepts of fuzzy time series from Song and Chissom (1993, 1994) and its forecasting frame work and some basic definitions.

Definition 1.A *fuzzy set* A is defined as an uncertain subset of the fundamental set X. A = $\{(x, \mu_A(x)) | x \epsilon X\}$. The uncertainty is assessed by the membership function $\mu_A(x)$.

Definition 2. Let Y(t) {t = 0,1,2,3,…}, a subset of ℝ, be the universe of discourse on which fuzzy sets $f_i(t)$ (i = 1,2,3…) are defined and let F(t) be the collection of $f_i(t)$. Then F (t) is defined as *fuzzy time series* on Y(t).

From this definition we can see that, (1) F(t) is the function of time

(2) F(t) can be regarded as a *linguistic variable*, which is a variable whose values are linguistic values represented by fuzzy sets.

(3) $f_i(t)$ (i = 1,2,3…) are possible linguistic values of F(t), where $f_i(t)$, i = 1,2,3,…. are represented by fuzzy sets.

Song and Chissom employed a fuzzy relational equation to develop their forecasting model under the assumption that the observations at time t are dependent only upon the accumulated results of the observation at previous times, which is defined as follows.

Definition 3. Suppose F(t) is caused only by F(t-1) and is denoted by F(t-1) → F(t) , then there is a fuzzy relationship between F(t) and F(t-1) and can be expressed as the fuzzy relational equation F(t) = F(t-1) ∘ R(t,t-1). Here '∘' is max- min composition operator. The relation R is called first – order model of F(t).

Further, if fuzzy relation R(t,t-1) of F(t) is independent of time t, that is to say, for different times t_1 and t_2, $R(t_1,t_1-1) = R(t_2,t_2-1)$, then F(t) is called a *time invariant* fuzzy time series otherwise F(t) is *time variant.*

Definition 4. Suppose F(t-1) = A_i and F(t) = A_j a *fuzzy logical relationship* can be defined as $A_i \rightarrow A_j$ where A_i and A_j are called the left hand side and the right hand side of the fuzzy logical relationship respectively.

Definition 5. If F(t) is caused by more fuzzy sets F(t-n), F(t–n+1), … , F(t-1) the fuzzy relationship is represented by A_{i1}, A_{i2}, A_{i3},… $A_{in} \rightarrow A_j$, where F(t-n) = A_{i1}, F(t-n+1) = A_{i2}, … , F(t-1) = A_{in}. This relationship is called n^{th} order fuzzy time series model.

Definition 6. A fuzzy subset A of the real line ℝ with membership function

μ_A : ℝ → [0 , 1] is called a fuzzy number if

a) A is normal, i.e., there exist an element x_0 such that $\mu_A(x_0)$ = 1.

b) A is fuzzy convex, i.e., $\mu_A[\lambda x_1 + (1 - \lambda)x_2] \geq \min[\mu_A(x_1),\ \mu_A(x_2)]$,

$\forall x_1, x_2 \in \mathbb{R}, \forall \lambda \in [0,1]$.

c) μ_A is upper semi continuous.

d) Supp A is bounded, where Supp A = cl ($\{x_0 \in \mathbb{R}\ ,\ \mu_A(x_0) > 0\}$), and cl is the closure operator.

The notion of fuzzy number was introduced by Dubois and Prade [5].It is known that for any fuzzy number A there exist four numbers a_1, a_2, a_3, $a_4 \in \mathbb{R}$ and two functions μ_A^L , μ_A^R : $\mathbb{R} \rightarrow [0\ ,\ 1]$, where μ_A^L is non decreasing and μ_A^R is non increasing, such that we can describe a membership function μ_A in the following manner:

$$\mu_A(x) = \begin{cases} \mu_A^L(x), & a_1 \leq x < a_2 \\ 1\ , & a_2 \leq x \leq a_3 \\ \mu_A^R(x), & a_3 < x \leq a_4 \\ 0\ , & \text{otherwise.} \end{cases}$$

Functions μ_A^L and μ_A^R are called the left side and right side of a fuzzy number A, respectively.

Definition 7. The r - cut of a fuzzy number A is non fuzzy set defined as

$$A_r = \{x \in \mathbb{R} : \mu_A(x) \geq r\}.$$

Every r - cut of a fuzzy number is a closed interval. We have

$A_r = [A_1(r), A_2(r)]$ where $A_1(r) = \inf\{x \in \mathbb{R} : \mu_A(x) \geq r\}$,

$A_2(r) = \sup\{x \in \mathbb{R} : \mu_A(x) \geq r\}$.

If the sides of the fuzzy number A are strictly monotone then one can easily see that $A_1(r)$ and $A_2(r)$ are inverse functions of μ_A^L and μ_A^R respectively, i.e., we have

$[\mu_A^L(x)]^{-1} = \inf\ \{x \in \mathbb{R}\ ,\ \mu_A(x) \geq r\} = A_1(r)$ and

$[\mu_A^R(x)]^{-1} = \sup\ \{x \in \mathbb{R}\ ,\ \mu_A(x) \geq r\} = A_2(r)$.

Definition 8[20]. The trapezoidal fuzzy number A = (x_0, y_0, σ, β) with two fuzzifier x_0, y_0 and left fuzziness $\sigma > 0$ and right fuzziness $\beta > 0$ is a fuzzy set where the membershipfunction is as

$$\mu_A(x) = \begin{cases} \frac{x - x_0 + \sigma}{\sigma}\ , & x_0 - \sigma \leq x \leq x_{0,} \\ 1, & x_0 \leq x \leq y_{0,} \\ \frac{y_0 - x + \beta}{\beta}\ , & y_0 \leq x \leq y_0 + \beta \\ 0\ , & \text{otherwise,} \end{cases}$$

and its parametric form is A = $[x_0 - \sigma + \sigma r,\ y_0 + \beta - \beta r]$.

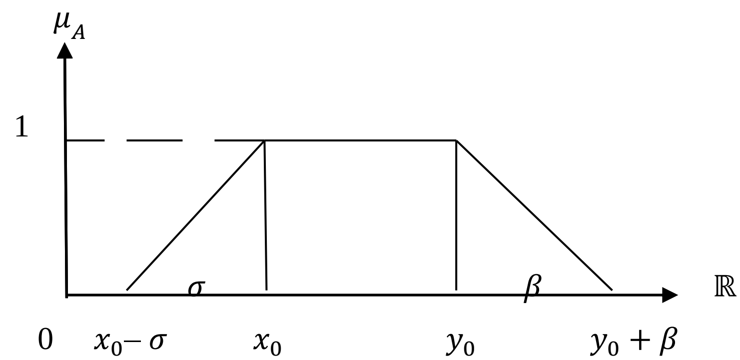

Figure 1: The trapezoidal fuzzy number A = (x_0, y_0, σ, β)

If $\sigma = \beta$, then the trapezoidal fuzzy number A is called symmetric trapezoidal fuzzy number and it is denoted as A = (x_0, y_0, σ).

Nearest symmetrical trapezoidal fuzzy number approximation (NSTFNA) [20]:

For arbitrary fuzzy numbers A and B, their parametric form is A = [A_1(r) , A_2(r)] and [B_1(r) and B_2(r)], r ∈ [0,1] respectively, then the distance between A and B is defined as

$$d(A, B) = \sqrt{\int_0^1 [A_1(r) - B_1(r)]^2 dr \; + \; \int_0^1 [A_2(r) - B_2(r)]^2 dr}.$$

Simply, D(A,B) = d^2(A,B).

To find NSTFNA, T(A) = $(x_{0\,,} y_{0.} \sigma)$ that approximate fuzzy number of A, where $[x_{0\,,} y_0\,]$ is core of T(A) and σ is a left and right width of T(A) we have to minimize D$(x_{0\,,} y_{0.} \sigma)$ = D(A, T(A)) $= \int_0^1 [A_1(r) - B_1(r)]^2 dr \; + \; \int_0^1 [A_2(r) - B_2(r)]^2 dr$

where the parametric form of T(A) is $[x_0 - \sigma + \sigma r, y_0 + \sigma - \sigma r]$. We then obtain

the nearest symmetric trapezoidal fuzzy number (NSTFN) for the symmetric trapezoidal fuzzy number as follows:

$x_0 = \int_0^1 A_1(r) dr + \frac{\sigma}{2}$

$y_0 = \int_0^1 A_2(r) dr - \frac{\sigma}{2}$

$\sigma = 6\int_0^1 [A_1(r) - A_2(r)](1 - r) dr - 3 \int_0^1 [A_1(r) - A_2(r)]\, dr$

i,e., NSTFNA, T(A) = $(x_{0\,,} y_{0.} \sigma)$ for any fuzzy number A, where $x_{0\,,} y_0 \; and \; \sigma$ are given as above.

Note : For the trapezoidal fuzzy number A = (t_1, t_2, t_3, t_4) where [t_2, t_3] is core of A, t_1 is left width and t_4 is right width, then the nearest symmetric trapezoidal fuzzy number for A is ($t_2 + \frac{t_4 - t_1}{4}$, $t_3 + \frac{t_4 - t_1}{4}$, $\frac{t_1 + t_4}{2}$).

Definition 9. Arithmetic Operations.
Let A_1 = (a_1 , b_1 , c_1 , d_1) and A_2 = (a_2 , b_2, c_2 , d_2) be trapezoidal fuzzy numbers defined on the universal set of real numbers ℝ. Then the arithmetic operations between $A_1$and A_2 are

i.$A_1 \oplus A_2$ = ($a_1 + a_2$, $b_1 + b_2$, $c_1 + c_2$, $d_1 + d_2$)

ii. $A_1 \ominus A_2$= ($a_1 - d_2$, $b_1 - c_2$, $c_1 - b_2$, $d_1 - a_2$)

iii. $A_1 \otimes A_2$ =($a_1 \times a_2$,$b_1 \times b_2$, $c_1 \times c_2$, $d_1 \times d_2$)

iv. $\lambda A_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1 : w_1), & \lambda \geq 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1 : w_1), & \lambda < 0 \end{cases}$

Definition 10. A ranking function is a map from the set of fuzzy numbers F(ℝ) into a real line and is defined by R(A) =$\frac{a+b+c+d}{4}$ where A = (a,b,c,d).

Remark. Basic method of Chen (1996) is used to develop the new method. Chen's method consists of the following steps:

i) Define the universe of discourse U.

ii) Divide U into equal length of intervals.

iii) Define the fuzzy sets on U and fuzzify the historical data.

iv) Derive the fuzzy logical relationships based on the historical data.

v) Classify the fuzzy logical relationship groups among the derived logicalrelationship.

vi) Utilize defuzzification rules to calculate the forecasted value.

3. Proposed Method

Step 1: Collect the historical data Av_t.
Step 2: Find the maximum D_{max} and the minimum D_{min} among all Av_t. To form the universe of discourse two small numbers D_1 and D_2 are assigned. Define the universe of discourse as U = [$D_{min} - D_1$, $D_{max} + D_2$].

Step 3: Divide the universe in to seven **[11]**equal length intervals U_i,i = 1 to 7.

According to the distribution of the historical data divideU_i into intervals of different length and denotes v_j.

Step 4: According to the intervals $v_1 = [d_1,d_2]$, $v_2 = [d_2, d_3]$,…,$v_m = [d_m, d_{m+1}]$ derived in step3the fuzzy sets which we define on trapezoidal fuzzy numbers are as follows:

$A_1 = [d_0, d_1, d_2, d_3]$

$A_2 = [d_1, d_2, d_3, d_4]$

$\vdots$

$A_{m-1} = [d_{m-2}, d_{m-1}, d_m, d_{m+1}]$

$A_m = [d_{m-1}, d_m, d_{m+1}, d_{m+2}]$.

Step 5: If the value of the historical data is located in the range of v_j, then it belongs to the fuzzy number A_j. All the data must be classified in to the corresponding fuzzy numbers.

Step 6: Derive the fuzzy logical relationships based on definition 4.

Step 7: Arrange the fuzzy logical relationships in to the fuzzy logical relationship groups based on the same fuzzy number on the left hand sides of the fuzzy logical relationships. If the transition happens to the same fuzzy set, make a separate logical relationship group. The fuzzy logical relationship groups are like the following:

$A_j \to A_{k1}, A_j \to A_{k2}, \ldots, A_j \to A_{kp}.$

Step 8: The forecasted value at time t, Fv_t, is determined by the following three heuristic rules. Assume the fuzzy number Av_t at time t – 1 is A_j.

Rule 1: If the fuzzy logical relationship group of A_j is empty, $A_j \to \varphi$ or $A_j \to A_j$, then the forecasted value Fv_t is R[NSTFN(A_j)].

Rule 2: If the fuzzy logical relationship group of A_j is one to one ,i.e., $A_j \to A_k$then the forecasted value Fv_t is R[NSTFN(A_k)].

Rule 3: If the fuzzy logical relationship group of A_j is one to many i.e., $A_j \to A_{k1}$, $A_j \to A_{k2}, \ldots, A_j \to A_{kp}$, then the value of Fv_t is calculated as $Fv_t = R[\frac{NSTFN(A_{k1}) + NSTFN(A_{k2}) + \ldots + NSTFN(A_{kp})}{p}]$.

4. **Numerical Example**

This method illustrates its forecasting method with the example of enrollments of the University of Alabama.

Step1: The example adopted from Chen (1996) concerns the number of student enrollments at the University of Alabama from 1971 to 1992 as shown in table 1.

Step 2: The maximum and minimum data of enrollments are 19337 and 13055 respectively. Let $D_1 = 5$ and $D_2 = 13$. Then U = [13050, 19350].

Step 3: Divide U in to seven intervals U_i, i = 1 to 7 with equal length of 900.

i.e., $U_1 = [13050, 13950], \ldots, U_7 = [18450, 19350]$.

By the following frequency distribution

Intervals	U_1	U_2	U_3	U_4	U_5	U_6	U_7
No. of historical enrollment data	3	1	7	3	3	1	4

divide the intervals U_i, i = 1 to 7 as follows:

v_1 = [13050, 13350),…,v_3 = [13650, 13950) with length of 300.

v_4 = [13950, 14850) with length of 900.

v_5 = [14850, 14979), …, v_{11} = [15621, 15750) with length 129 and 128 alternatively.

v_{12} = [15750, 16050) , … , v_{17} = [17250, 17550) with length of 300.

v_{18} = [17550, 18450) with length of 900.

v_{19} = [18450, 18675),…, v_{22} = [19125, 19350] with length of 225.

Step 4: The fuzzy trapezoidal numbers can be then defined by

A_1= [12750, 13050, 13350, 13650]; A_2 = [13050, 13350, 13650, 13950];

A_3 = [13350, 13650, 13950, 14850]

⋮

A_{20} = [18450, 18675, 18900, 19125]

A_{21} = [18675, 18900, 19125, 19350], A_{22} = [18900, 19125, 19350, 19575].

Step 5: Fuzzify the enrollments as shown in Table 1.

Step6: According to the fuzzified enrollments, the fuzzy logical relationships are derived as follows:

$A_1 \to A_2$ $A_2 \to A_3$ $A_3 \to A_4$ $A_4 \to A_9$ $A_9 \to A_8$ $A_8 \to A_{10}$

$A_{10} \to A_{12}$ $A_{12} \to A_{15}$ $A_{15} \to A_{15}$ $A_{15} \to A_{14}$ $A_{14} \to A_9$ $A_9 \to A_{10}$

$A_{10} \to A_7$ $A_7 \to A_7$ $A_7 \to A_{12}$ $A_{12} \to A_{15}$ $A_{15} \to A_{18}$ $A_{18} \to A_{21}$

$A_{21} \to A_{22}$ $A_{22} \to A_{22}$ $A_{22} \to A_{20}$

Step 7: Generate the fuzzy logical relation groups as follows:

$A_1 \to A_2$ $A_2 \to A_3$ $A_3 \to A_4$ $A_4 \to A_9$ $A_7 \to A_7$

$A_7 \to A_{12}$ $A_8 \to A_{10}$ $A_9 \to A_8, A_{10}$ $A_{10} \to A_{12}, A_7$ $A_{12} \to A_{15}$

$A_{14} \to A_9$ $A_{15} \to A_{15}$ $A_{15} \to A_{14}, A_{18}$ $A_{18} \to A_{21}$ $A_{21} \to A_{22}$

$A_{22} \to A_{22}$ $A_{22} \to A_{20}$

Table 1. Fuzzified and Forecasted enrollments

Year	Actual Enroll- -ment Av_t	Fuzzified Enroll- -ment	Forecasted Enrollment Fv_t	$\left\lvert\frac{Av_t-Fv_t}{Av_t}\right\rvert \times 100$
1971	13055	A_1	–	--
1972	13563	A_2	13500	0.4645
1973	13867	A_3	13950	0.5985
1974	14696	A_4	14357.25	2.3050
1975	15460	A_9	15428.5	0.2038
1976	15311	A_8	15428.5	0.7674
1977	15603	A_{10}	15557	0.2948
1978	15861	A_{12}	15557.125	1.9159
1979	16807	A_{15}	16800	0.0416
1980	16919	A_{15}	16800	0.7034
1981	16388	A_{14}	17240.625	5.2027
1982	15433	A_9	15428.5	0.02916
1983	15497	A_{10}	15428.5	0.4420
1984	15145	A_7	15557.125	2.7212
1985	15163	A_7	15171.5	0.0561
1986	15984	A_{12}	15942.75	0.2580
1987	16859	A_{15}	16800	0.3499
1988	18150	A_{18}	17240.625	5.0103
1989	18970	A_{21}	19012.5	0.2240
1990	19328	A_{22}	19237.5	0.4682
1991	19337	A_{22}	19237.5	0.5146
1992	18876	A_{20}	18787.5	0.4688

Step 8: Calculate the forecasted enrollments. For example

[1974]: The fuzzified enrollment of the year 1973 is A_3 and the corresponding fuzzy logical relationship group is $A_3 \rightarrow A_4$.

A_4 = [13650, 13950, 14850, 14979]. By note t_2 = 13950; t_3 = 14850;

t_1 = 13950 – 13650 = 300; t_4 = 14979 – 14850 = 129

$$\frac{t_4-t_1}{4} = -42.75\,, \qquad \frac{t_4+t_1}{2} = 214.5$$

NSTFN(A_4) = [13907.25 – 214.5, 13950 – 42.75, 14850 – 42.75, 14807.25 + 214.5]

= [13692.75, 13907.25, 14807.25, 15021.75]

According to rule 2, the Fv_t for 1974 is given by

$$Fv_t = R[NSTFN(A_4)] = 14357.25$$

[1978] and [1984]: The fuzzified enrollment of the year 1977 and 1983 is A_{10}. The fuzzy logical relationship group is $A_{10} \rightarrow A_{12}, A_7$.

$$NSTFN(A_{12}) = [15578.25, 15792.75, 16092.75, 16307.25]$$

$$NSTFN(A_7) = [14979, 15107, 15236, 15364].$$

According to rule 3, the Fv_t for 1978 and 1984 is

$$Fv_t = R\left[\frac{15578.25+14979}{2}, \frac{15792.75+15107}{2}, \frac{16092.75+15236}{2}, \frac{16307.25+15364}{2}\right]$$

$$= 15557.125.$$

5. Experimental results

To evaluate the forecasting performance, four fuzzy time series methods are adopted for comparing of their forecasting results with those obtained by the proposed method. The mean absolute percentage error (MAPE) is used to evaluate the forecasting accuracy. The formula is

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Av_t - Fv_t}{Av_t}\right| \times 100.$$

Index	Chen (1996)	Hwang et. al (1998)	Lee and Chou's method (2004)	Hao-Tien Liu (2007)	Proposed method
MAPE	3.08	2.94	2.69	1.33	1.097

6. Conclusion

In this paper we have developed a new fuzzy time series forecasting method based on nearest symmetric trapezoidal fuzzy numbers. Frequency distribution is used to divide the intervals with different length. By means of transforming the trapezoidal fuzzy numbers in to nearest symmetric trapezoidal fuzzy numbers and by using the ranking function the forecasting accuracy is improved. Comparing to other four methods the research goal is reached.

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